Structural Analysis

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1. Basic Concepts

1.1 Introduction:

Structural Engineering.

- Analysis (SOM, SA)
  - Equilibrium
  - Compatibility
  - Energy
- Design (RCC, PSC, STEEL)
  - Safety
  - Serviceability
  - Durability
  - Economy
  - Aesthetic

1.2 What is a Structure?

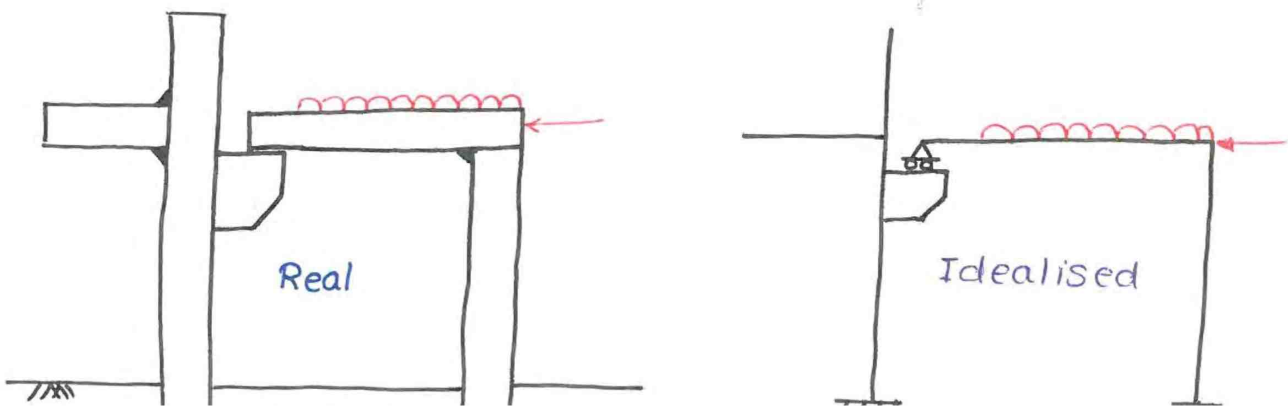
Any arrangement of members that can transfer load acting upon it to the supports safely can be termed as Structure.

1.3 Meaning of Structural Analysis:

SA: Support Reactions → BMD & SFD

SOM: → Deflection and Rotation, → Stress, → Strain

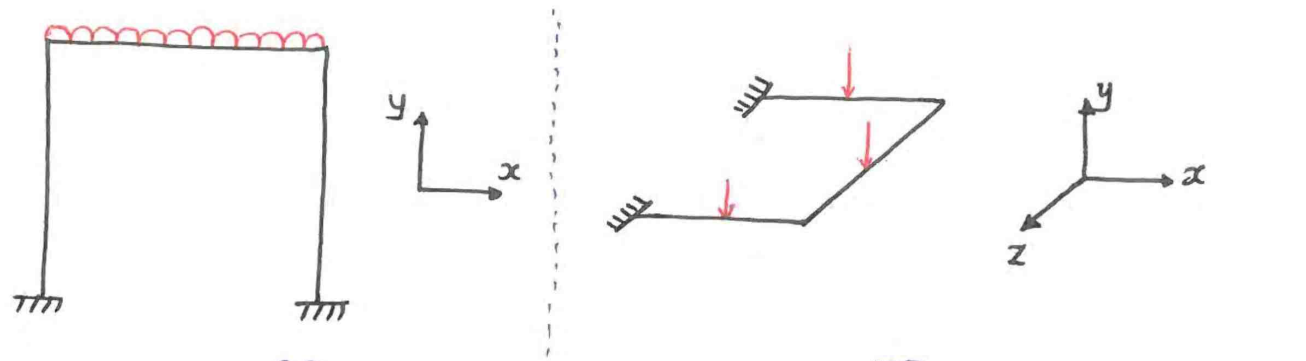
1.4 Idealisation of Structure:



1.5 Planar and 3D Structure:

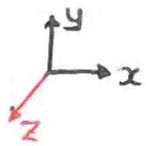
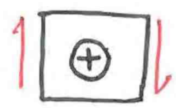
If 2-axes are sufficient to define geometry and loading of a structure then that structure is called 2D structure/ planar structure.

If 3-axes are required to define geometry and loading of a structure then that structure is called 3D structure.



2D 3D

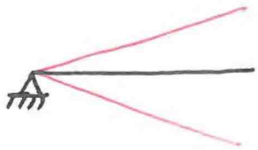
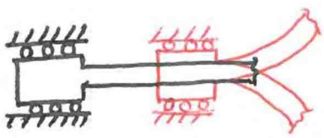
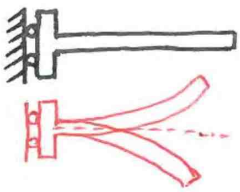
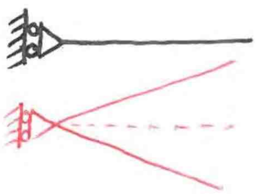
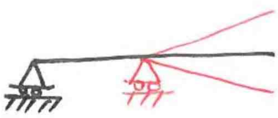
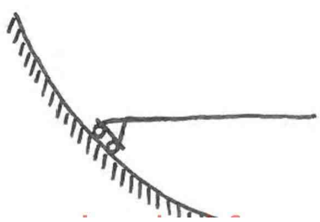
1.6 Sign Convention:

	Positive
x-axis	⟶
y-axis	↑
z axis	$\vec{x} \times \vec{y}$ 
Rotation	Clockwise
Forces	Along axis
Moment	Clockwise
SF	
BM	

1.7 Types of Support :

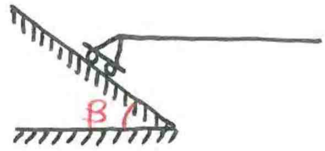
Any arrangement that can restrict movement of any point of a structure is called as support.

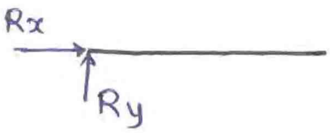
Reaction at support is always due to restriction of movement so direction of reaction is always in opposite direction to expected movement.

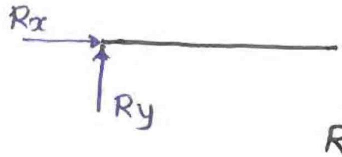
Movement Restricted	Representation	Reaction	Remarks.
x, y. rotation			Fixed
x, y			Pin / Hinge
y. rotation			Guided Roller
x, rotation			Guided Roller
x			Roller
y			Roller
			Inclined Roller.

* Note:

Inclined roller support and hinge support both don't provide any movement in x and y direction but we get one reaction for inclined roller and two reactions for hinge support.

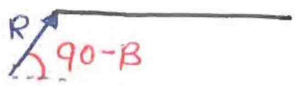






$R_x = R\sin\beta$

$R_y = R\cos\beta$

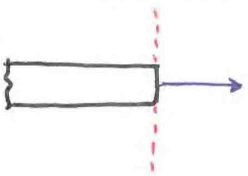


Two unknowns are either (R_x and R_y) or (R and θ)

Unknown is only R because β is known.

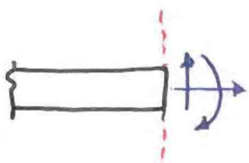
1.8 Types of Structural Member:

1) Axial Member.



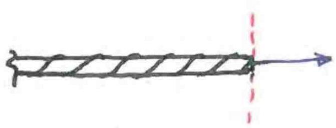
Unknown member forces = 1
(Axial Tension or compression)

2) Beam/Frame Member.



Unknown Member forces = 3
(Axial Tension or compression, Shear, Moment)

3) Cable:



Unknown Member Forces = 1
(Axial Tension)

1.9 Equilibrium and Static Equilibrium:

• **Equilibrium:**

If net force (force and moment) acting on a body is zero in all directions then body is in equilibrium.

For e.g. Bodies in space, vehicle moving with constant speed

For 3D :-

$\Sigma F_x = 0 \quad \Sigma M_x = 0$

$\Sigma F_y = 0 \quad \Sigma M_y = 0$

$\Sigma F_z = 0 \quad \Sigma M_z = 0$

For 2D (x-y plane)

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma M_z = 0$

• **Static Equilibrium:**

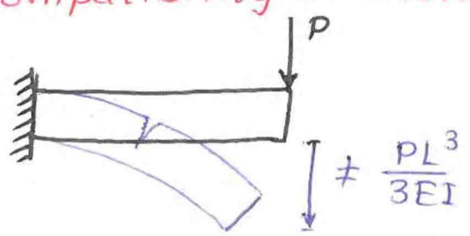
If net force acting on a body is zero and body is in rest/static state then only body can be classified under static equilibrium condition.

For e.g. Buildings, Bridges etc.

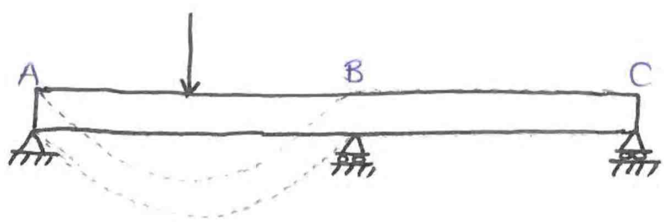
1.10 Compatibility:

The continuity or good-fit of material or member or components while being deformed under loading is called compatibility of structure.

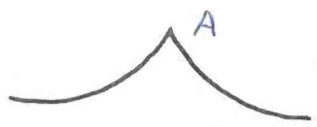
1.10.1 Compatibility of Member:



Compatibility is not maintained due to fracture in member



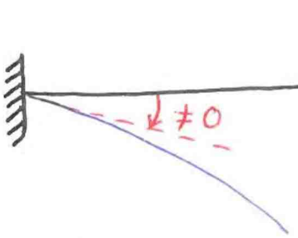
Compatibility is not maintained because of sudden change in slope at B.



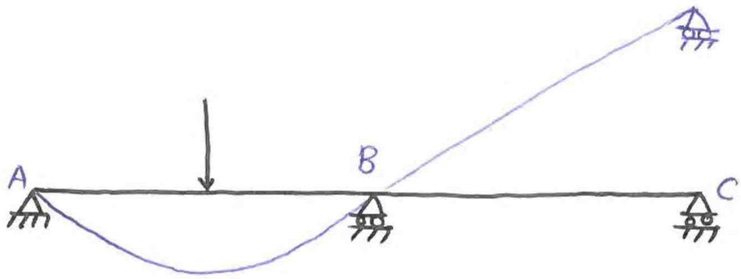
Sudden change in slope at A

Smooth Curve.

1.10.2 Compatibility of Support:

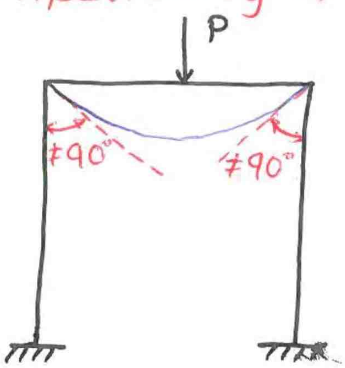


compatibility is not maintained because of non-zero slope at fixed support.

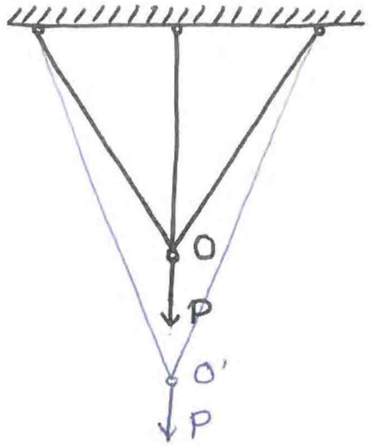


compatibility is not maintained because of non-zero vertical deflection at C.

1.10.3 Compatibility of Joint:



Compatibility is not maintained because of change in angle between members at rigid joints B & C



Compatibility is not maintained because all 3 members are not intact at O!

1.11 Free Body Diagram:

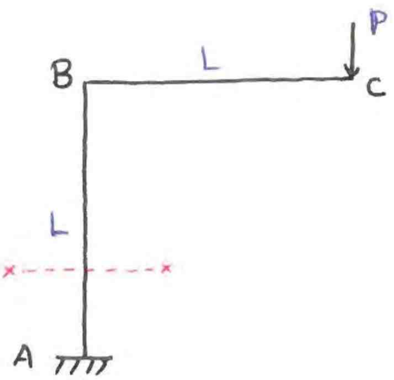
It is a graphical representation of body with all forces (internal and external) acting upon it.

1.11.1 Statically Determinate Structure:

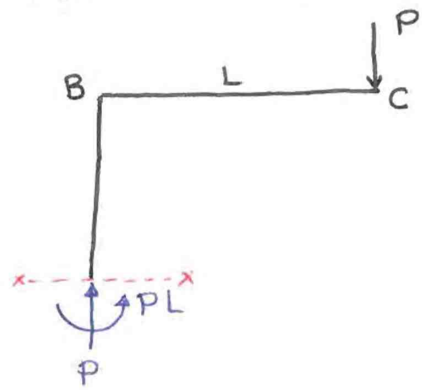
Step I: Make body free from all loads and reactions.

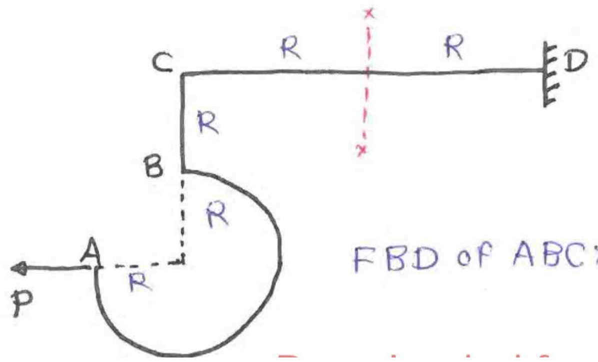
Step II: Apply all loads and support reactions.

Step III: Apply all internal forces at cut section to satisfy conditions of equilibrium.

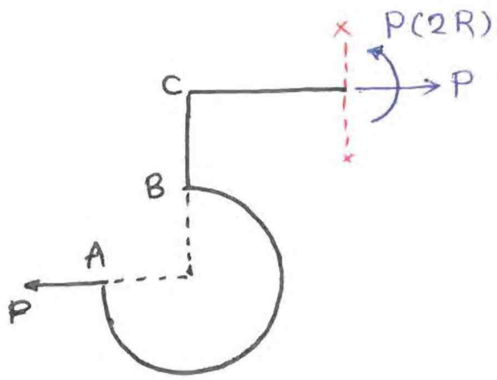


FBD of XBC ?





FBD of ABCX ?

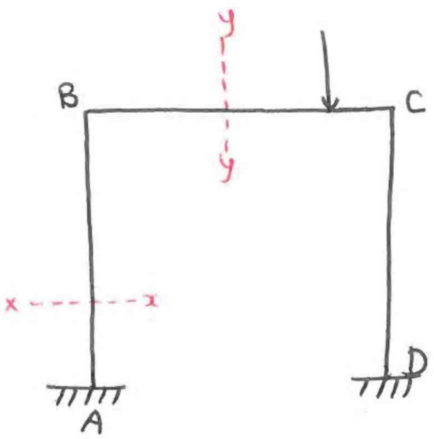


1.11.2 Statically Indeterminate Structure:

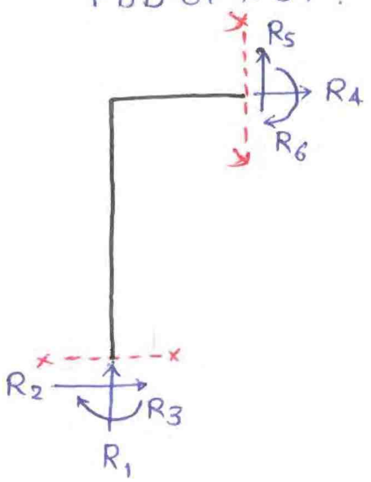
Step I: Make body free from all loads and reactions.

Step II: Apply all external loads and all possible support reactions. (because support reactions are not known).

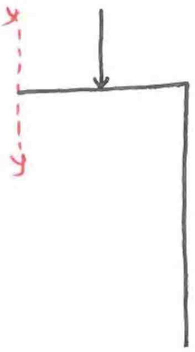
Step III: Apply all possible internal forces at cut section (because member forces are not known)

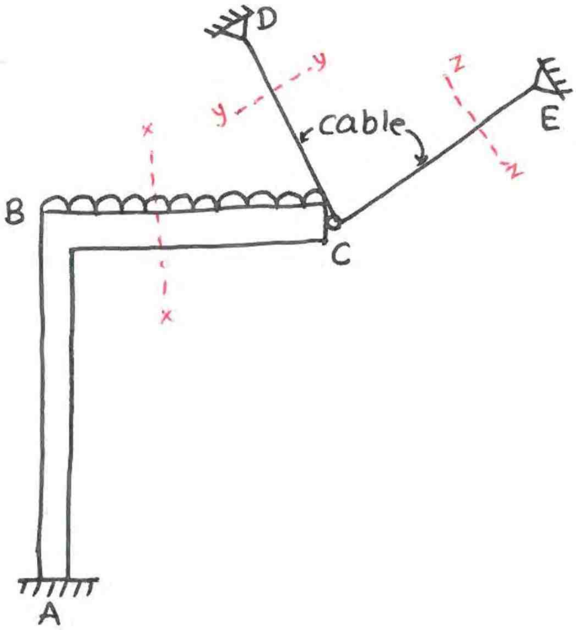


FBD of XBY ?

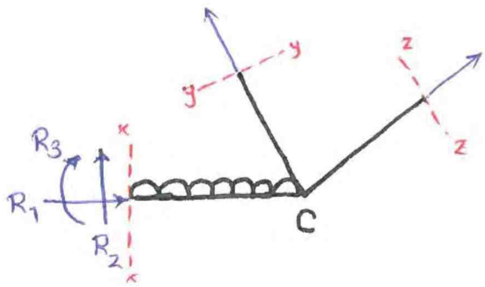


FBD of YCD ?





FBD of XCYZ ?



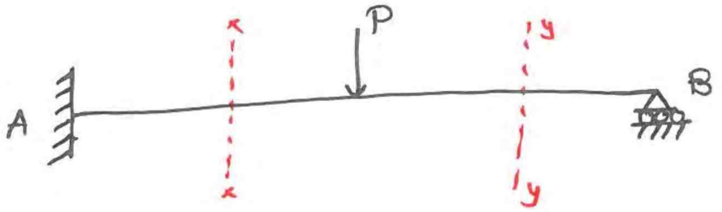
1.12 Static Indeterminacy:

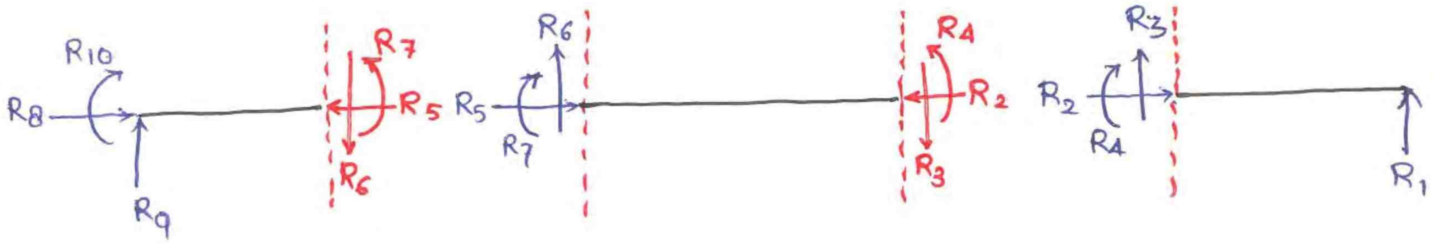
If available equations from conditions of equilibrium are not sufficient to analyse the structure completely then such type of structure is called statically indeterminate structure.

1.12.1 Degree of Static Indeterminacy:

Total number of unknowns (internal forces and support reactions) − Total number of available equations from conditions of equilibrium is called degree of static indeterminacy.

1.12.2 Meaning of DSI





- Observation:

  i) In above FBD, total number of unknown are 10 (R_1 to R_{10})

  ii) Total number of available equations from conditions of static equilibrium are 9 for 3 portions ($3 \times 3 = 9$)

- Conclusion:

  From above FBD, it is clear that only 1 unknown (R_1) is required to be known to calculate all other unknowns (R_2 to R_{10}) so degree of static indeterminacy is 1.

In general, minimum number of unknown forces (support reaction or member forces) are required to be known to calculate all other unknown forces (support reactions and member forces) of a structure is Dsi of that structure.

1.12.3 Procedure:

StepI: Identify start joint.

StepII: Calculate Total number of unknowns (member forces and support reactions) at that joint.

StepIII: Calculate total number of available equations (including extra equations due to release) at that joint.

StepIV: Calculate StepII − Step III. This is static indeterminacy at that joint. It may be positive or negative.

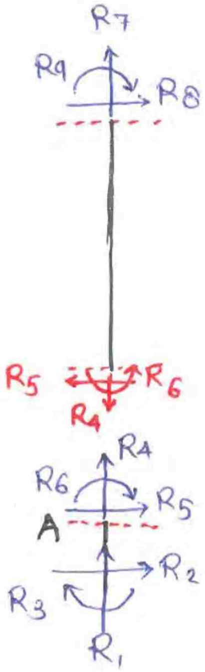
StepV: Make mark on each member meeting at that joint.

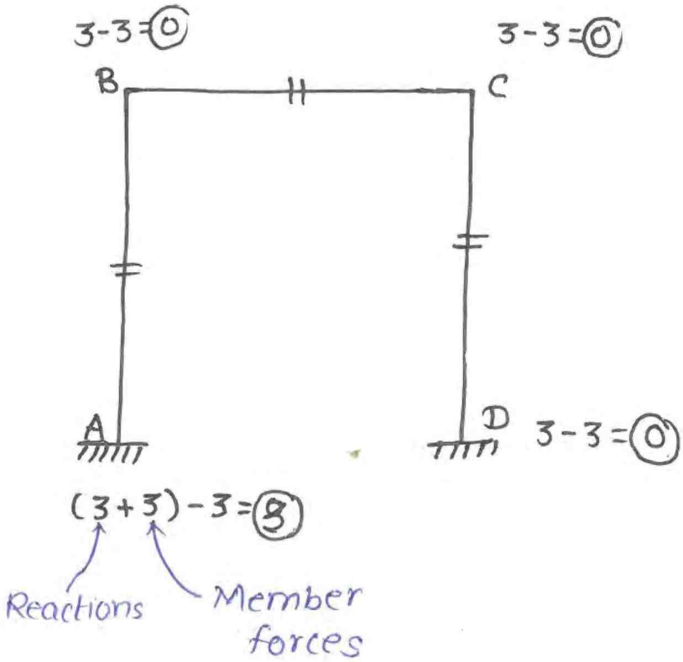
StepVI: Repeat stepII to StepV at all joints.

StepVII: Add up values obtained from stepIV for all joints. This is Dsi of entire structure.

*** Note:**
Free end of overhang portion should also be considered as joint.

Ex.1.

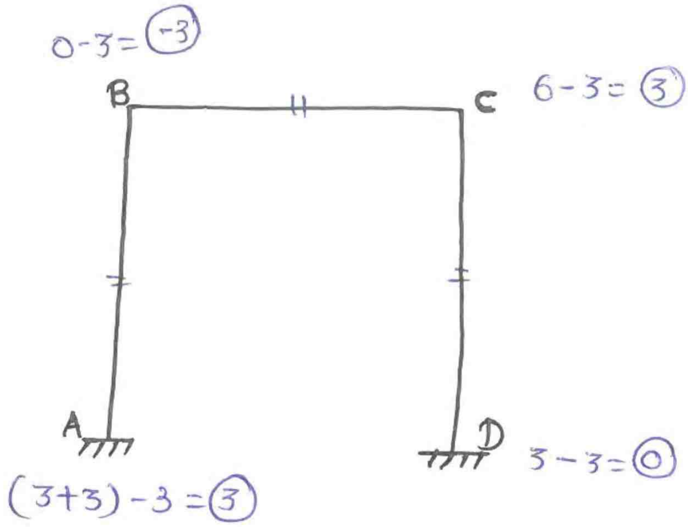




Seq → ABCD

$DSI = 3+0+0+0$

$= 3$

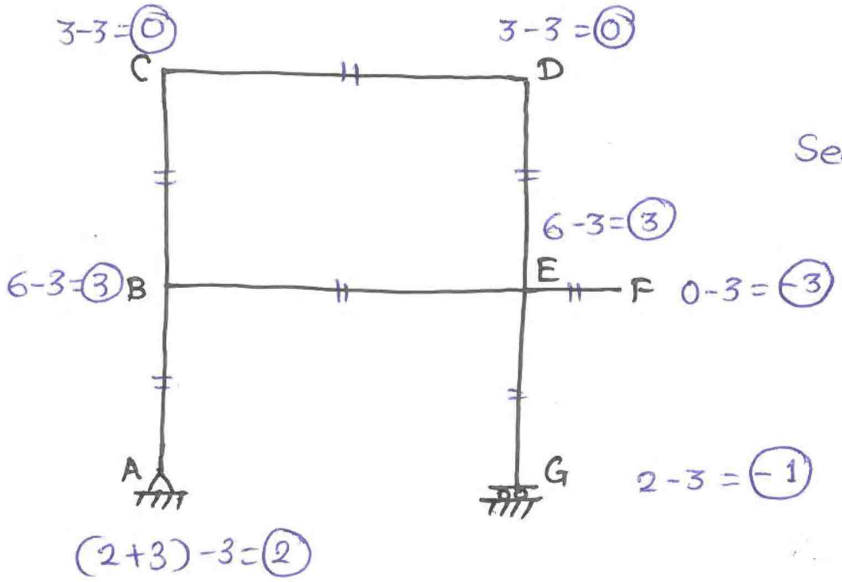


Seq - CABD

DSI = 3+3-3+0

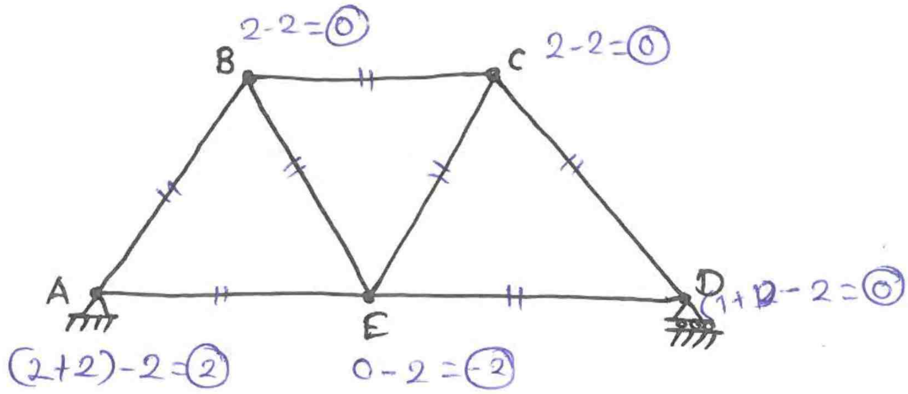
= 3

Ex.2



DSI = 2+3+3-3-1 = 4

Ex.3.

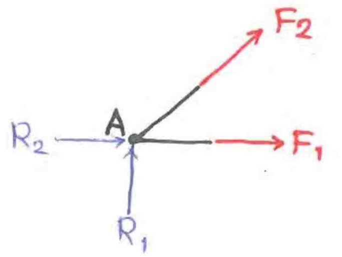


Seq. ABCDE

DSI = 2 - 2 = 0

* Note:

$\Sigma M_z = 0$ does not provide any equilibrium equation at joint of truss because all forces are concurrent at joint of truss.

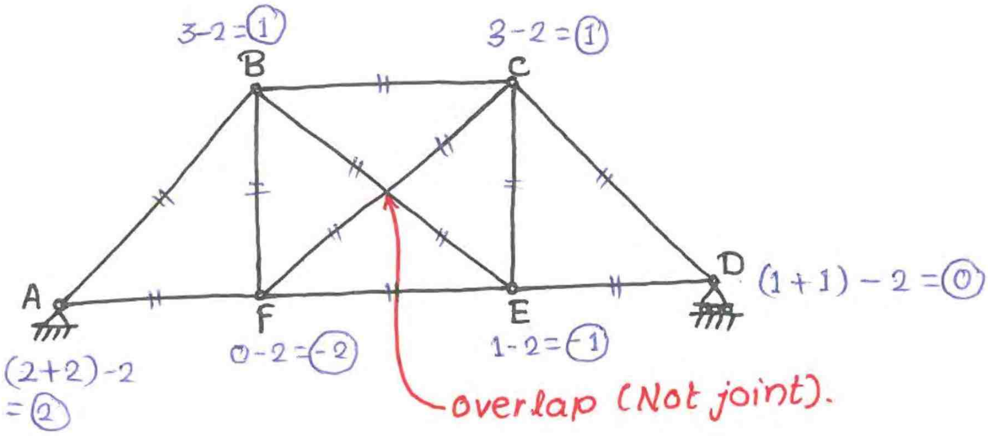


$\Sigma M_Z = 0$

$\Rightarrow \Sigma M_A = 0$

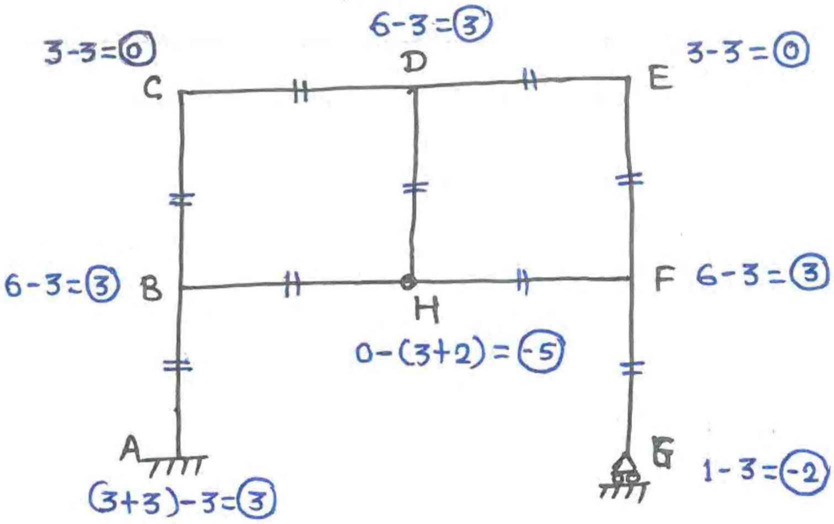
$\Rightarrow 0 = 0$ (useless)

Ex. 4.



Seq. A to F

$DSI = 2 + 1 + 1 + 0 - 1 - 2 = 1$

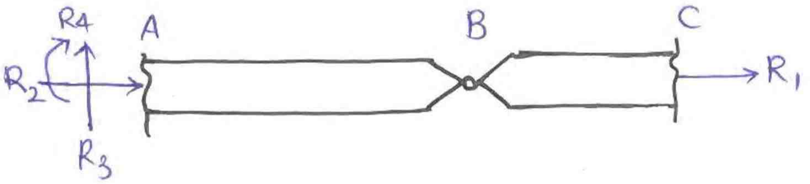


seq. A to H

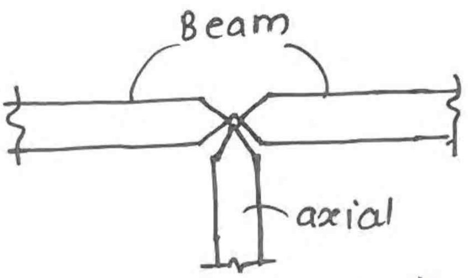
DSI = 12 - 7 = 5

*Note:

If axially loaded member is connected through hinge then that does not provide any extra equation at hinge.

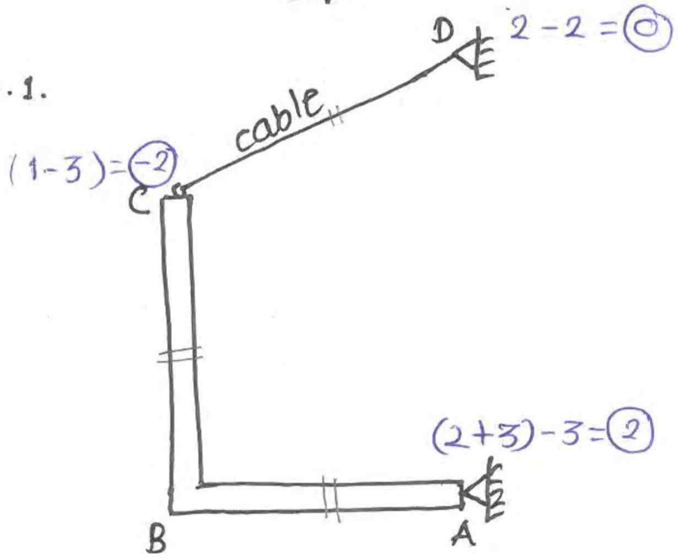


No extra equation



One extra equation.

Ex. 1.



seq- ABCD

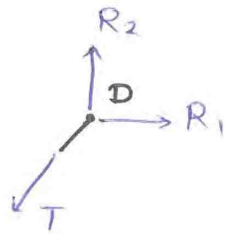
DSI = 0

At joint C:

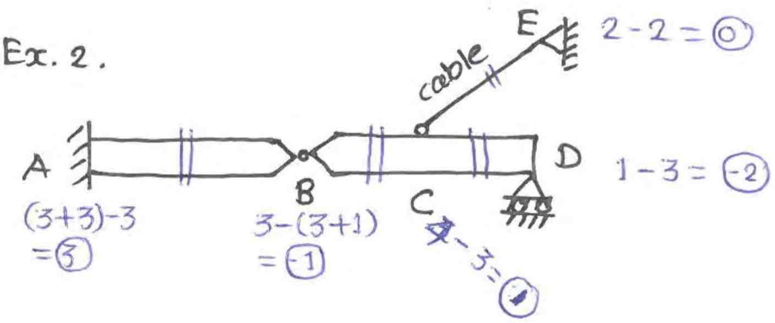
Hinge doesn't provide any extra equation because CD is axial (cable)

At joint D:-

$\Sigma M_z=0$ doesn't provide any equation because all forces are concurrent at D.



Ex. 2.



seq. ABCDE

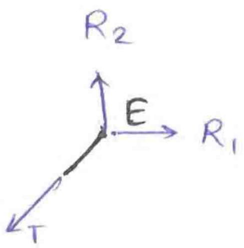
DSI = 1

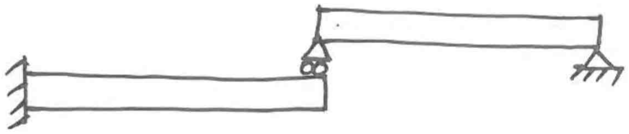
At joint C:

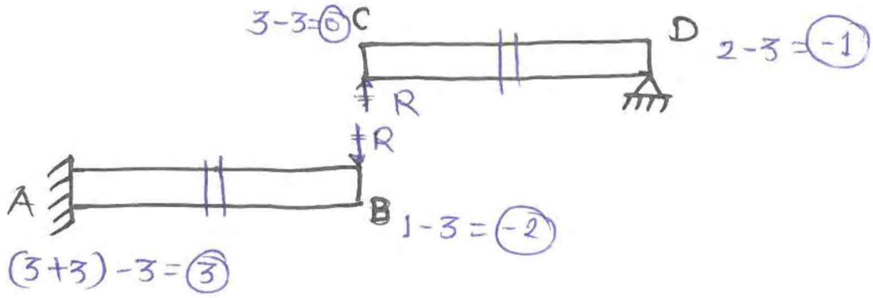
Hinge doesn't provide any extra equation because CE is axial (cable)

At joint E:-

$\Sigma M_z=0$ doesn't provide any equation because all forces are concurrent at E.







Seq.- ABCD

DSI = 0

1.12.5 External and Internal Static Indeterminacy:

DSI

External | Internal

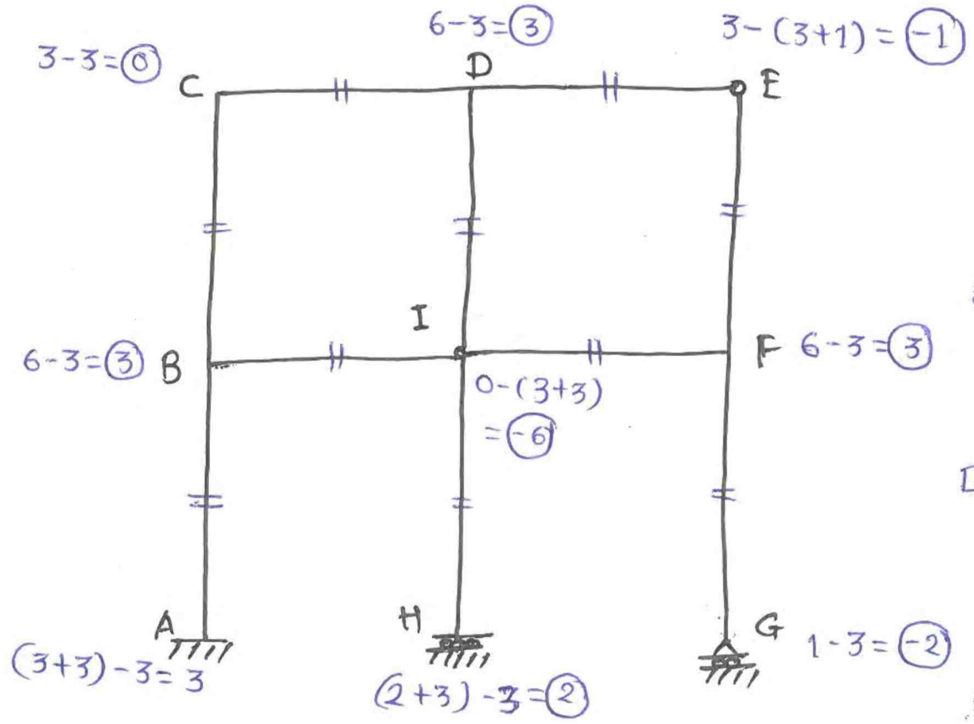
- For 2D Structure:

DSI_e = Total No. of Reactions − 3

$DSI_i = DSI_{Total} - DSI_e$

- For 3D Structure

DSI_e = Total No. of Reactions − 6



Seq. A to I

$DSI_{Total} = 5$

DSI_e = Total No of reactions − 3

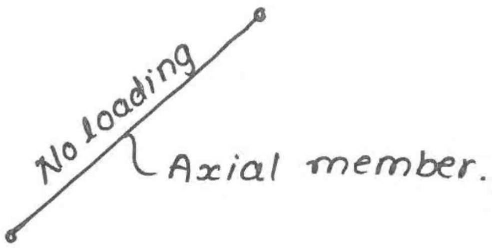
$= 6 - 3 = 3$

$DSI_i = DSI_{Total} - DSI_e$

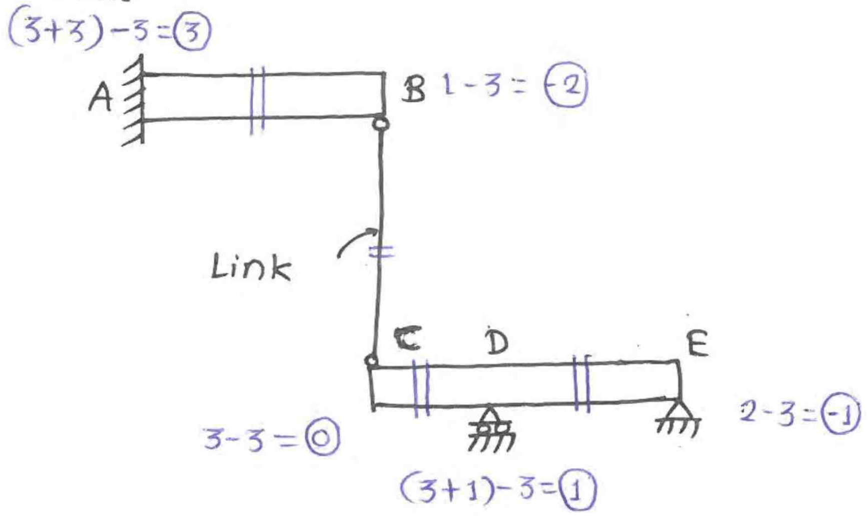
$= 5 - 3 = 2$

1.12.6 Concept of Link:

Link is a structural member connected through hinges at both ends and subjected to no loading in between then it has axial force only.



Ex.1

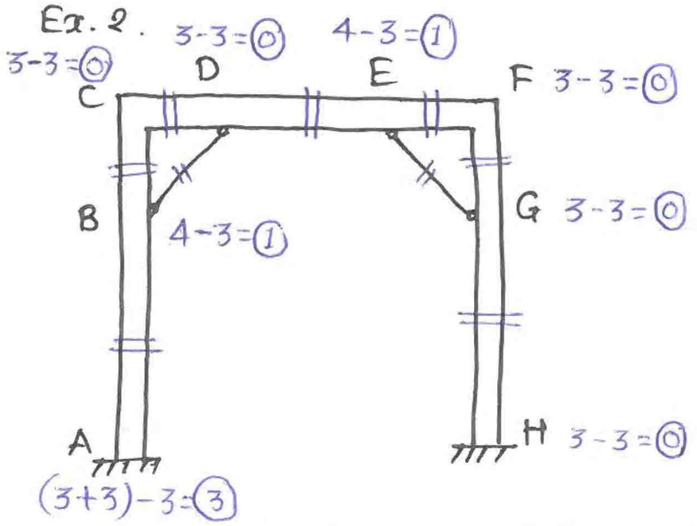


Seq A to E

$DSI_{Total} = 1$

Hinges of B and C don't provide any extra equation because BC is axial (Link).

Ex. 2.

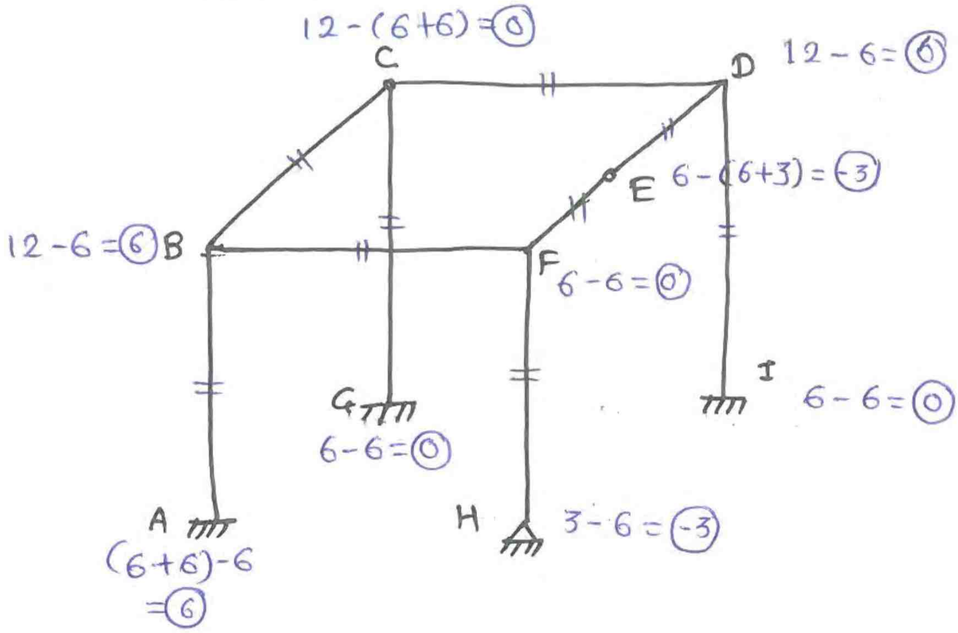


seq A to H

$DSI_{Total} = 5$

Hinges of B, D, E and G don't provide any extra equation because BD and EG are axial (Link)

1.12.7 3D Structure:



Seq. A to I

DSI = 12

1.12.8. Formula for DSI

DSI = Total No. of Unknowns − Total No. of available equations

- For 2D:-
  - Beams & Frames:-

    DSI = (3m + r) − (3j + Extra eq^{n}s due to release)

  - Truss:-

    DSI = (m + r) − (2j)

- For 3D:-
  - Beams & Frames:-

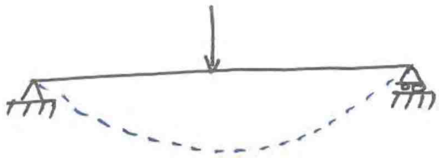
    DSI = (6m + r) − (6j + Extra eq^{n}s due to release)

  - Truss:-

    DSI = (m + r) − (3j)

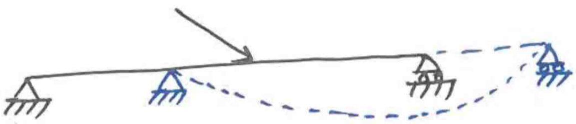
1.13 Stability of a Structure:

1.13.1 Difference between Movement and Deformation:



Deformation

Movement



Deformation + Movement.

* Note:
Structure is considered as stable if there is no movement due to application of load.

1.13.2 Stability Check:

Step I: Calculate DSI of a structure.

Step II: If DSI < 0 then structure is unstable.

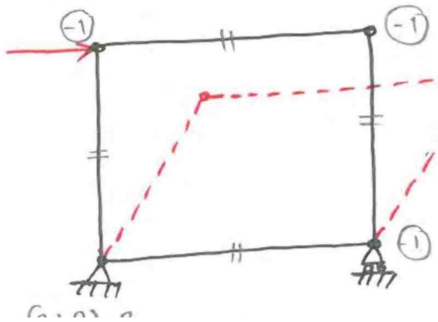
If DSI ≥ 0 then structure is unstable if

a) All reactions are parellel

b) All reactions are concurrent.

c) By visual inspection.

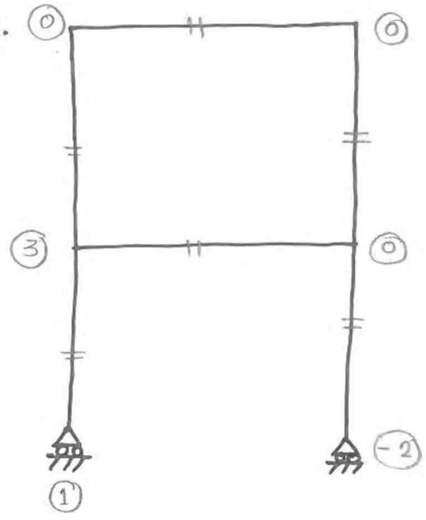
Ex.1. Check stability of given truss.



DSI = -1 < 0

so unstable.

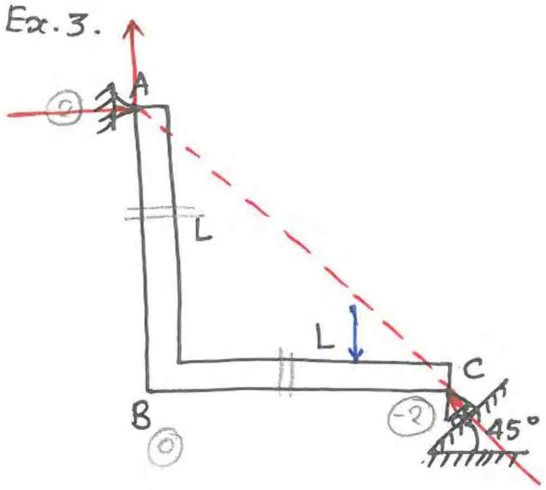
Ex. 2.



DSI = 2 >0

Unstable because all reactions are parellel.

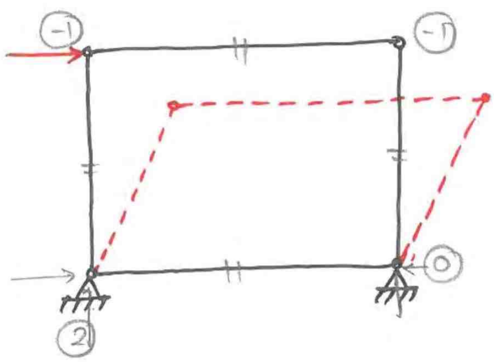
Ex. 3.



DSI = 0

Unstable because all reactions are concurrent

Ex. 4. Check stability of given truss:

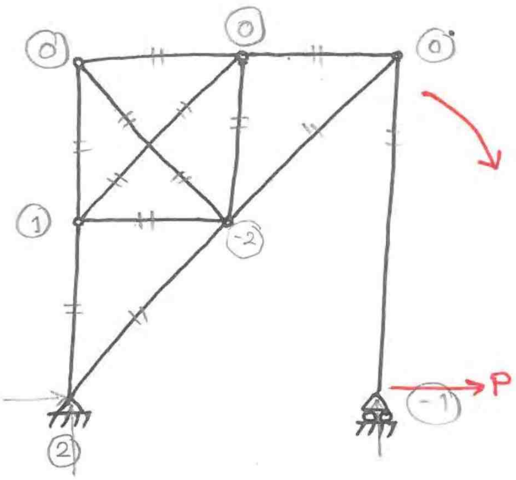


DSI = 0

Unstable by visual inspection.

Rectangular panel of truss makes truss unstable.

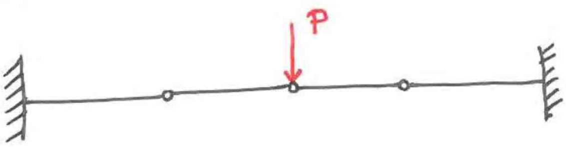
Ex.5.



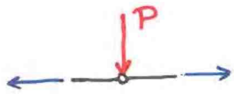
DSI = 0

Unstable by visual inspection.

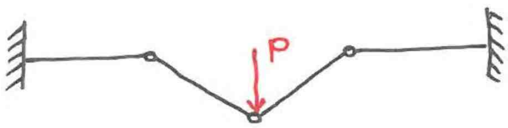
Ex. 6.

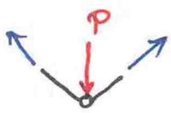


DSI = 0



$\Sigma F_y \neq 0 \Rightarrow$ Unstable.

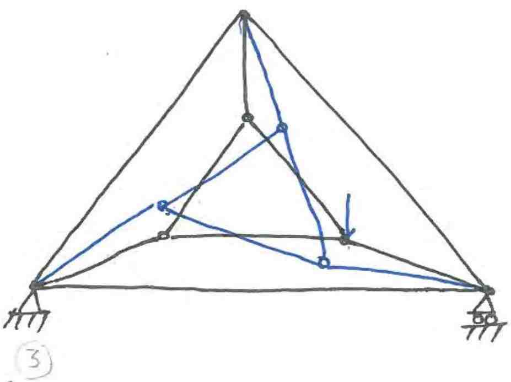




$\Sigma F_y = 0 \Rightarrow$ Stable

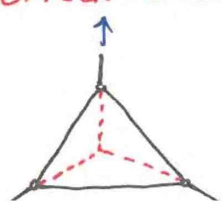
- Note: Three collinear hinges makes structure unstable.

Ex.7.

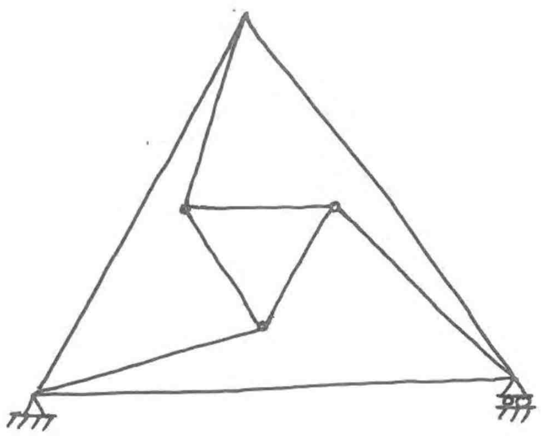


DSI = 0

Unstable because supporting forces of internal triangle are concurrent.



Ex. 8.



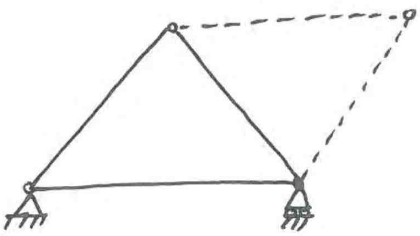
DSI = 0

Stable.

• Note:

- Externally Unstable → Car without brake.
- Internally Unstable → swing.

1.13.3 Deficient Truss:



$m = 2j - 3$ (Perfect Truss)

$m < 2j - 3$ (Deficient Truss) Unstable

$m > 2j - 3$ (Redundant)

1.14 Kinematic Indeterminacy / Degree of Freedom:

Sum of all independent displacements of all joints of a structure is called KI/DOF of structure.

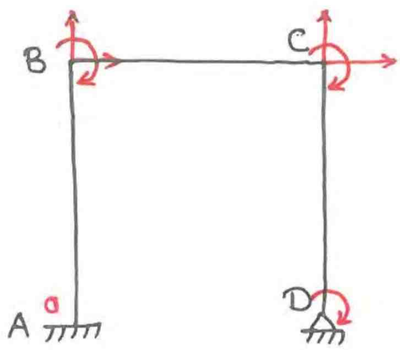
For visualisation only

Rigid ⟶ Stone

Inextensible ⟶ Reinforcing Bar

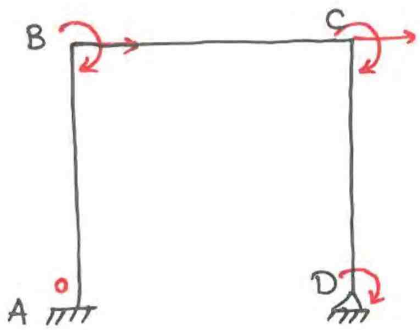
Extensible ⟶ A bar of rubber.

Ex. 1.a) If members are Extensible



KI = 7 (Δ_{xB}, Δ_{YB}, θ_B, Δ_{xC}, Δ_{YC}, θ_C, θ_D)

Ex. 1.b) If members are Inextensible.

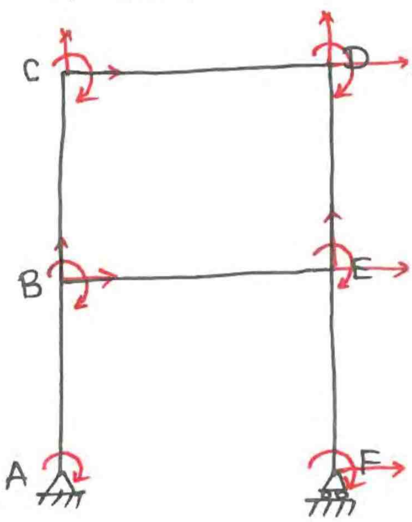


Dependent

KI = 4 (Δ_{xB}, θ_B, $\boxed{\Delta_{xC} = \Delta_{xB}}$ θ_C, θ_D)

Ex. 2.

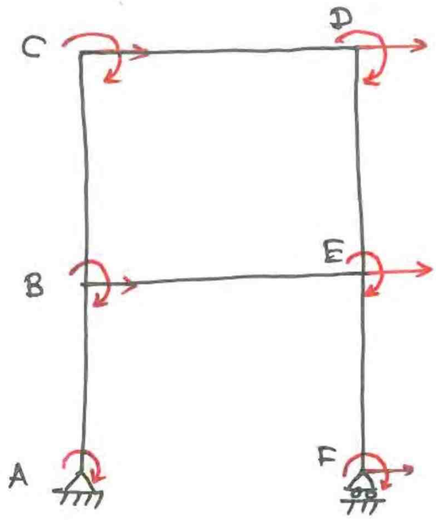
a) Extensible



KI = 15 (θ_A, Δ_{xB}, Δ_{YB}, θ_B, Δ_{xC}, Δ_{YC}, θ_C, Δ_{xD}, Δ_{YD}, θ_D, Δ_{xE}, Δ_{YE}, θ_E, Δ_{xF}, θ_F)

Ex. 2.

b) Inextensible.



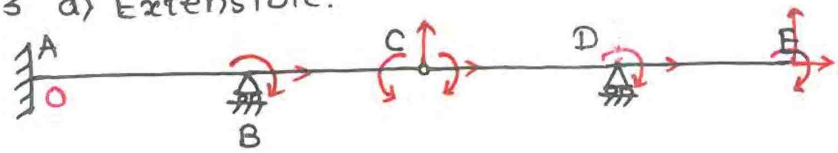
KI = 9 (θ_A , Δ_{xB} , θ_B , Δ_{xC}, θ_C

$\cancel{\Delta_{xD} = \Delta_{xC}}$, θ_D ,

$\cancel{\Delta_{xE} = \Delta_{xB}}$, θ_E ,

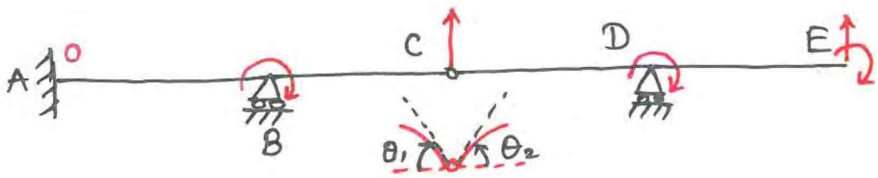
Δ_{xF} , θ_F

Ex.3 a) Extensible.



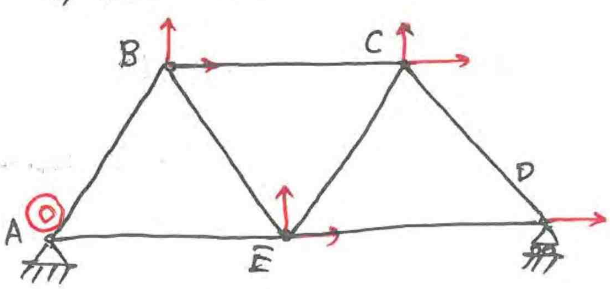
KI = 11 (θ_B , Δ_{xB} , Δ_{xC} , Δ_{YC} , θ_{CD} , θ_{CB} , Δ_{xD} , θ_D , Δ_{xE} , Δ_{YE} , θ_E)

b) Inextensible.



KI = 7 (θ_B , Δ_{YC} , θ_{CB} , θ_{CD} , θ_D , Δ_{YE} , θ_E)

Ex. 4. a) Extensible



KI = 7 (Δ_{xB} , Δ_{YB} , Δ_{xC} , Δ_{YC} , Δ_{xD} , Δ_{xE} , Δ_{YE})

- Formula for KI :-
  - For 2D :-
    - Beams and Frames :-

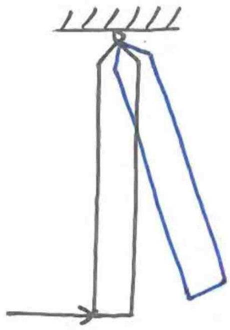
$$KI = \left(3j + \text{extra displacements due to release}\right) - \left(r + \text{No. of inextensible members}\right)$$

    - Truss :-

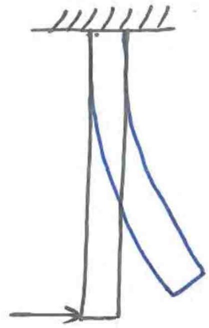
$$KI = (2j) - (r + \text{No. of inextensible members})$$

1.15 Elastic Curve

1.15.1. Difference between elastic curve and deflected shape:-



Deflected Shape



Elastic Curve

All elastic curves are deflected shape but all deflected shapes are not elastic curve.

1.15.2 How to Draw Elastic Curve ?

StepI: By visual inspection.

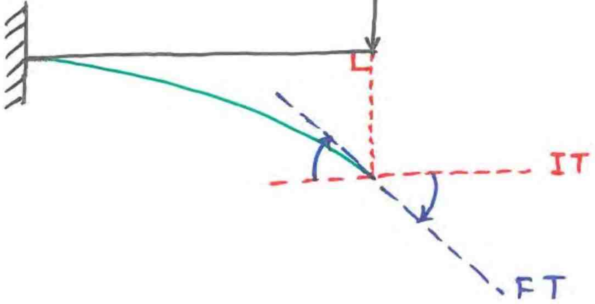
StepII: By satisfying compatibility conditions.

StepIII: By making BMD and elastic curve consistent.

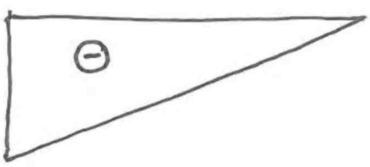
*Note:
In all coming examples, all members are assumed to be axially inextensible.

- Statically Determinate Beams:

Ex.1.

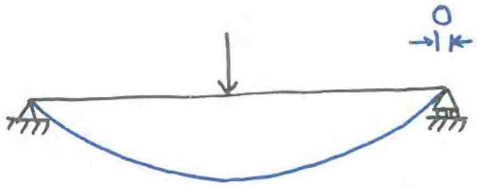


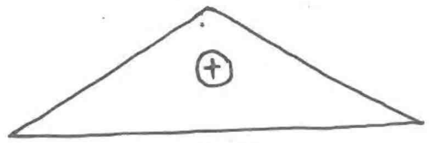
BMD



*Note:
Member always deflects in $\perp^{lar}$ direction to its longitudinal length provided member is inextensible & displacement is small.

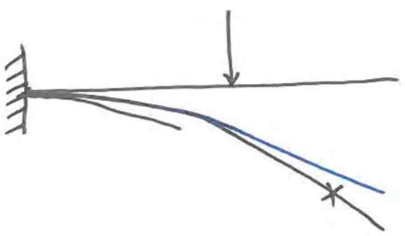
Ex. 2.

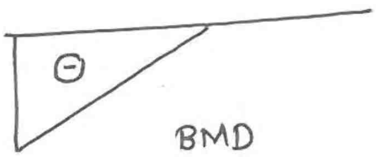




BMD.

Ex. 3.

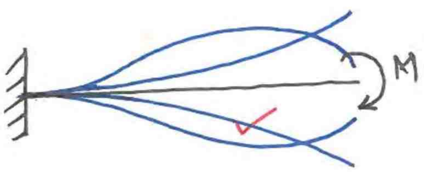


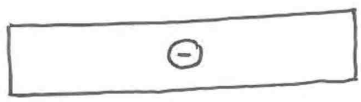


BMD

*Note: Member never bends without bending moment.

Ex. 4.





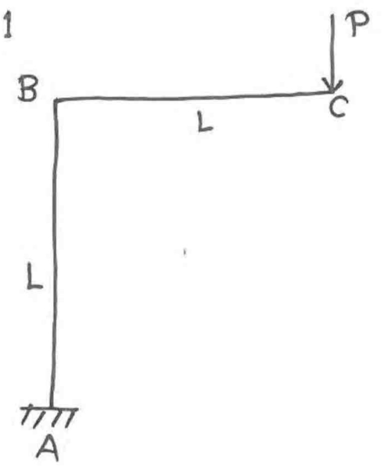
1.16 BMD for Frames:

Step I: Select reference face.

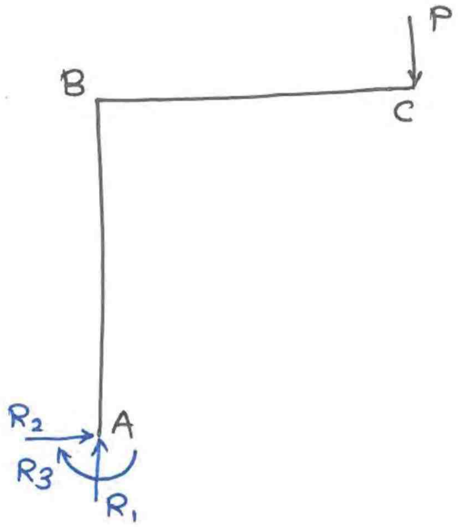
Step II Write Bending Moment at any section by considering moment producing compression on reference face as positive

Step III: Positive Bending Moment is plotted on reference face side and negative BM on opposite reference face.

Ex. 1



FBD:-



$\Sigma F_x = 0$

$\Rightarrow R_2 = 0$ ---- (i)

$\Sigma F_y = 0$

$\Rightarrow R_1 - P = 0$

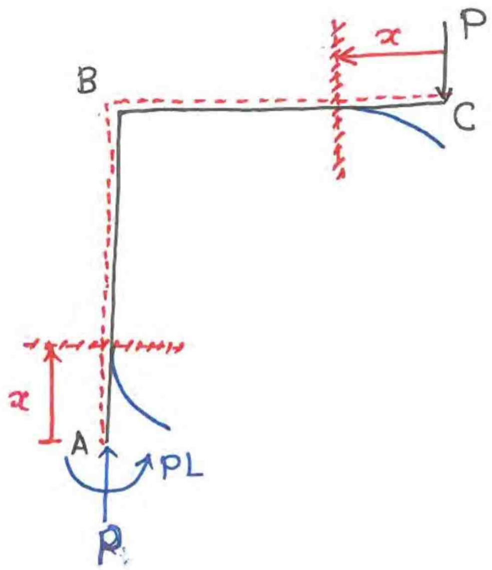
$\Rightarrow R_1 = P$ ---- (ii)

$\Sigma M_z = 0$

$\Rightarrow \Sigma M_A = 0$

$\Rightarrow R_3 + PL = 0$

$\Rightarrow R_3 = -PL$



Considering outer face as reference face.

For CB:-

$BM_x = -Px$ (-ve becoz tension on reference face)

At $x = 0$, $BM = 0$

at $x = L$, $BM = -PL$

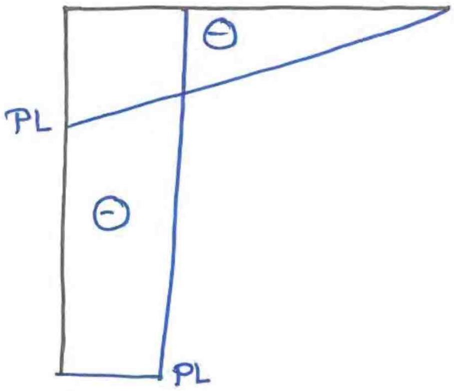
For AB :-

$BM_x = -PL$ (-ve becoz tension on reference face)

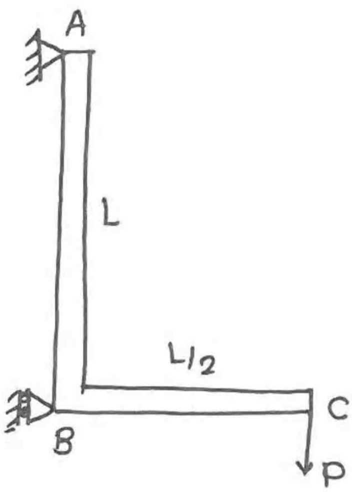
At $x=0$, $BM=-PL$

$x=L$, $BM=-PL$

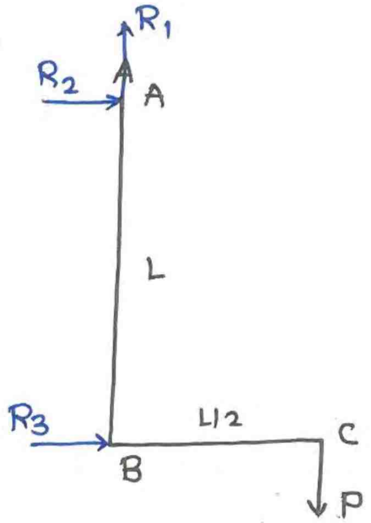
BMD:-



Ex.2.



FBD:



$\Sigma F_x = 0$

$\Rightarrow R_2 + R_3 = 0$

$\Rightarrow R_3 = -R_2$

$\Sigma F_y = 0$

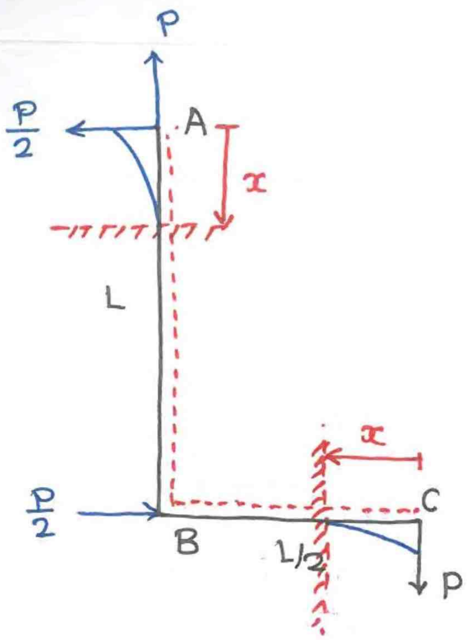
$\Rightarrow R_1 - P = 0$

$\Rightarrow R_1 = P$

$\Sigma M_z = 0$

$\Rightarrow \Sigma M_A = 0$

$-R_3 L + P \times \frac{L}{2} = 0 \quad \Rightarrow R_3 = \frac{P}{2}$



Considering outer face as reference face.

For CB:-

$BM_x = -Px$ (-ve becoz tension on ref face)

At $x=0$, $BM=0$

$x = L/2$, $BM = -\frac{PL}{2}$

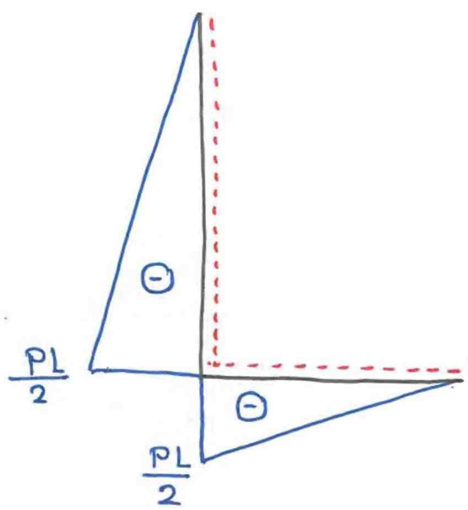
For AB:-

$BM_x = -\frac{P.x}{2}$ (-ve becoz tension on ref. face).

At $x=0$, $BM=0$

$x=L$, $BM = \frac{-PL}{2}$

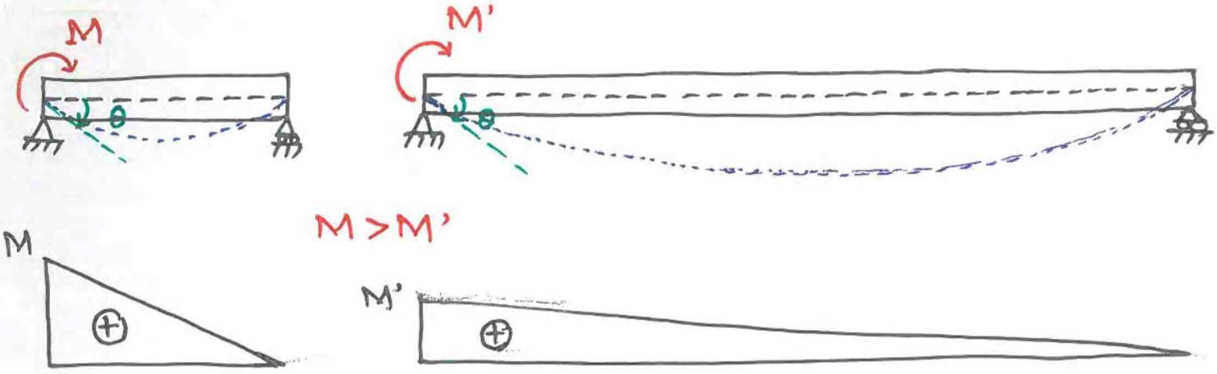
BMD:-



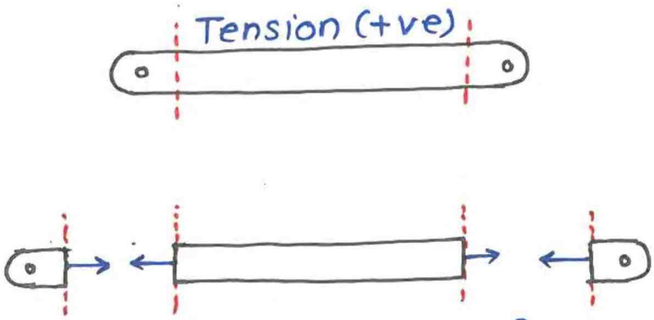
Truss

*** Note:**

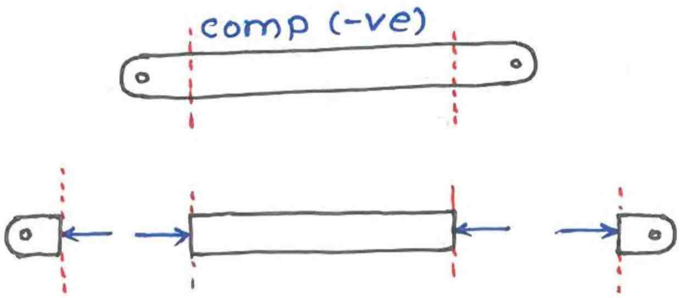
Practically all joints of truss are rigid so rotation of joint produces BM in members. Since members of truss are very slender so BM due to rotation of joints is very less. That is why this BM is neglected in analysis.



3.3 Sign Convention:



Force is always away from cross-section.

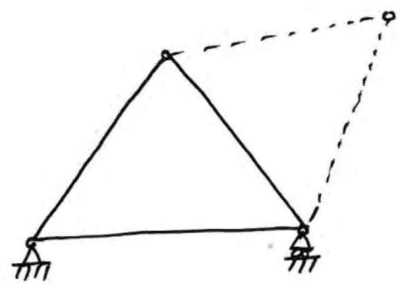


Force is always towards cross-section.

- Types of Truss:-

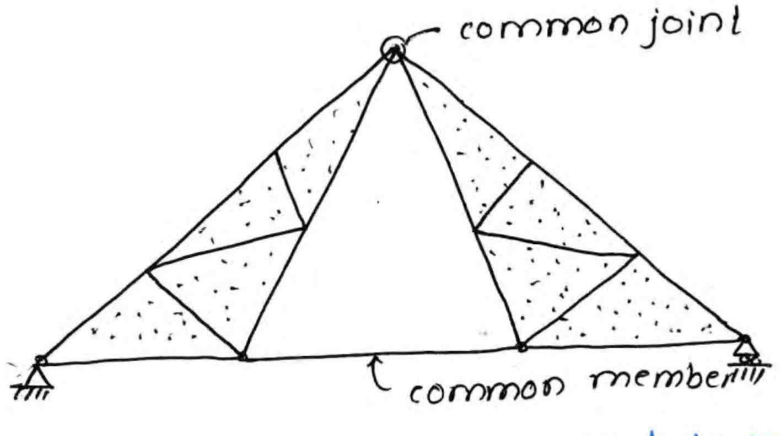
1) Simple Truss:-

If two members are required to make the additional joint in simple triangular truss then such truss is classified as simple truss.

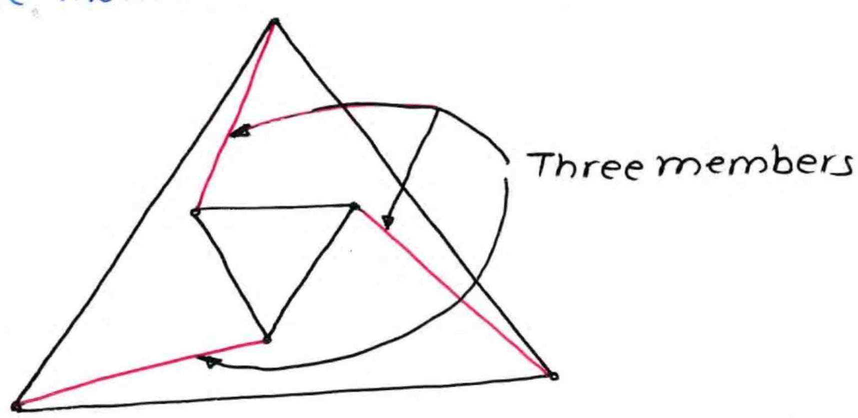


2) Compound Truss:-

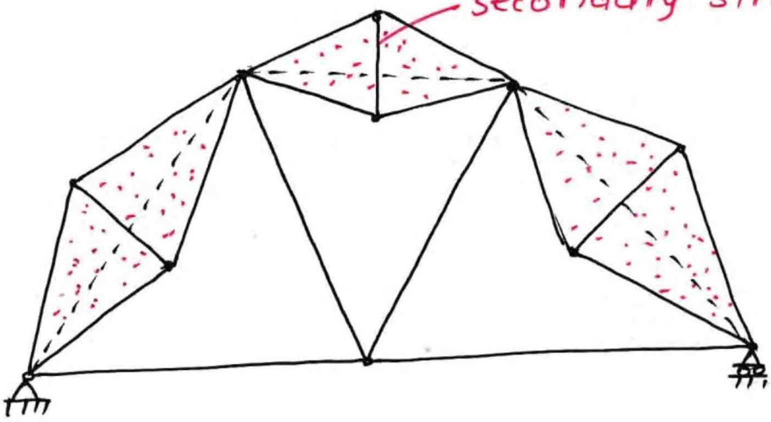
a) If 2 simple trusses are joined by a common joint and a common member.



b) If three members are used to join two simple trusses.

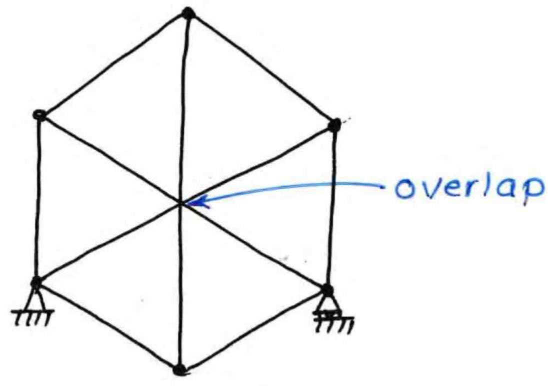


c) If any member of simple truss is replaced by secondary simple truss:-



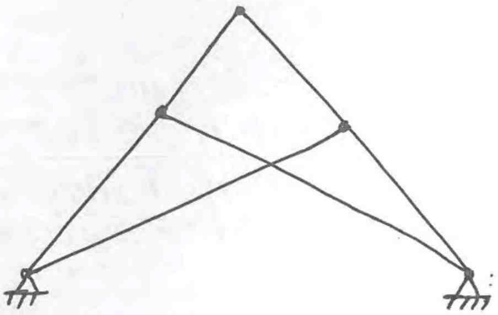
3) Complex truss:-

Trusses that can not be classified as simple and compound is called complex truss.

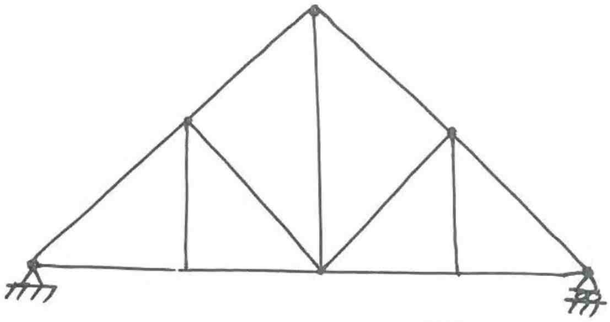


Complex Truss.

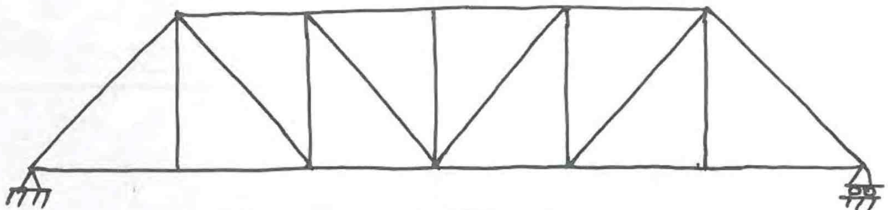
3.4 Types of Truss:



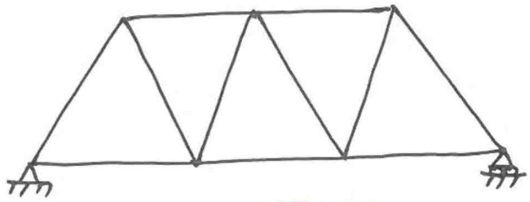
Scissor Truss



Pratt Building Truss



Pratt Bridge Truss



Warren Truss.

3.5 Methods of Analysis

1) Method of Joint
2) Method of Section
3) Graphical Method
4) Tension Coefficient Method.

*Note:
Willot Mohr's method is a graphical method which is used for calculation of deflection of truss.

3.5.1 Method of Joints:

Step I: Calculate support reactions using overall equilibrium of truss.

Step II: Identify joint of sart in such a way that number of unknowns at that joint should not be more than 2.

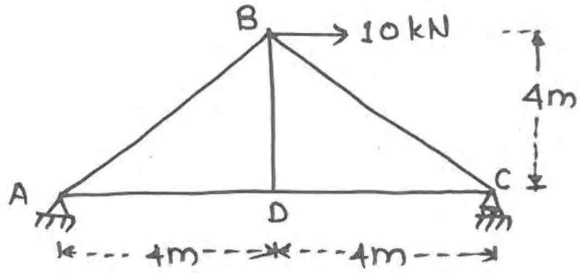
Step III: Apply equations of equilibrium at joint of step II ($\Sigma F_x = 0$ and $\Sigma F_y = 0$) and calculate unknown forces at that joint.

Step IV: Move to another joint where again not more than 2 unknowns are present.

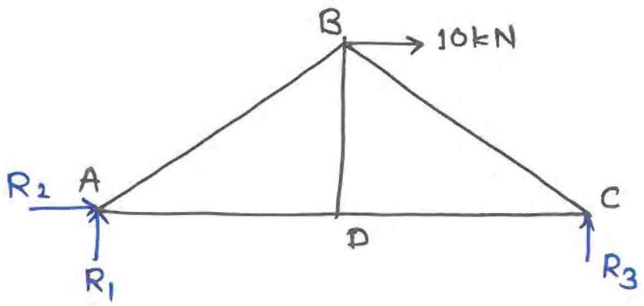
Step V: Repeat step III and step IV till force in all members are known.

Step VI: Represent member force in tabular formate.

Ex. 1.



⇒



$\Sigma F_x = 0$

$\Rightarrow R_2 + 10 = 0$ ----(i)

$R_2 = -10$

$\Sigma F_y = 0$

$\Rightarrow R_1 + R_3 = 0$ ------(ii)

$\Sigma M_z = 0$

$\Rightarrow \Sigma M_A = 0$

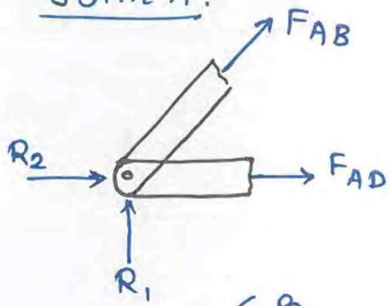
$\Rightarrow 10 \times 4 - R_3 \times 8 = 0$

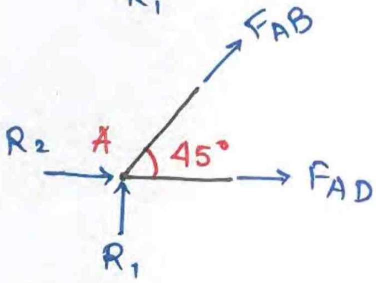
$R_3 = 5\,kN$ ----(iii)

from equation (ii)

$R_1 = -5\,kN$

Joint A:





$$\Sigma F_x = 0$$

$$\Rightarrow R_2 + F_{AB}\cos 45 + F_{AD} = 0$$

$$\Rightarrow -10 + F_{AB} \times \frac{1}{\sqrt{2}} + F_{AD} = 0 \quad \cdots (iv)$$

$$\Sigma F_y = 0$$

$$\Rightarrow R_1 + F_{AB}\sin 45 = 0$$

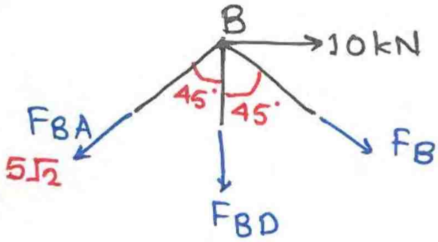
$$\Rightarrow -5 + F_{AB} \times \frac{1}{\sqrt{2}} = 0 \quad \cdots\cdots (v)$$

From eq^n (iv) & (v)

$$F_{AB} = 5\sqrt{2}\ kN$$

$$F_{AD} = 5\ kN.$$

Joint B:



$$\Sigma F_x = 0$$

$$\Rightarrow -F_{BA}\sin 45° + F_{BC}\sin 45° + 10 = 0$$

$$\Rightarrow -5\sqrt{2} \times \frac{1}{\sqrt{2}} + F_{BC} \times \frac{1}{\sqrt{2}} + 10 = 0$$

$$\Rightarrow \quad F_{BC} = -5\sqrt{2}$$

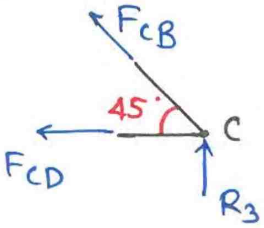
$$\Sigma F_y = 0$$

$$\Rightarrow -F_{BA}\cos 45° - F_{BD} - F_{BC}\cos 45° = 0$$

$$\Rightarrow -(5\sqrt{2}) \times \frac{1}{\sqrt{2}} - F_{BD} - (-5\sqrt{2}) \times \frac{1}{\sqrt{2}} = 0$$

$$\Rightarrow \quad F_{BD} = 0$$

Joint C:



$$\Sigma F_x = 0$$

$$-F_{CD} - F_{CB}\sin 45 = 0$$

$$\Rightarrow -F_{CD} - (-5\sqrt{2}) \times \frac{1}{\sqrt{2}} = 0$$

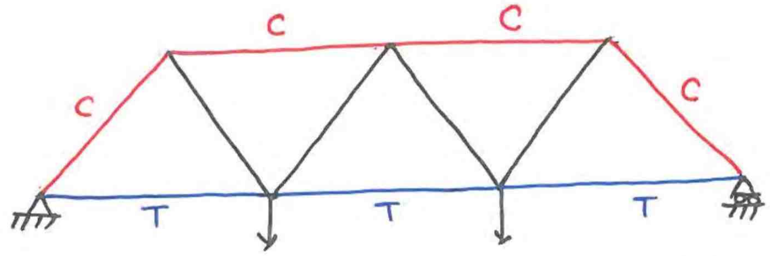
$$\Rightarrow F_{CD} = 5kN.$$

Member	Force (kN)
AB →	$5\sqrt{2}$
AD →	5
BD →	0
BC →	$-5\sqrt{2}$
CD →	5

(+) → Tension

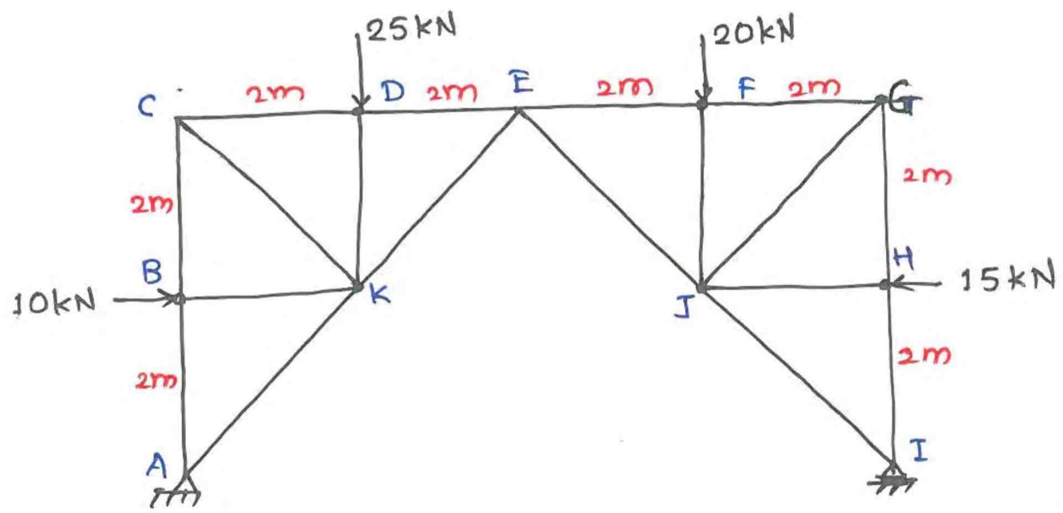
* Note:

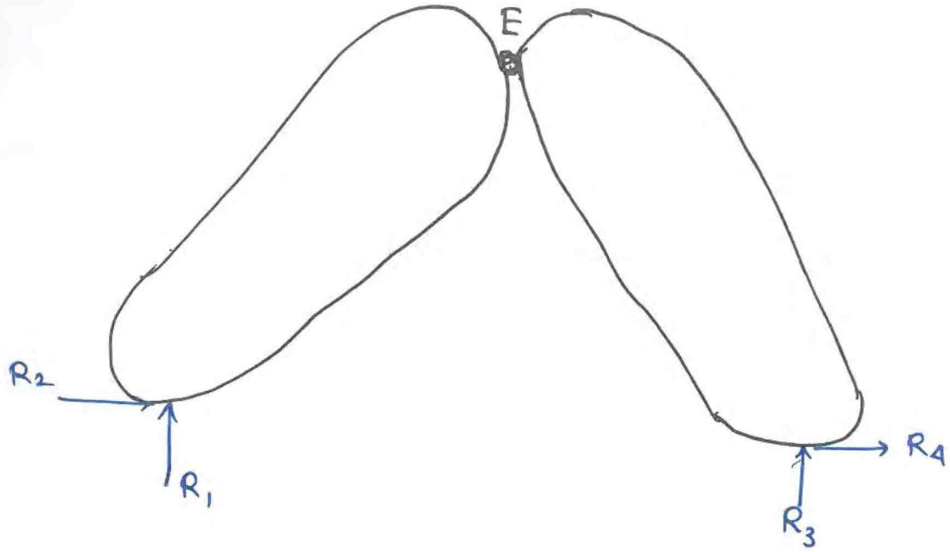
IF bridge truss is subjected to downward loading then tope chord and bottom chord members are subjected to compression and tension respectively.



Top chord → compression

Bottom chord → Tension.





Equations:

$\Sigma F_x = 0$

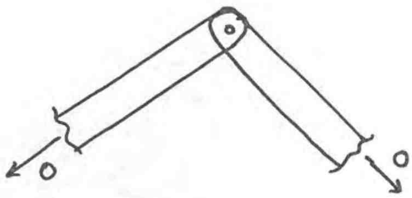
$\Sigma F_y = 0$

$\Sigma M_z = 0$

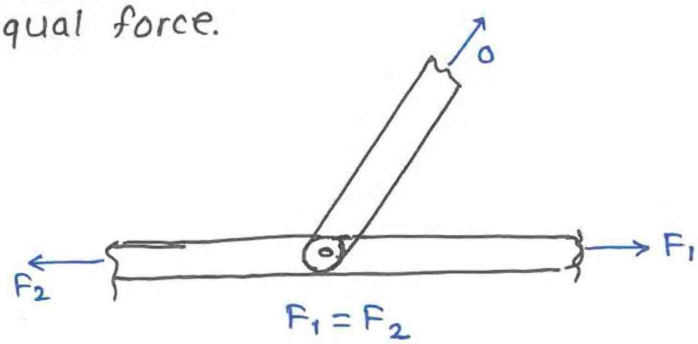
$M_E = 0$ (R.H.S.)

3.5.2 Some Tricks:

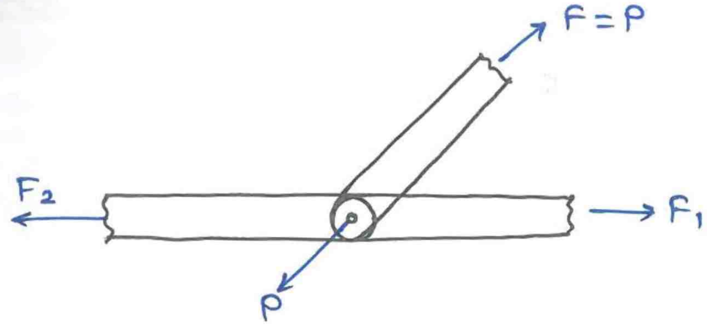
Trick 1: If two non collinear members are meeting at a joint and subjected to no external force or reaction then both members have zero force



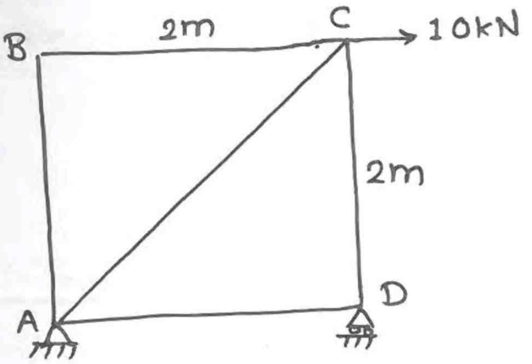
Trick 2: If three members are meeting at a joint out of which two are collinear and joint is subjected to no external force or reaction then non-collinear member has zero force and collinear members have equal force.



Trick 3: If three members are meeting at a joint out of which two are collinear and 1 external force or reaction is along non-collinear member then non-collinear member has force equal to applied loading or reaction.

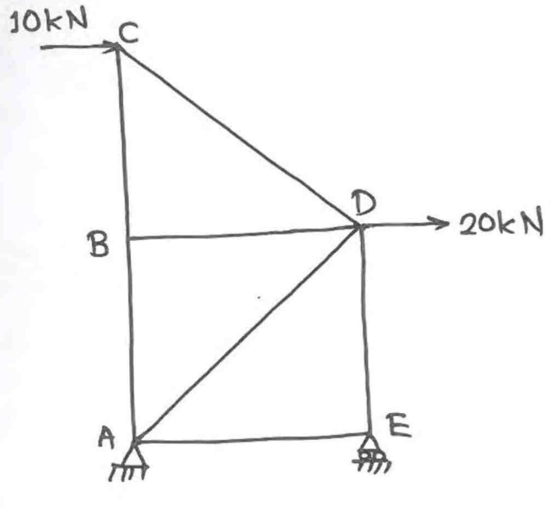


$F_1 = F_2$



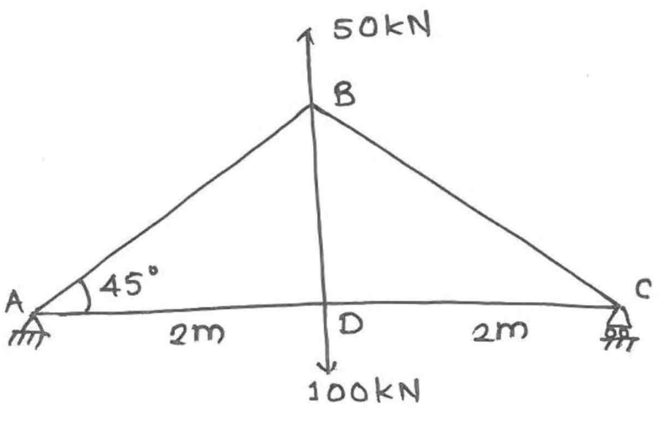
$F_{BC} = ?$

$F_{BC} = 0$



$F_{BD} = ?$

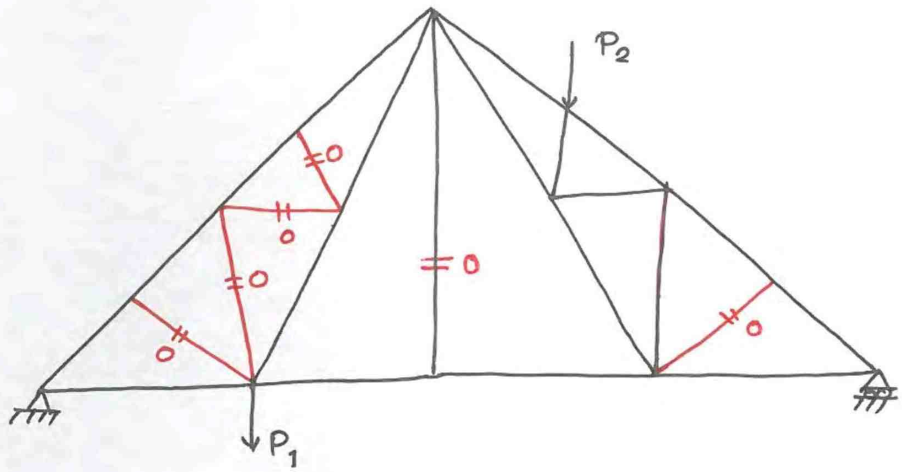
$F_{BD} = 0$



$F_{BD} = ?$

$F_{BD} = 100$ kN

Q. How many members have zero force ?



Note:

Zero force member is provided to reduce effective length of other members.

3.5.3 Method of Section

Procedure:

Step I: Calculate support reactions if required.

Step II: Decide cut section

Step III: Use equations of equilibrium to calculate force in desired member.

Guidelines to decide cut Section:

i) Section should pass through the member in which force is to be calculated.

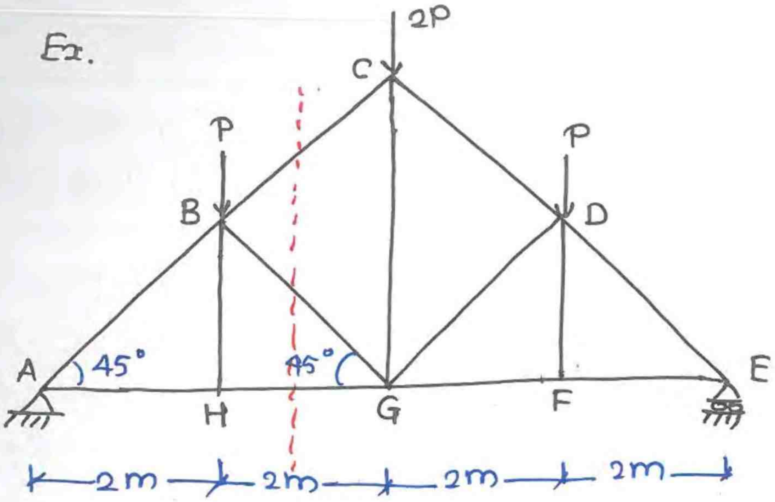
~~ii) Section should cut minimum number of members to calculate force in desired m-~~

ii) Section may cut any number of members.

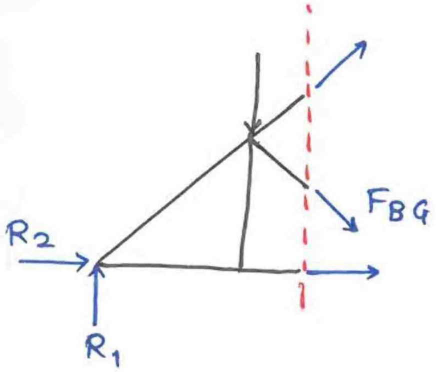
iii) Section should be such that force in desired member can be calculated using either of following equations.

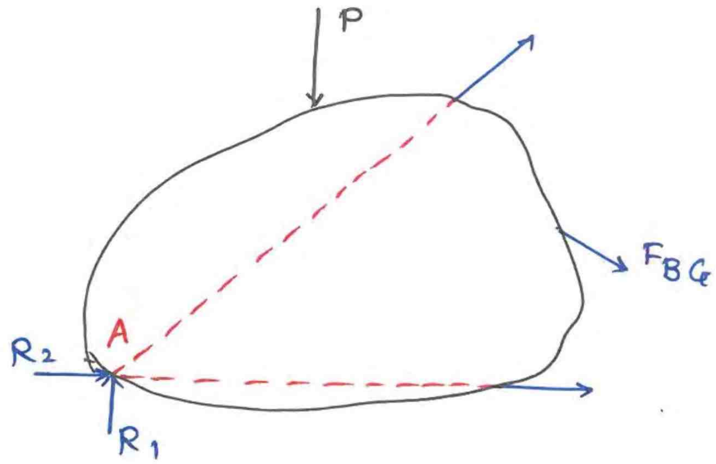
$$\Sigma F_x = 0, \ \Sigma F_y = 0, \text{ and } \Sigma M_z = 0.$$

Ex.



$F_{BG} = ?$





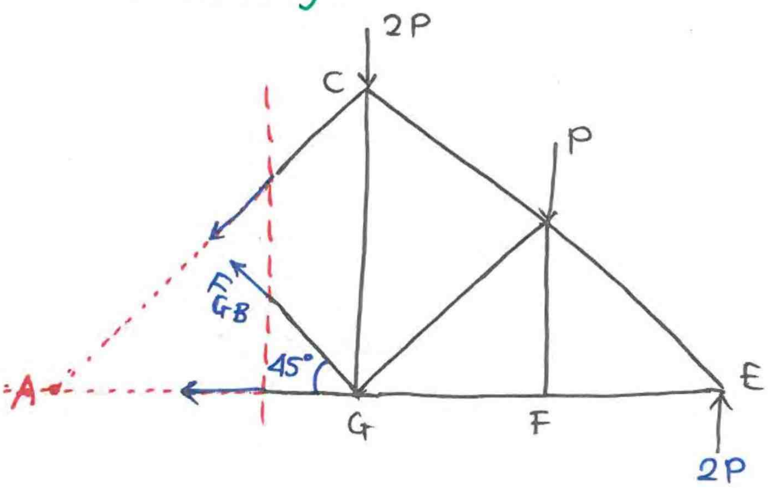
$$\Sigma M_z = 0$$

$$\Rightarrow \Sigma M_A = 0$$

$$\Rightarrow \quad P \times 2 + F_{BG} \times 2\sqrt{2} = 0$$

$$F_{BG} = \frac{-P}{\sqrt{2}}$$

Alternatively:



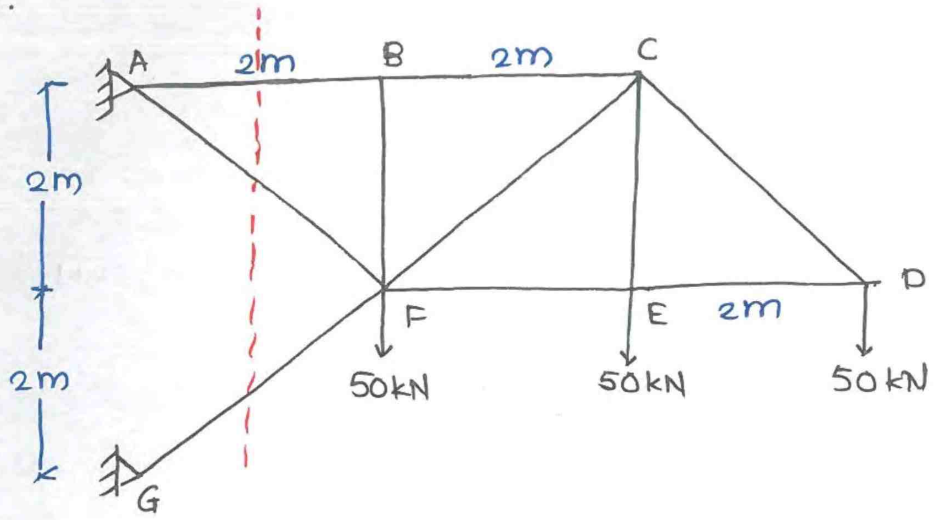
$$\Sigma M_z = 0$$

$$\Rightarrow \Sigma M_A = 0$$

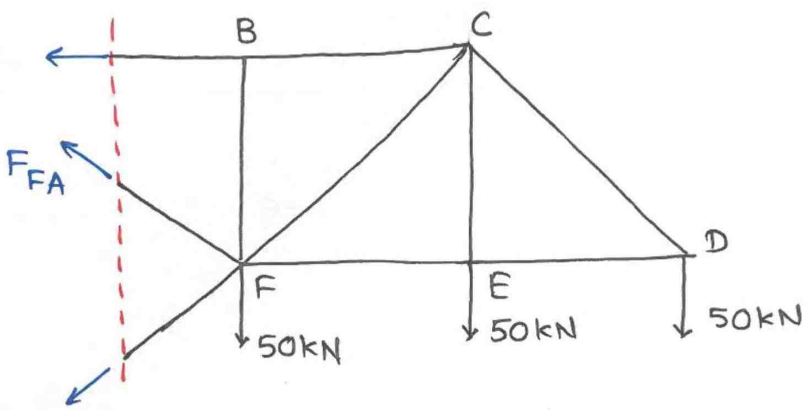
$$\Rightarrow \quad 2P \times 4 + P \times 6 - 2P \times 8 - (F_{GB} \sin 45°) \times 4 = 0$$

$$\Rightarrow \quad F_{GB} = -P/\sqrt{2}$$

Ex. 2.



$F_{AF} = ?$



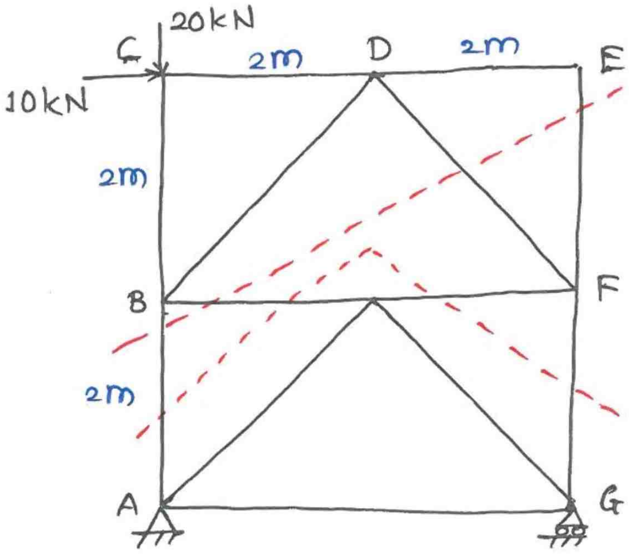
$\Sigma M_z = 0$

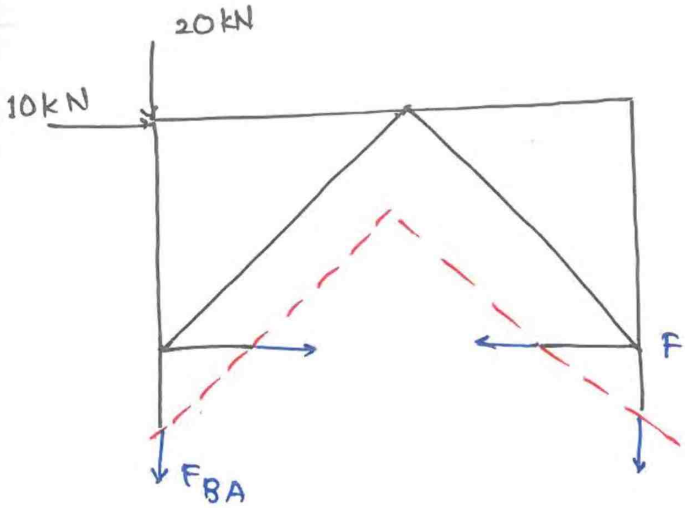
$\Rightarrow \Sigma M_C = 0$

$-50 \times 2 + 50 \times 2 + F_{FA} \times 2\sqrt{2} = 0$

$F_{FA} = 0$

Ex. 3.





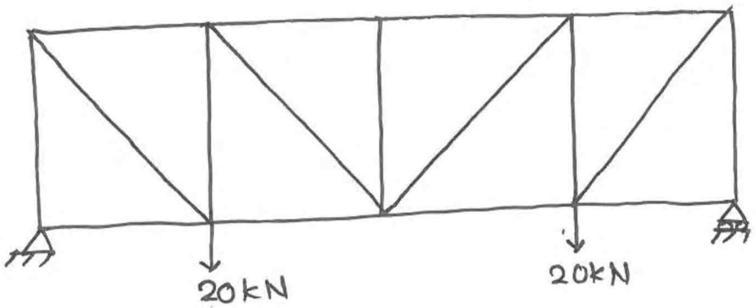
$\Sigma M_z = 0$

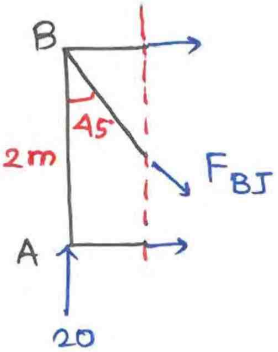
$\Rightarrow \Sigma M_F = 0$

$\Rightarrow -20\times4 + 10\times2 - F_{BA}\times4 = 0$

$F_{BA} = -15\,kN$

Ex.





$\Sigma F_y = 0$

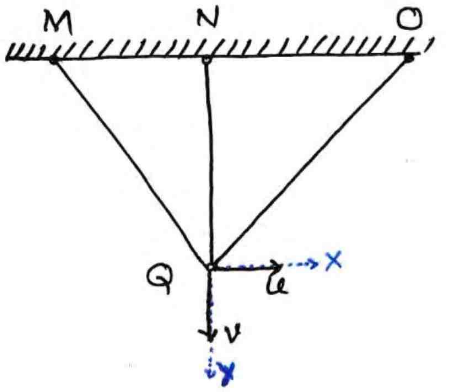
$20 - F_{BJ}\cos45^\circ = 0$

$F_{BJ} = 20\sqrt{2}\ kN$

* Note:

- If all unknown forces of FBD are parallel except a force which needs to be calculated then either $\Sigma F_x = 0$ or $\Sigma F_y = 0$ is used.
- If all unknown forces of FBD are concurrent at any point except a force which need to be calculated then $\Sigma M_z = 0$ is used.

Q4. In a redundant joint model, three bar members are pin connected at Q as shown in the figure. Under some load placed at Q, the elongation of members MQ and OQ are found to be 48mm and 35mm respectively. Then the horizontal displacement 'u' and the vertical displacement 'v' of the node Q, in mm, will be respectively



MN = 400mm

NO = 500mm

NQ = 500mm

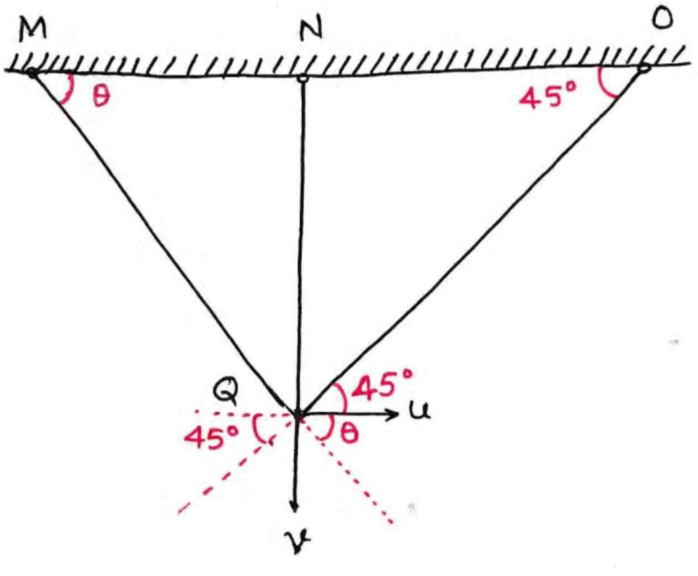
a) -6.64 and 56.14

b) 6.64 and 56.14

c) 0.0 and 59.41

d) 59.41 and 0.0

[2003 : 2 marks]



Taking components of u and v along members,

$$u\cos\theta + v\sin\theta = 48 \quad \cdots (i)$$

$$-u\cos 45^\circ + v\sin 45^\circ = 35 \quad \cdots (ii)$$

From (i) and (ii)

$u = 6.65$ mm

$v = 56.14$ mm.

$$\cos\theta = \frac{400}{\sqrt{400^2 + 500^2}}$$

$$\sin\theta = \frac{500}{\sqrt{400^2 + 500^2}}$$

4 Influence Line Diagram.

ILD

4.1. Need of ILD:

ILD is used for analysis of a structure if load changes its position over the structure.

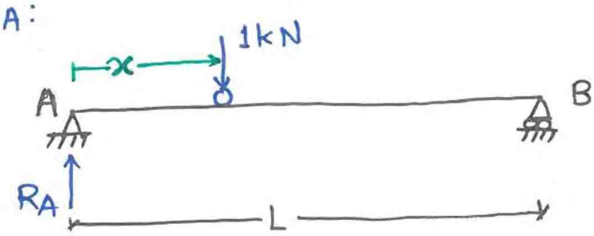
4.2 What is ILD?

It is a graphical representation of variation of reaction, shear force, bending moment, axial force, deflection etc. due to different positions of moving unit point load over span.

4.3 Derivation of ILD:

Considering a simply supported beam of span L.

For R_A:



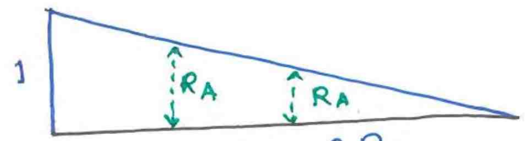
$$\Sigma M_B = 0$$

$$\Rightarrow R_A \times L - 1(L-x) = 0$$

$$\Rightarrow R_A = 1 - \frac{x}{L}$$

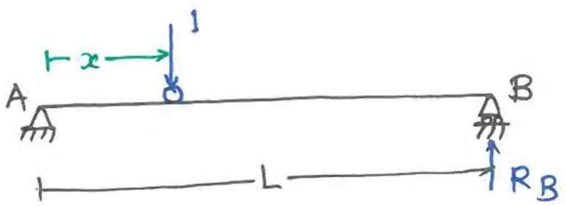
at $x=0$, $R_A = 1$

$x = L$, $R_A = 0$



ILD of R_A

For R_B:



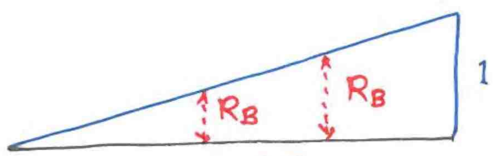
$$\Sigma M_A = 0$$

$$\Rightarrow -R_B \times L + 1 \times x = 0$$

$$R_B = \frac{x}{L}$$

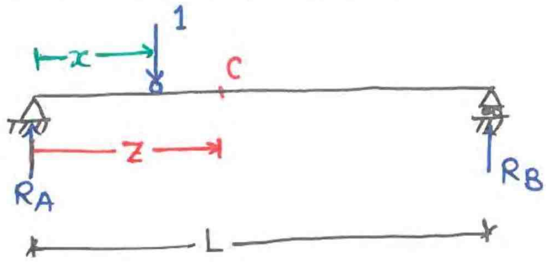
at $x=0$, $R_B = 0$

$x = L$, $R_B = 1$



ILD of R_B

• For SF_c :- case I: $0 \leq x < z$

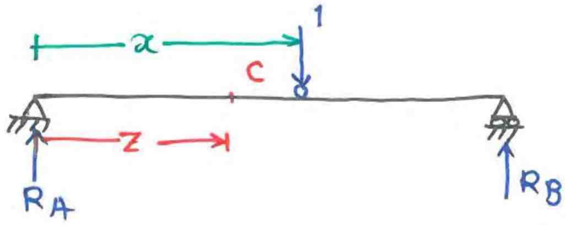


$$SF_c = R_A - 1$$
$$= \frac{L-x}{L} - 1$$
$$= -\frac{x}{L}$$

at $x = 0$, $SF_c = 0$

$x = z$, $SF_c = -\frac{z}{L}$

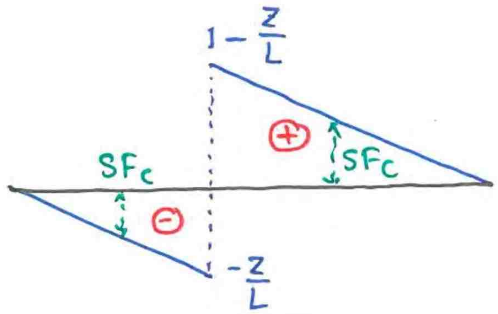
case II: $z < x \leq L$



$$SF_c = R_A$$
$$= \frac{L-x}{L}$$

at $x = z$, $SF_c = 1 - \frac{z}{L}$

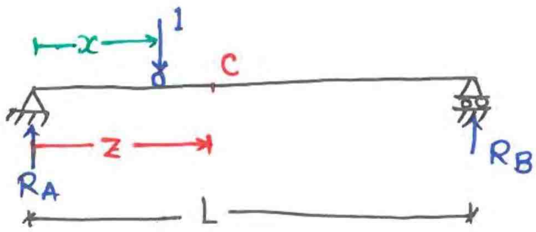
$x = L$, $SF_c = 0$



ILD of SF_c

• For BM_c :-

case I: $0 \leq x \leq z$

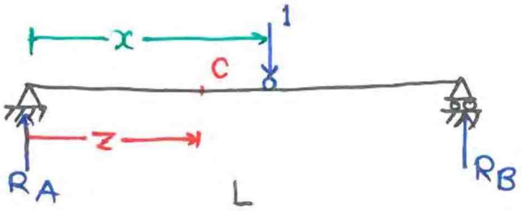


$$BM_c = R_A \times z - 1(z - x)$$
$$= \left(\frac{L-x}{L}\right) \times z - (z - x)$$
$$= \frac{x(L-z)}{L}$$

at $x = 0$, $BM_c = 0$

$x = z$, $BM_c = \frac{z(L-z)}{L} = \frac{ab}{L}$

case II: $z \leq x \leq L$

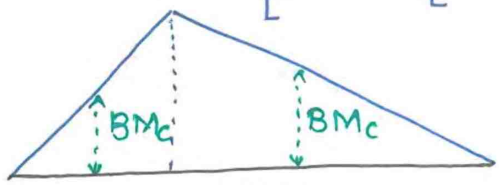


$$BM_c = R_A \times z$$
$$= \left(\frac{L-x}{L}\right) z$$

at $x = z$, $BM_c = \frac{(L-z)z}{L}$

$x = L$, $BM_c = 0$

$$\frac{z(L-z)}{L} = \frac{ab}{L}$$



ILD of BMc

- **Note:**

  Difference between SFD and ILD of SF at C.

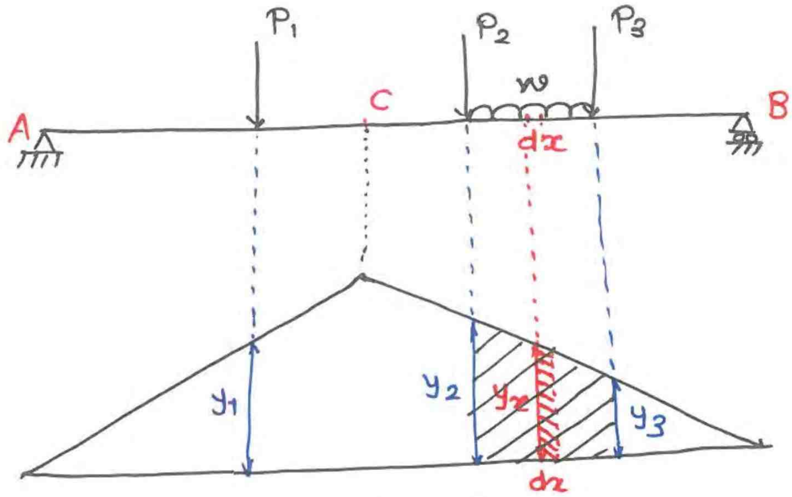
  - SFD:-
    It is the variation of shear force along a span due to particular position of load.
  - ILD of SF:-
    It is the variation of shear force at particular section due to different position of load along span.

4.4 Muller Breslau's Principle:

It states that ILD of any stress function (Reaction, SF, BM) is the deflected shape of a structure after removing a stress function from the structure and applying unit displacement (deflection or rotation) in the positive direction of stress function.

- **Note:**
  - It is valid for all types of statically determinate and linearly elastic indeterminate structure (truss, arch, frame, cable structure etc)
  - It is not valid for moving unit point moment.
  - Proof of this principle is by virtual work method.
  - ILD of deflection is not plotted by Muller Breslau's principle.

4.5. Application of ILD:



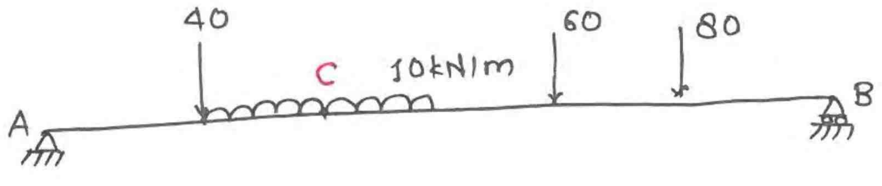
ILD of BM_C

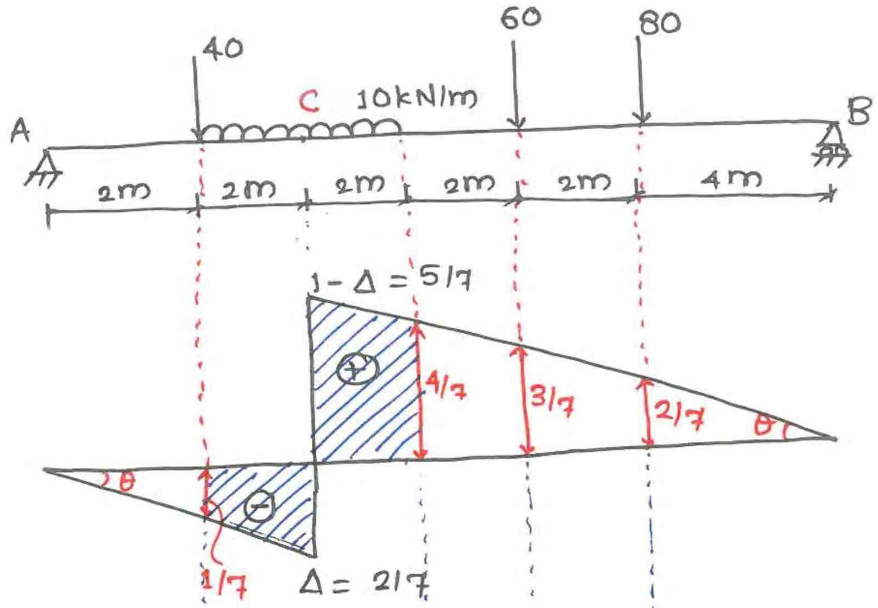
$$BM_C = P_1 y_1 + P_2 y_2 + P_3 y_3 + \Sigma(\omega dx)\cdot y_x$$

$$= P_1 y_1 + P_2 y_2 + P_3 y_3 + \omega \Sigma y_x \cdot dx$$

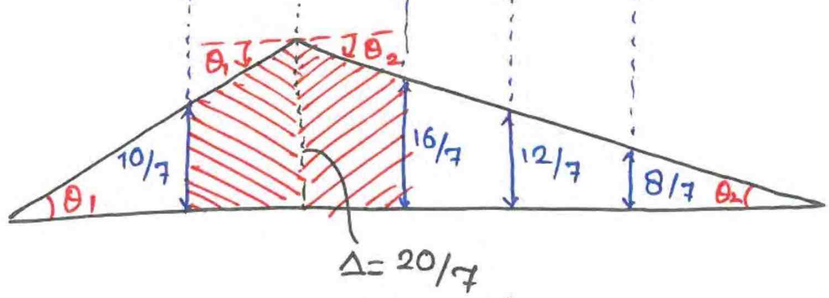
$$= P_1 y_1 + P_2 y_2 + P_3 y_3 + \omega \times \text{Shaded Area.}$$

Ex. Calculate SF and BM at C.





ILD of SF_c



ILD of BM_c.

From ILD of SF_c :-

$$\theta = \theta$$

$$\frac{\Delta}{4} = \frac{1-\Delta}{10}$$

$$\Delta = 2/7$$

$$SF_c = 40\left(-\frac{1}{7}\right) + 60\left(\frac{3}{7}\right) + 80\left(\frac{2}{7}\right) + 10 \times \frac{1}{2} \times 2\left(-\frac{1}{7} - \frac{2}{7}\right) + 10 \times \frac{1}{2} \times 2\left(\frac{5}{7} + \frac{4}{7}\right)$$

$$SF_c = 51.42\,kN$$

From ILD of BM_c :-

$$\Rightarrow \quad \theta_1 + \theta_2 = 1$$

$$\Rightarrow \quad \frac{\Delta}{4} + \frac{\Delta}{10} = 1 \qquad \Rightarrow \Delta = 20/7$$

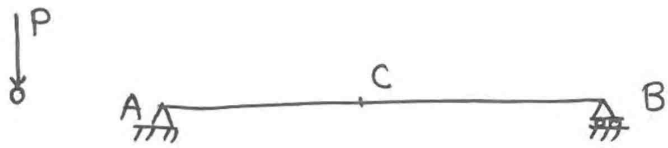
$$BM_C = 40\left(\frac{10}{7}\right) + 60\left(\frac{12}{7}\right) + 80\left(\frac{8}{7}\right) + 10 \times \frac{1}{2} \times 2\left(\frac{10}{7} + \frac{20}{7}\right)$$

$$+ 10 \times \frac{1}{2} \times 2\left(\frac{20}{7} + \frac{16}{7}\right)$$

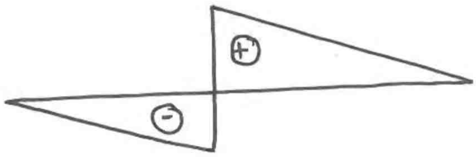
$BM_C = 345.7\,kNm$

4.6 Effect of Moving Load:

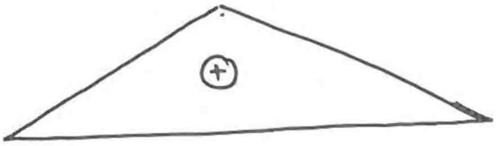
4.6.1 Single point load:



ILD of SF_C



ILD of BM_C

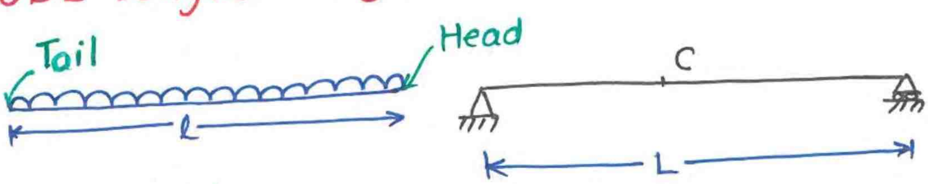


Max^m +ve SF_C = Load is placed just right to C

Max^m −ve SF_C = Load is placed just left to C.

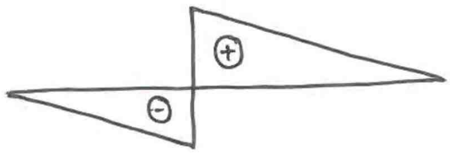
Max^m sagging BM_C = Load is placed at C.

4.6.2 UDL length longer than span:



$l > L$

ILD of SF_C



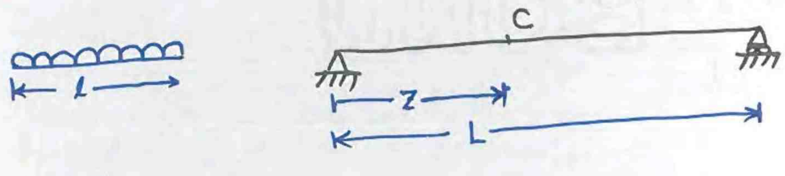
ILD of BM_C

Max^m +ve SF_c = Tail is placed just right to C.

Max^m -ve SF_c = Head is placed just Left to C.

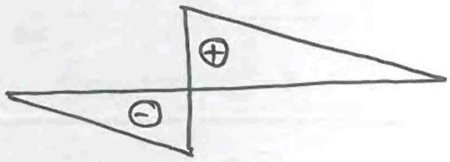
Max^m Sagging BM_c = Load is placed over entire span.

4.6.3 UDL length shorter than span:

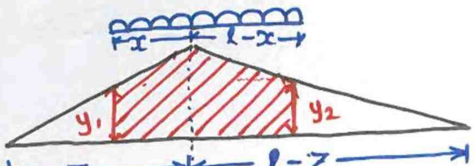


$l < L$

ILD of SF_c



ILD of BM_c



$\boxed{y_1 = y_2}$

Max^m +ve SF_c = Tail is placed just right to C

Max^m -ve SF_c = Head is placed just left to C.

Max^m sagging BM_c = Load is placed in such a way that section should divide load length and span in equal ratio.

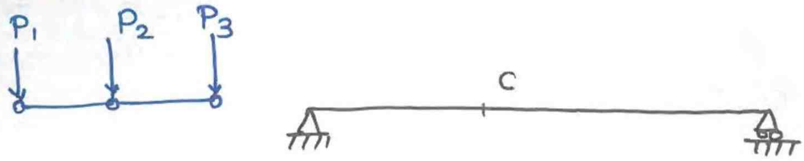
$$\frac{x}{l-x} = \frac{z}{L-z}$$

$$\Rightarrow \frac{x}{z} = \frac{l-x}{L-z}$$

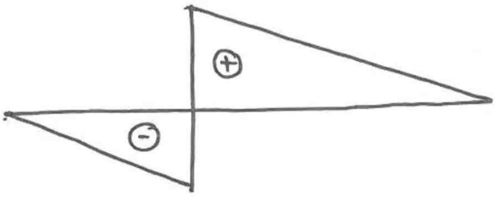
$$\Rightarrow \frac{\omega x}{z} = \frac{\omega(l-x)}{L-z}$$

$\Rightarrow$ Avg. Load on Left side of the section = Avg. Load on Right side of the section.

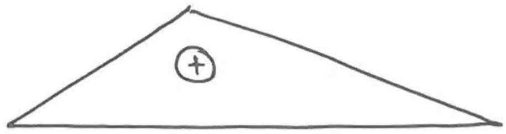
4.6.4 Series of Point Load:

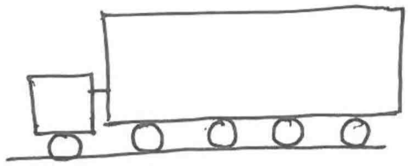


ILD of SF_c



ILD of BM_c



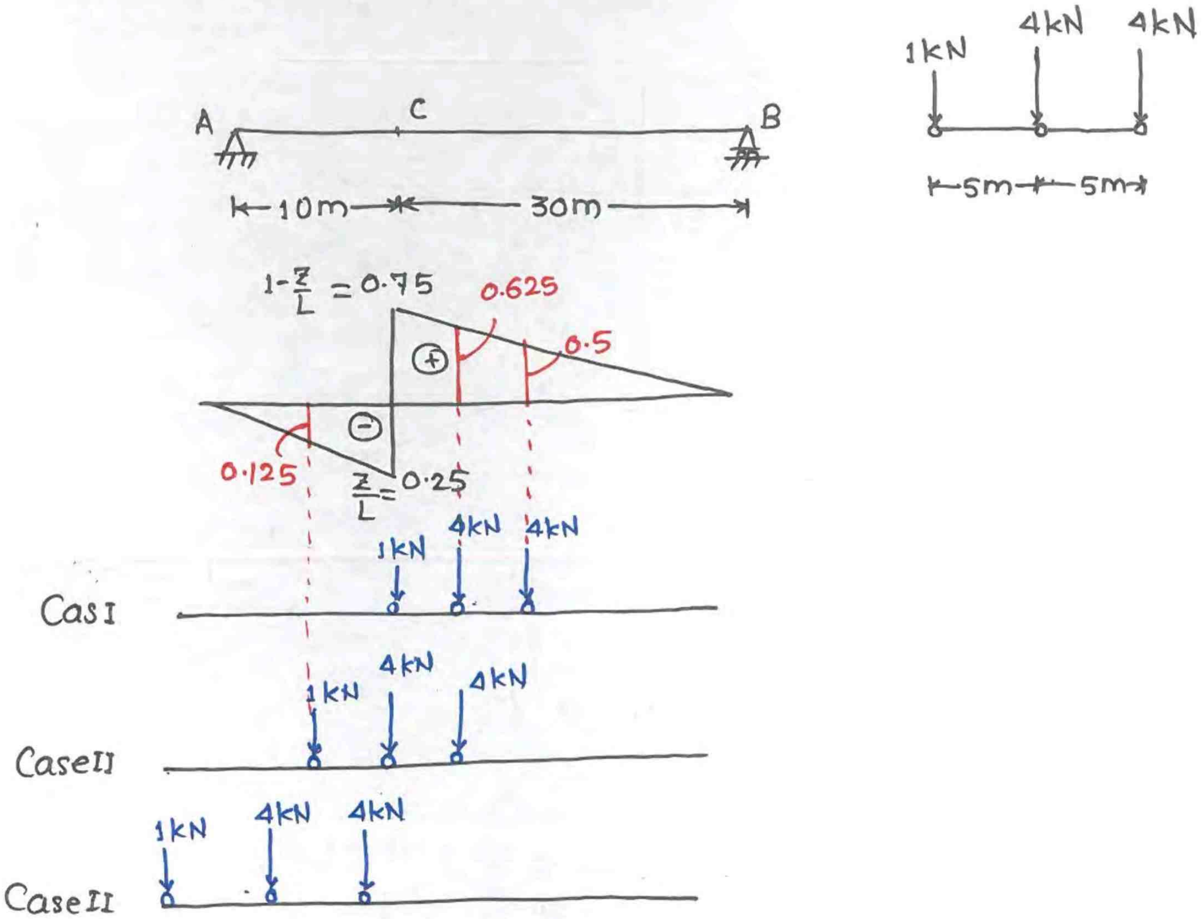


Max^m +ve SF_c = By Trials

Max^m −ve SF_c = By Trials

Max^m sagging BM_c = It can be achieved by placing equal avg. load on either side of section but this cannot be done in the case of series of point load so trials are performed corresponding to approx equal avg. load on either side of section. In all trials, atleast one point load must be placed exactly at section

Ex. A simply supported beam is subjected to series of point load moving from right to left as shown in fig. Calculate maximum +ve SF and maximum sagging BM at 10m from left support.



- Case I: 1 kN is just right to C.

$$SF_c = 1 \times (0.75) + 4 \times (0.625) + 4 \times (0.5) = 5.25 \text{kN}$$

- Case II: Middle 4kN is just right to C.

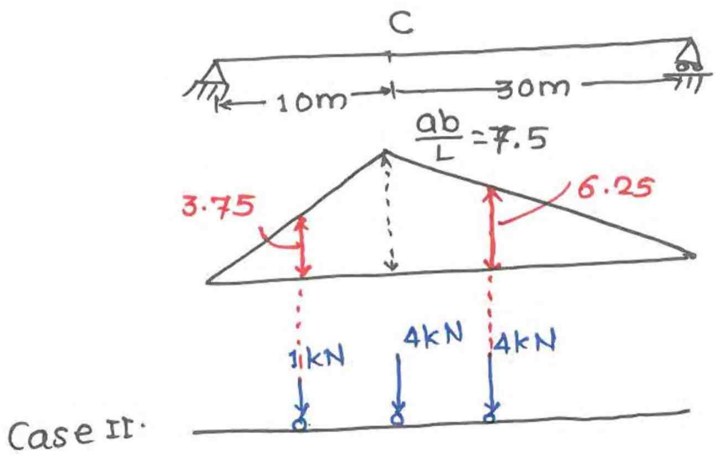
$$SF_c = 1 \times (-0.125) + 4 \times (0.75) + 4 \times (0.625) = 5.375 \text{ kN}$$

- Case III: Rear 4 kN is just right to C.

$$SF_c = 1 \times (0) + 4 \times (-0.125) + 4 \times (0.75) = 2.5 \text{kN}$$

so maximum +ve SF_c is 5.375 kN corresponding to case II.

By visual inspection, we can say that case II will give maximum sagging BM_c.

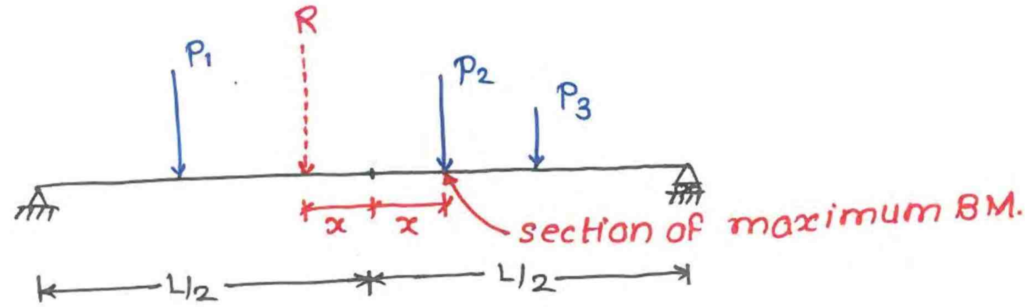


Case II: Middle 4 kN is just right to C.

$$BM_c = 1(3.75) + 4(7.5) + 4(6.25) = 58.75 \text{ kN}$$

4.7 Absolute Maximum Bending Moment:

If series of point load is moving over simply supported beam then absolute maximum BM is achieved by placing load in such a way that resultant of all point loads and one adjacent load should be at equidistance from centre of span and maximum BM is obtained just below to adjacent load considered.

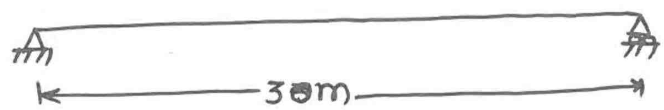


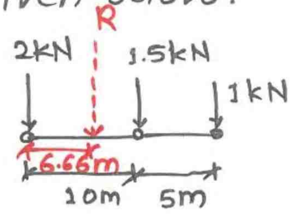
Case I: If resultant coincides with any point load then resultant is placed at centre of span and absolute maximum BM is at centre of span.

Case II: If resultant is closer to heavier adjustent load then, only heavier load is considered as adjucent load.

Case III: If resultant is closer to lighter load then calculation is done corresponding to both adjucent loads.

Ex. Determine absolute maximum BM for beam given below.





⇒

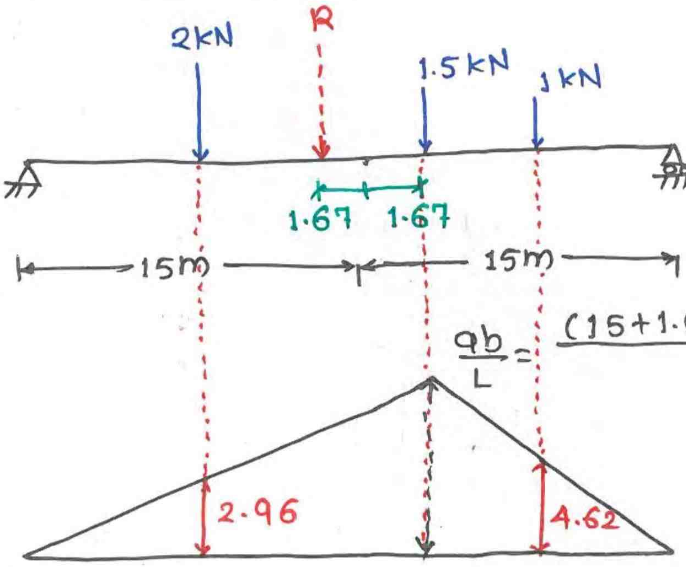
$R = 2 + 1.5 + 1 = 4.5\,kN$

Position of R from 2kN $= \dfrac{P_1 x_1 + P_2 x_2 + P_3 x_3}{P_1 + P_2 + P_3}$

$= \dfrac{2(0) + 1.5 \times (10) + 1 \times 15}{4.5}$

$= 6.66\,m$

Case I: Considering 1.5kN as adjucent load.

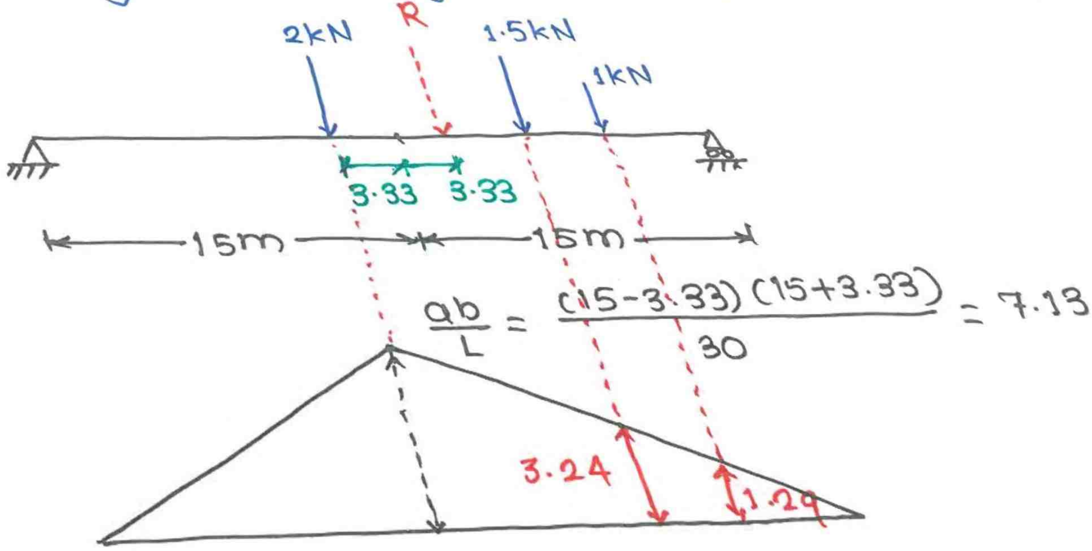


$\dfrac{ab}{L} = \dfrac{(15+1.67)(15-1.67)}{30} = 7.4$

$BM_{max} = 2(2.96) + 1.5(7.4) + 1(4.62)$

$= 21.64\,kNm.$

Case II: Considering 2kN as adjucent load.



$$BM_{max} = 2(7.1) + 1.5(3.24) + 1(1.29)$$

$$= 20.41 \text{ kNm}$$

So, absolute maximum BM is 21.64 kNm.

Statically indeterminate structure

5.2 Difficulty in Analysis:

Equations of equilibrium are not sufficient to analyze the structure so extra equations from compatibility conditions are formulated. This makes analysis of statically indeterminate structure relatively difficult.

5.3 Methods of Analysis:

Analysis of indeterminate structure (static and kinematic both) is done by following methods:

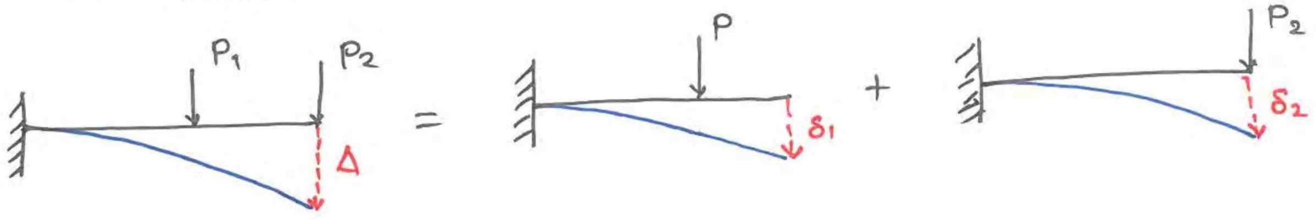
1) Force Method / Compatibility Method / Flexibility Method
2) Displacement Method / Equilibrium Method / Stiffness Method.

5.4 Difference between

Force Method	Displacement Method.
i) Forces are taken as unknown.	i) Displacements are taken as unknown.
ii) Compatibility conditions are used to calculate unknown forces.	ii) Equilibrium conditions are used to calculate unknown displacements.
iii) If DSI < KI then force method is preferable.	iii) If KI < DSI then displacement method is preferable.
iv) Methods: • Method of consistent Deformation • Strain Energy Method • 3-Moment Method / Clapeyron's Method. • Flexibility Matrix Method. • Column Analogy Method	iv) Methods: • Slope Deflection Method • Moment Distribution Method • Stiffness Matrix Method • Kani's Method.

5.5 Principle of Superposition:

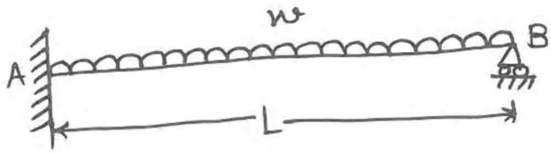
For linearly elastic structure, resultant BM, SF, Deflection, stress, strain etc due to multiple loadings is the algebraic sum of effect due to individual loading.



$$\Delta = \delta_1 + \delta_2$$

5.6 Method of Consistent Deformation:

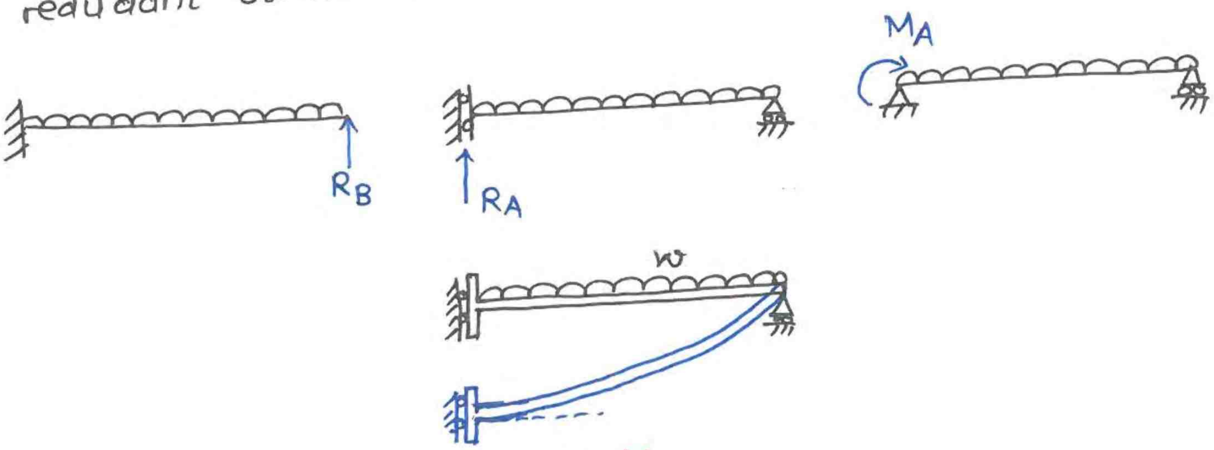
Considering a propped cantilever subjected to udl as given below,



Step I: Calculate DSI.

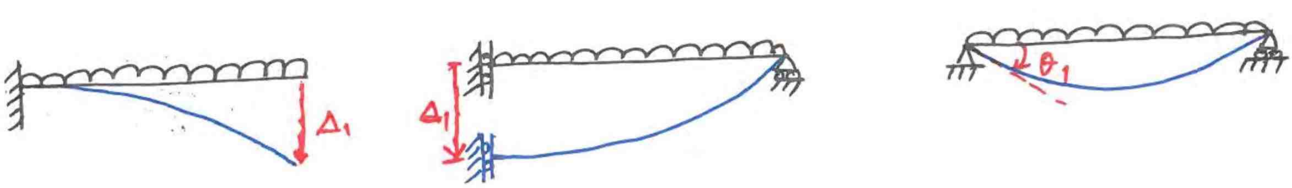
DSI = 1

Step II: Identify Redundants of the Structure. Redundant should be selected in such a way that after removal of redundant structure must remain stable.

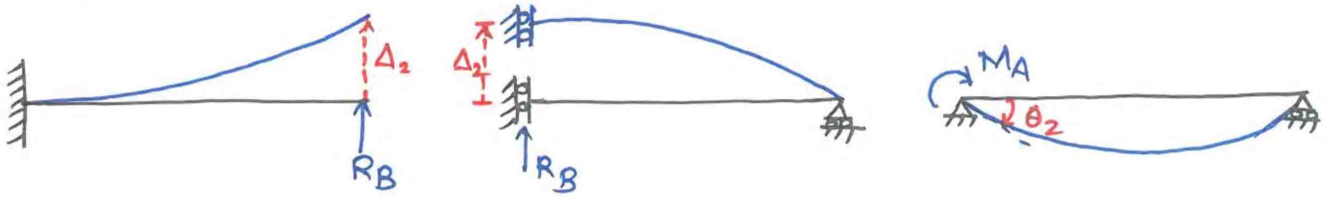


stable.

Step III: Remove all Redundants and make primary structure.



Step IV: Apply redundants on primary structure (without loading), one at a time.



Step V: Write compatibility equations corresponding to each redundant.

$$\Delta_B = 0$$

$$\Rightarrow \quad -\Delta_1 + \Delta_2 = 0$$

$$\Rightarrow \quad -\frac{wL^4}{8EI} + \frac{R_B L^3}{3EI} = 0$$

$$\Rightarrow \quad R_B = \frac{3}{8} wL$$

$$\Delta_A = 0$$

$$\Rightarrow -\Delta_1 + \Delta_2 = 0$$

No standard formula for Δ_1 & Δ_2 so it is not preferable to consider R_A as redundant.

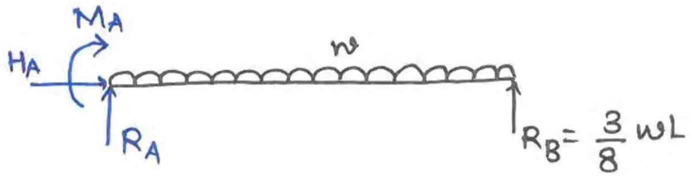
$$\theta_A = 0$$

$$\Rightarrow \quad \theta_1 + \theta_2 = 0$$

$$\Rightarrow \quad \frac{wL^3}{24EI} + \frac{M_A L}{3EI} = 0$$

$$\Rightarrow \quad M_A = -\frac{wL^2}{8}$$

Step VI: Calculate other reactions using equations of equilibrium.



$$\Sigma F_x = 0$$

$$\Rightarrow \quad H_A = 0 \text{ ---- (i)}$$

$$\Sigma F_y = 0$$

$$\Rightarrow \quad R_A + R_B - wL = 0$$

$$\Rightarrow \quad R_A + \frac{3}{8} wL - wL = 0$$

$$\Rightarrow \quad R_A = \frac{5}{8} wL \text{ --- (ii)}$$

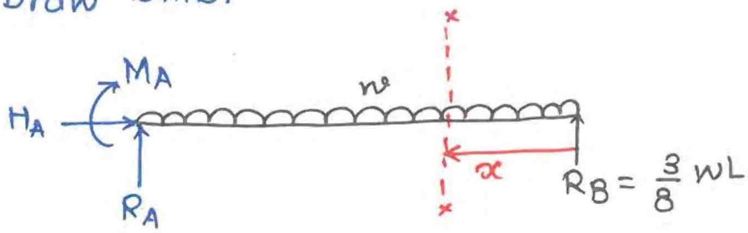
$$\Sigma M_z = 0$$

$$\Rightarrow \quad \Sigma M_A = 0$$

$$\Rightarrow \quad M_A + w \times L \times \frac{L}{2} - R_B \times L = 0$$

$$\Rightarrow \quad M_A = -\frac{wL^2}{8}$$

Step VII: Draw BMD:

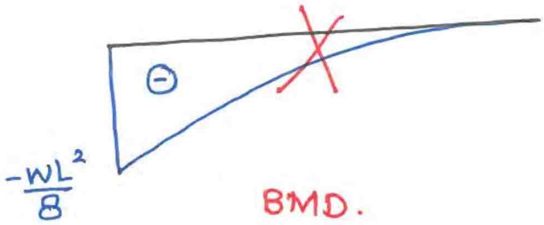


$$BM_x = R_B \cdot x - \frac{wx^2}{2}$$

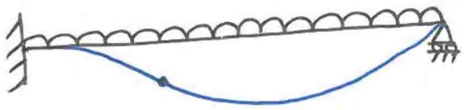
$$= \frac{3}{8} wL \cdot x - \frac{wx^2}{2}$$

At $x = 0$, $BM = 0$

$x = L$, $BM = -\frac{wL^2}{8}$



BMD.



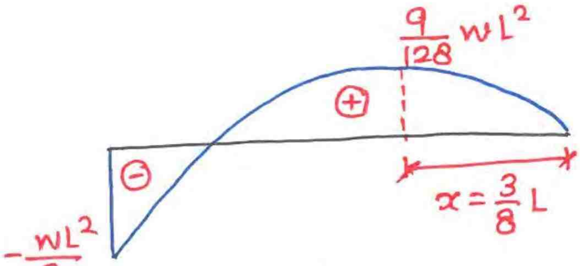
For maximum BM:-

$$\frac{d(BM_x)}{dx} = 0$$

$$\Rightarrow \frac{3}{8} wL - wx = 0$$

$$\Rightarrow x = \frac{3}{8} L$$

At $x = \frac{3}{8} L$, $BM = \frac{9}{128} wL^2$



BMD

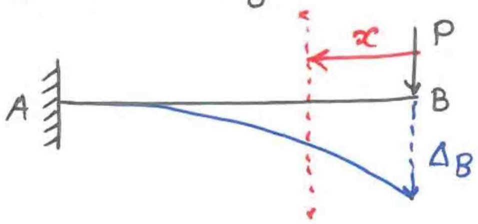
5.7 Principle of Virtual Work:

5.7.1 Derivation:

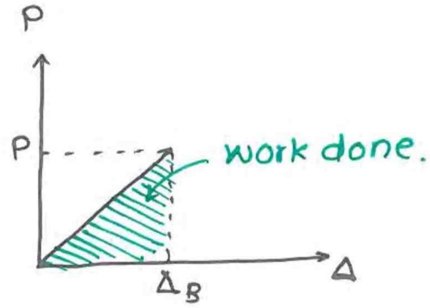
From Work · Energy Theorem

- External Workdone = Internal Workdone
- External Work done = Strain Energy.

Ex. Calculate Δ_B



External workdone = $\frac{1}{2} \times P \times \Delta_B$



Strain Energy = U

$$= \int \frac{M_x^2 \, dx}{2EI}$$

$$= \int_0^L \frac{(Px)^2 \, dx}{2EI}$$

$$= \frac{P^2}{2EI} \left[\frac{x^3}{3} \right]_0^L$$

$$= \frac{P^2 L^3}{6EI}$$

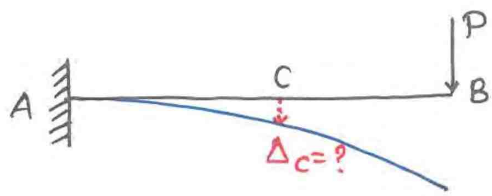
From Strain Energy Method:-

External Workdone = Strain Energy

$$\frac{1}{2} \cdot P \cdot \Delta_B = \frac{P^2 L^3}{6EI}$$

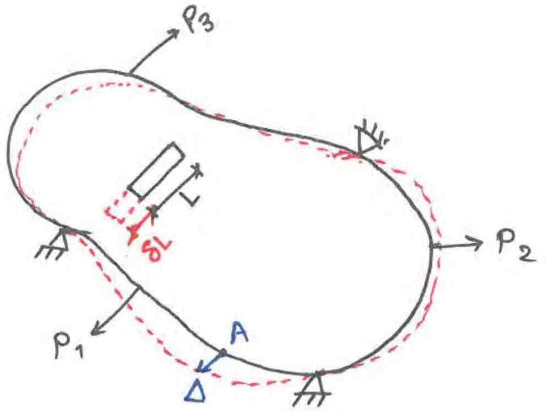
$$\Rightarrow \boxed{\Delta_B = \frac{PL^3}{3EI}}$$

Above procedure can be used if deflection is need to be calculated in the direction of applied force. It means, following example can not be solved by above procedure.



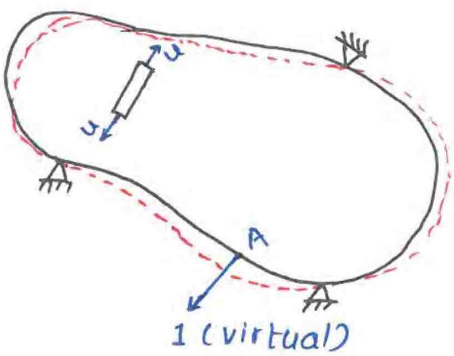
So this problem is solved using virtual work principle:-

Considering an arbitary elastic body subjected to P force system as given below



Work energy thereom cannot be used to calculate Δ because no term of external workdone comprises Δ.

To solve this problem, unit virtual load is applied on the arbitary body in the direction of Δ.



Now apply P force system on orbitary body which is already subjected to unit virtual load.

from principle of virtual work:

External virtual workdone = Internal virtual workdone

$$\boxed{1\cdot \overbrace{\Delta = \sum u\, \delta L}^{\text{real}}}$$

- Case I: Statically stable Elastic Body:

External virtual workdone = Internal Virtual Workdone.

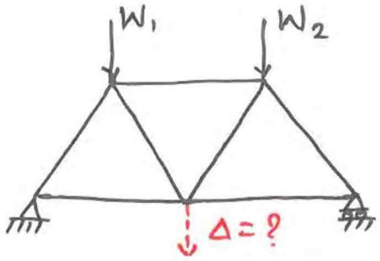
- Case II: Statically stable Rigid Body:

External virtual workdone = Internal virtual workdone = 0

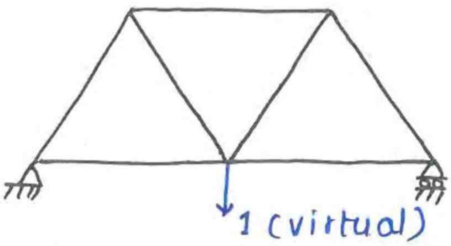
- Case III: Statically Unstable Elastic / Rigid Body:

External workdone ≠ Internal Workdon.

5.7.2 Principle of Virtual Work for Truss:-



Member force = P



Member force = k

from principle of virtual work.

$$\Rightarrow \quad 1 \cdot \Delta = \sum u \cdot \delta L$$

$$\Rightarrow \quad 1 \cdot \Delta = \sum k \frac{PL}{AE}$$

$$\Rightarrow \quad \boxed{\Delta = \sum \frac{kPL}{AE}}$$

Considering effect of temperature change and fabrication defect also.

$$\boxed{\Delta = \sum \frac{kPL}{AE} + \sum k(L\alpha T) + \sum k \cdot \delta}$$

T = +ve if temp. increases

δ = +ve if member is too long.

Procedure:

StepI: Calculate force in each member due to applied loading.(P

StepII: Apply unit load on the truss (without applied loading) in the direction of displacement need to be calculated and calculate force in each member. (k).

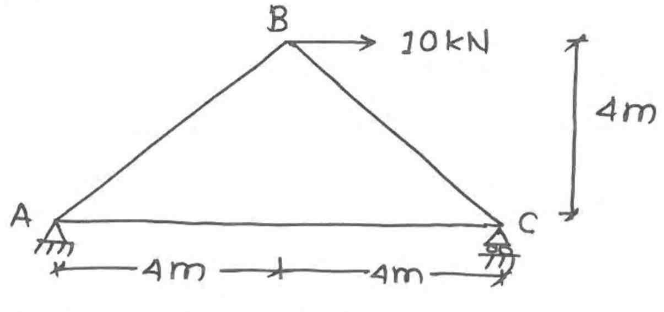
StepIII: Arrange all calculated values in tabular format as given below.

Member	P (kN)	K (kN)	L (m)	AE (kN)	$\frac{KPL}{AE}$	$K(L\alpha T)$	$K \cdot \delta$
					$\Sigma = ?$	$\Sigma = ?$	$\Sigma = ?$

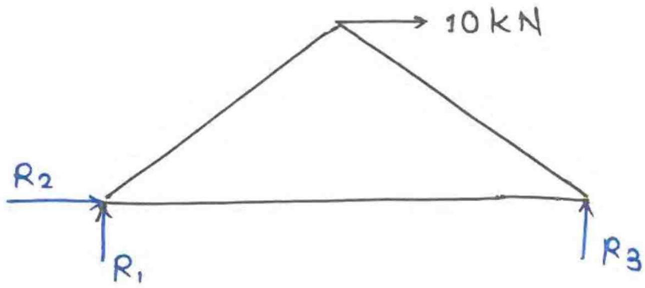
StepIV: Calculate deflection using expression given below.

$$\Delta = \Sigma \frac{KPL}{AE} + \Sigma k(L\alpha T) + \Sigma k \cdot \delta$$

Ex. Calculate vertical deflection of joint B. $A = 300\ mm^2$, $E = 200\ kN/mm^2$, $\alpha = 12 \times 10^{-6}/°C$, member AC is 5mm too short, increase in temp. of AB is 40°C.



⇒ StepI: Calculation of P

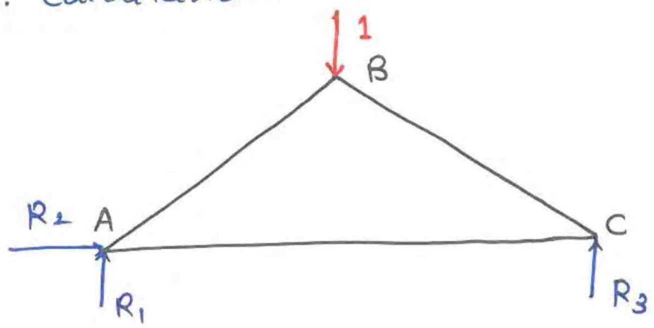


$F_{AB} = 7.07$ kN

$F_{BC} = -7.07$ kN

$F_{CA} = 5$kN

Step II: Calculation of k.



$F_{AB} = -0.707$ kN

$F_{BC} = -0.707$ kN

$F_{CA} = 0.5$ kN.

Step III:

Member	P(kN)	k(kN	L(m)	AE(kN)	$\frac{kPL}{AE}$	$k(L\alpha T)$	$k \cdot \delta$
AB	7.07	-0.707	$4\sqrt{2}$	6×10^4	-4.71×10^{-4}	-1.91×10^{-3}	0
BC	-7.07	-0.707	$4\sqrt{2}$		4.7×10^{-4}	0	0
CA	5	0.5	8		3.33×10^{-4}	0	0.5(-0.005)
				$\Sigma =$	3.33×10^{-4}	-1.91×10^{-3}	2.5×10^{-3}

$$\Delta = \Sigma \frac{kPL}{AE} + \Sigma k(L\alpha T) + \Sigma k \cdot \delta.$$

$$= 3.33\times10^{-4} - 1.91\times10^{-3} - 2.5\times10^{-3}$$

$$= -4.07\times10^{-3} \text{ m}$$

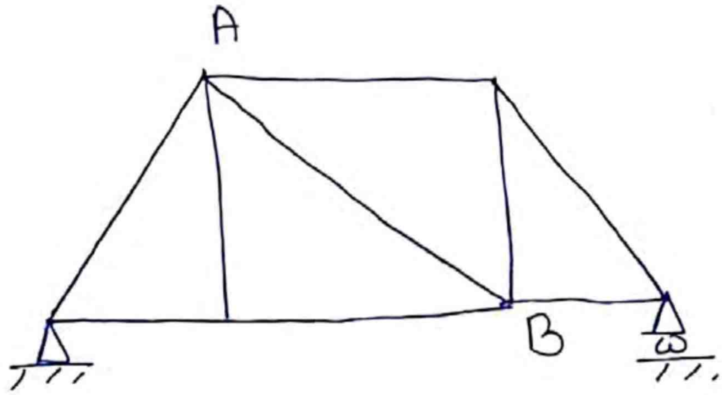
$$\boxed{\Delta = -4.07 \text{ mm}}$$

-ve value shows deflection opposite to the applied unit load. It means upward.

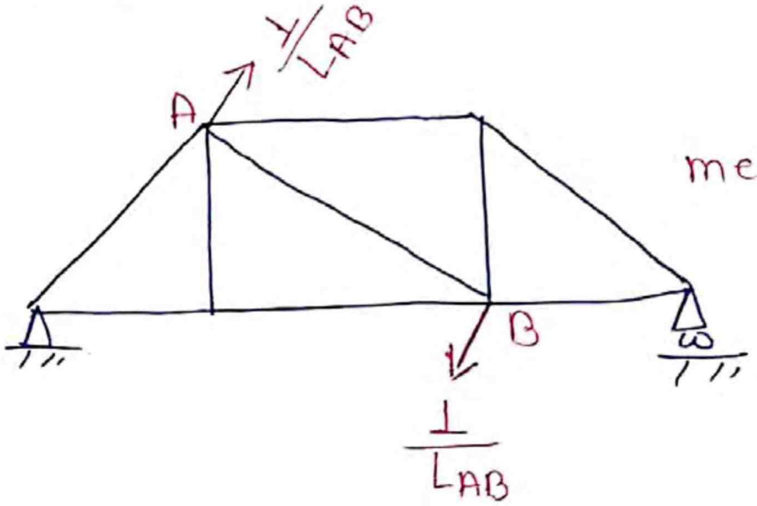
Case III :- If rotation of member is to be calculated then apply unit load divided by length of member, perpendicular to member as ~~Calculated~~ given below.

~~Calculation~~

Calculate rotation of member AB :



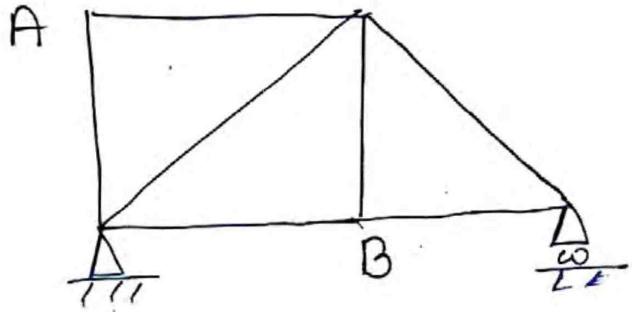
Applying unit load for member force (k).



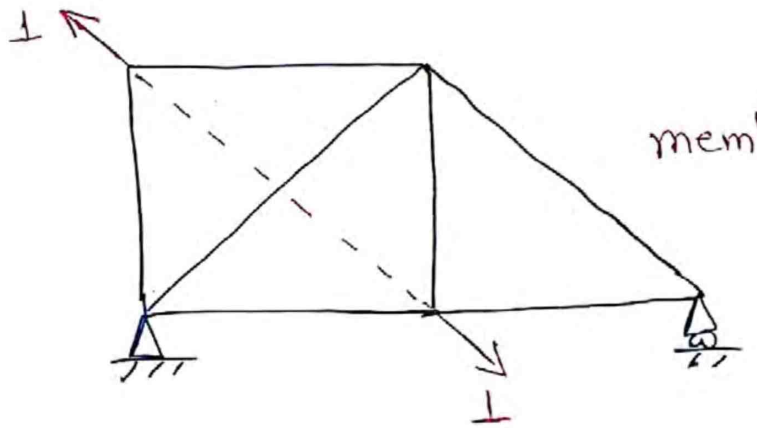
member force (K).

Case II :- If relative movement between two joints need to be calculated. then apply unit load at both joints together.

For example, calculate relative movement of joint AB in the direction of AB.



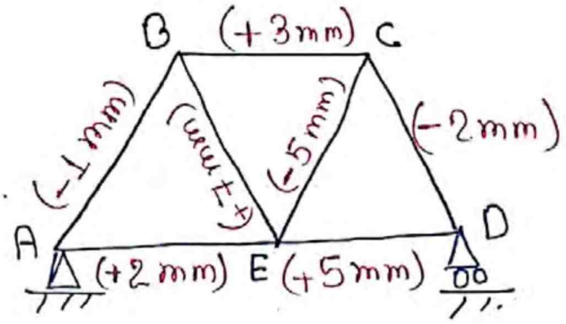
Applying unit virtual load as follows to Calculate force in member (K).



member force (k)

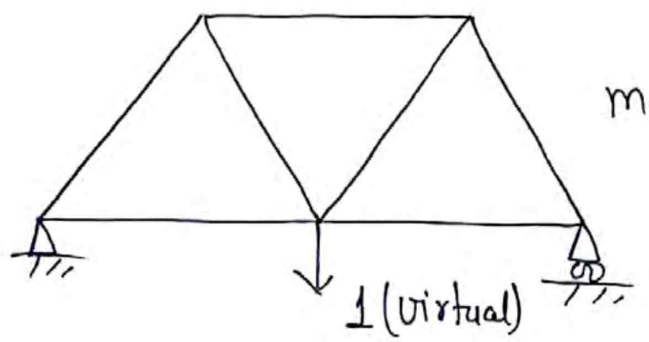
Calculation of Deflection of truss for different cases.

Case I :- If elongation of members due to applied loading is directly given.



Calculate vertical deflection at E.

Apply unit load at E and calculate force in each member. (K)

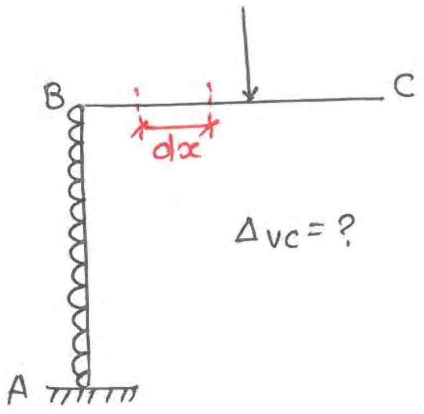


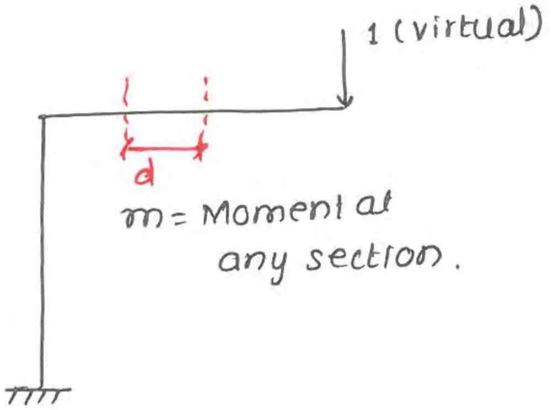
member force (K)

$$\Delta = \sum K\left(\frac{PL}{AE}\right)$$

This is elongation and Comp of member due to applied loading.

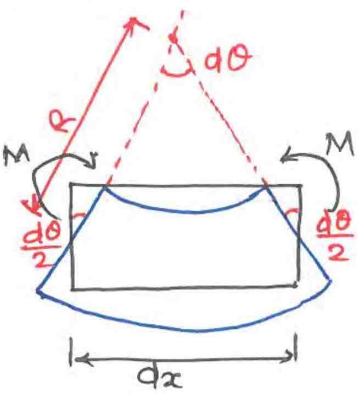
5.7.3 Principle of Virtual Work for Beams and Frames.





m = Moment at any section.

For dθ:–



$$d\theta = \frac{dx}{R}$$

$$\Rightarrow \frac{1}{R} = \frac{d\theta}{dx}$$

Now from flexure formula :-

$$\frac{M}{I} = \frac{E}{R}$$

$$\Rightarrow \frac{M}{I} = E\left(\frac{d\theta}{dx}\right)$$

$$\Rightarrow d\theta = \frac{M}{EI}\,dx$$

from principle of virtual work :-

$$1 \cdot \Delta = \Sigma u \cdot \delta L$$

$$\Rightarrow 1 \cdot \Delta = \int m \cdot d\theta$$

$$\boxed{\Delta = \int \frac{m\,M\,dx}{EI}}$$

For Rotation:-

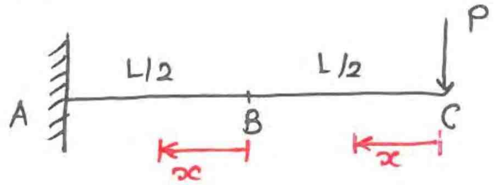
$$\boxed{\theta = \int \frac{mMdx}{EI}}$$

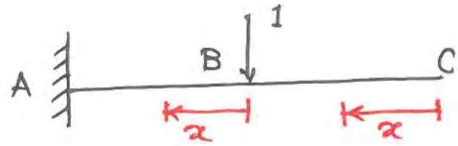
m= Moment at any section due to unit force/moment

M= Moment at any section due to applied loading.

m&M= +ve if clockwise/sagging/comp. on ref. face.

Ex. Calculate Δ_B and θ_B





⇒

For CB:-

M= Px (+ve becoz clockwise)

For BA:-

$M = P\left(\frac{L}{2}+x\right)$ (+ve becoz clockwise)

For CB:-

m=0

For BA:-

$m = 1\cdot x$ (+ve becoz clockwise)

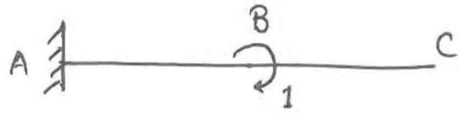
$$\Delta = \int \frac{mMdx}{EI}$$

$$= \underbrace{\int_0^{L/2} \frac{mMdx}{EI}}_{CB} + \underbrace{\int_0^{L/2} \frac{mMdx}{EI}}_{BA}$$

$$= \int_0^{L/2} \frac{0\,(Px)dx}{EI} + \int_0^{L/2} \frac{x\left[P\left(\frac{L}{2}+x\right)\right]dx}{EI}$$

$$\boxed{\Delta_B = \frac{5PL^3}{48EI}}$$

For θ_B :-



For CB :-

$m = 0$

For BA :-

$m = 1$

$$\theta = \int \frac{m M dx}{EI}$$

$$= \int_0^{L/2} \frac{m M dx}{EI} + \int_0^{L/2} \frac{m M dx}{EI}$$

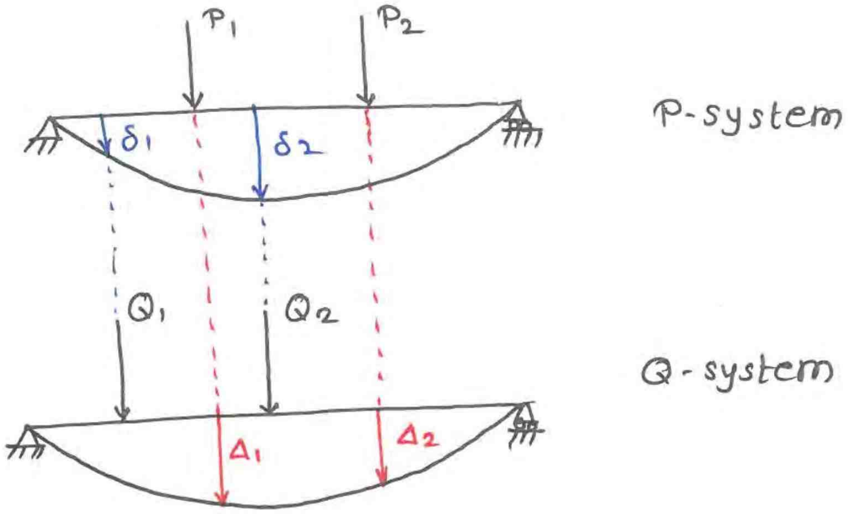
$$= \int_0^{L/2} \frac{0 \times (Px) dx}{EI} + \int_0^{L/2} \frac{1 \cdot P\left(\frac{L}{2} + x\right) dx}{EI}$$

$$= \frac{P}{EI}\left[\frac{L}{2}x + \frac{x^2}{2}\right]_0^{L/2}$$

$$\boxed{\theta_B = \frac{3PL^2}{8EI}}$$

5.8 Betti's Law:

Virtual workdone by P force system in going through deformation of Q force system is equal to virtual workdone by Q force system in going through deformation of P force system.



from Betti's Law:-

$$P_1\Delta_1 + P_2\Delta_2 = Q_1\delta_1 + Q_2\delta_2$$

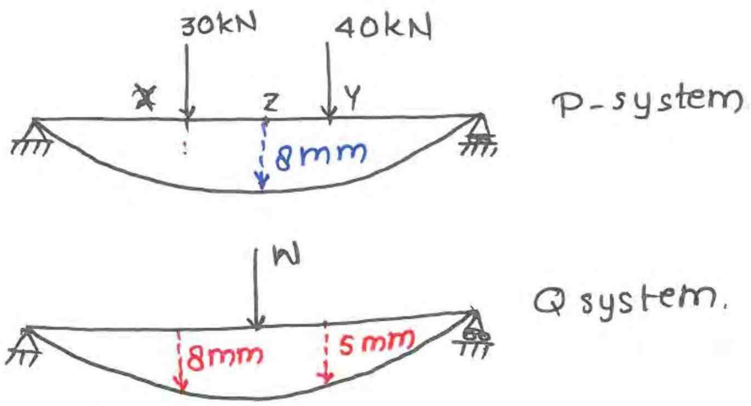
$$\Rightarrow \quad \Sigma P\Delta = \Sigma Q\delta$$

Δ = Displacement by Q system corresponding to P-system

δ = Displacement by P-system corresponding to Q-system.

Ex. The beam given below produces deflection of 8 mm at Z. To produce deflection of 8 mm and 5 mm at x and y respectively, load required at z would be

a) 20 kN b) 40 kN c) 55 kN d) 80 kN.



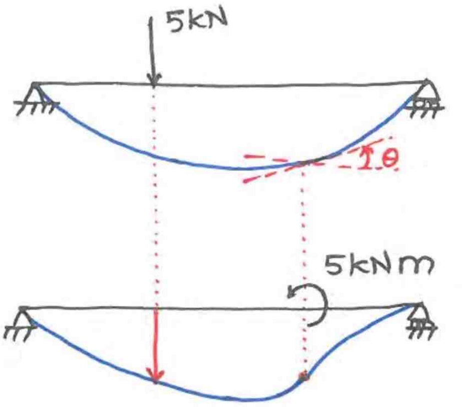
from Betti's Law :-

$$P_1\Delta_1 + P_2\Delta_2 = Q_1\delta_1$$

$$30\times 8 + 40\times 5 = W\times 8$$

5.9 Maxwell's Reciprocal Theorem:-

It is the special case of Betti's law where a single force/moment in P-force system and a single force/moment in Q-force system of equal magnitude are present.



from Betti's law:-

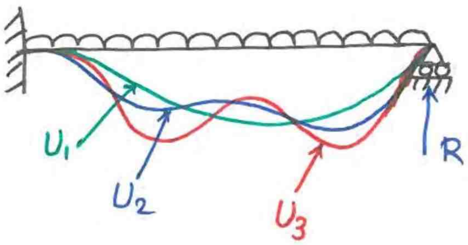
$$P_1\Delta_1 = Q_1\delta_1$$

$$\Rightarrow 5\times\Delta = 5\times\theta$$

$$\Rightarrow \Delta = \theta$$

5.10 Theorem of Least Work / Strain Energy Method.

In any statically indeterminate structure the redundant should be such as total internal energy of a structure is minimum.



U_1 is minimum.

$$\frac{\partial U}{\partial R} = 0$$

$$\Rightarrow \frac{\int \frac{M^2 dx}{2EI}}{\partial R} = 0$$

$$\Rightarrow \int \frac{M\frac{\partial M}{\partial R}dx}{EI} = 0$$

In case of settlement of support:-

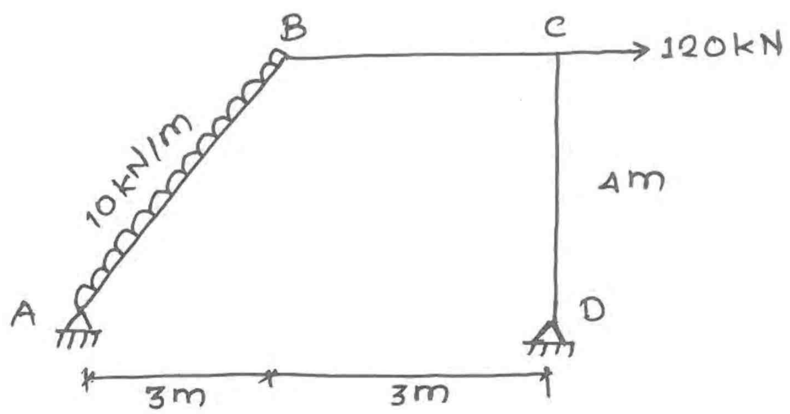
$$\frac{\partial U}{\partial R} = \Delta$$

$$\int \frac{M\frac{\partial M}{\partial R}dx}{EI} = \Delta$$

M = +ve if clockwise /sagging/ comp on ref. face

Δ = +ve if along R

Ex. Analyze the given frame using strain energy method. Horizontal settlement of support D is $\frac{10}{EI}$ towards right.

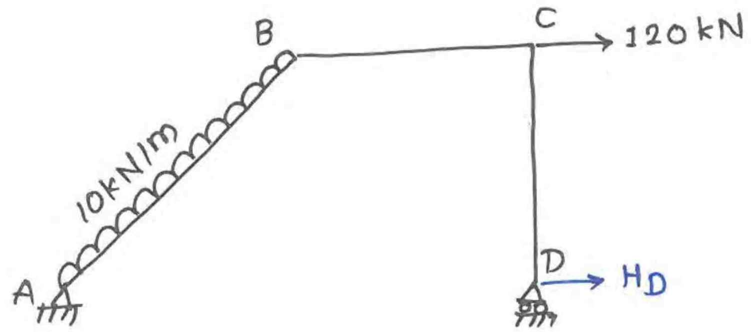


Step I: Calculate DSI.

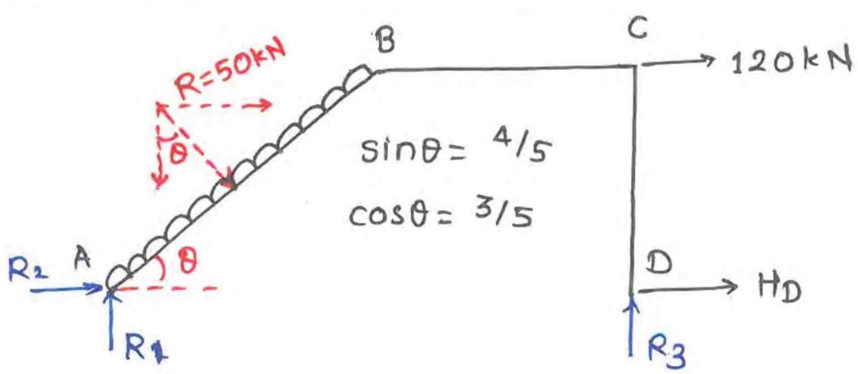
DSI = 1

Step II: Identify redundants.

In this case, redundant must be in the direction of settlement of support to consider the effect of settlement of support at D as redundant. So considering horizontal reaction at D as redundant.



Step III: Calculate other support reactions



$\Sigma F_x = 0$

$\Rightarrow R_2 + H_D + 120 + 50\sin\theta = 0$

$\Rightarrow R_2 = -H_D - 160 \quad \cdots (i)$

$$\Sigma F_y = 0$$

$$\Rightarrow R_1 + R_3 - 50\cos\theta = 0$$

$$\Sigma M_z = 0$$

$$\Rightarrow \Sigma M_A = 0$$

$$\Rightarrow 50 \times \frac{5}{2} + 120\times 4 - R_3 \times 6 \quad \text{---(iii)}$$

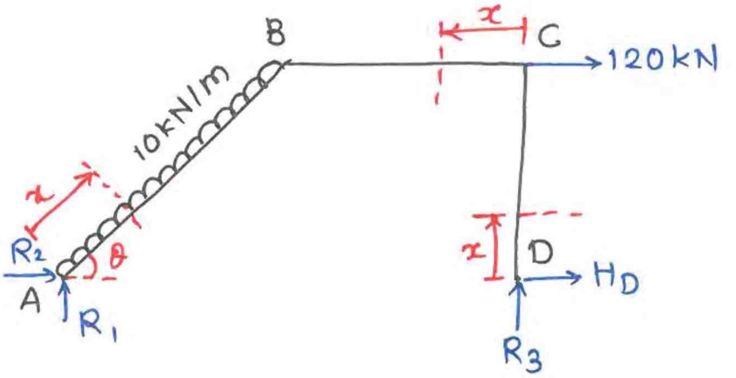
from eq^n (i), (ii) and (iii)

$$R_1 = -70.83 \text{ kN}$$

$$R_2 = -H_D - 160$$

$$R_3 = 100.83 \text{ kN}$$

Step IV: Make strain energy of structure minimum.



For DC:-

$M = -H_D \cdot x$ (-ve becoz anticlockwise)

$$\frac{\partial M}{\partial H_D} = -x$$

For CB:-

$$M = -R_3 \cdot x - H_D \times 4$$

$$= -100.83x - H_D \times 4$$

$$\frac{\partial M}{\partial H_D} = -4$$

For AB:-

$M = R_1\, x\cos\theta - R_2\, x\sin\theta - \frac{wx^2}{2}$ (+ve if clockwise)

$$= -70.83 \times x \times \frac{3}{5} - (-H_D - 160)\times x \times \frac{4}{5} - \frac{10x^2}{2}$$

$$= 85.51x - 5x^2 + 0.8 H_D x$$

$$\frac{\partial M}{\partial H_D} = 0.8x$$

$$\frac{\partial U}{\partial H_D} = \Delta_D$$

$$\int_0^4 \frac{M \frac{\partial M}{\partial H_D} dx}{EI} + \int_0^3 \frac{M \frac{\partial M}{\partial H_D} dx}{EI} + \int_0^5 \frac{M \frac{\partial M}{\partial H_D} dx}{EI} = \frac{10}{EI}$$ (+ve becoz along H_D)

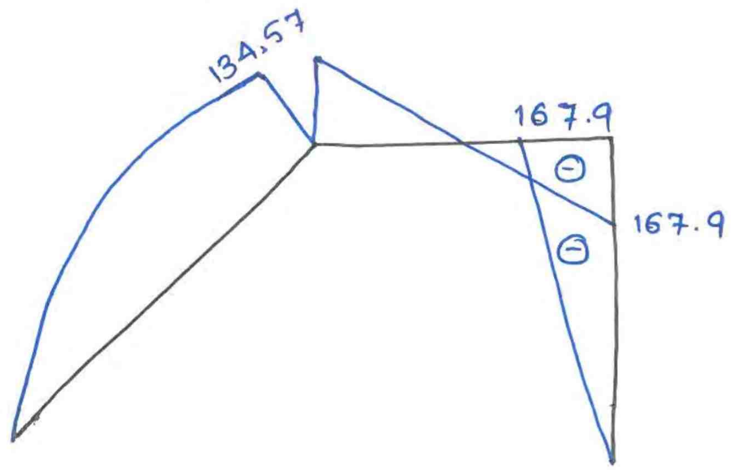
$$\Rightarrow \int_0^4 (-H_D x)(-x)\,dx + \int_0^3 (-100.83x - 4H_D)(-4)\,dx$$

$$+ \int_0^5 (85.51x - 5x^2 + 0.8H_D x)(0.8x)\,dx = 10$$

$$\Rightarrow \quad H_D = -41.98 \text{ kN}$$

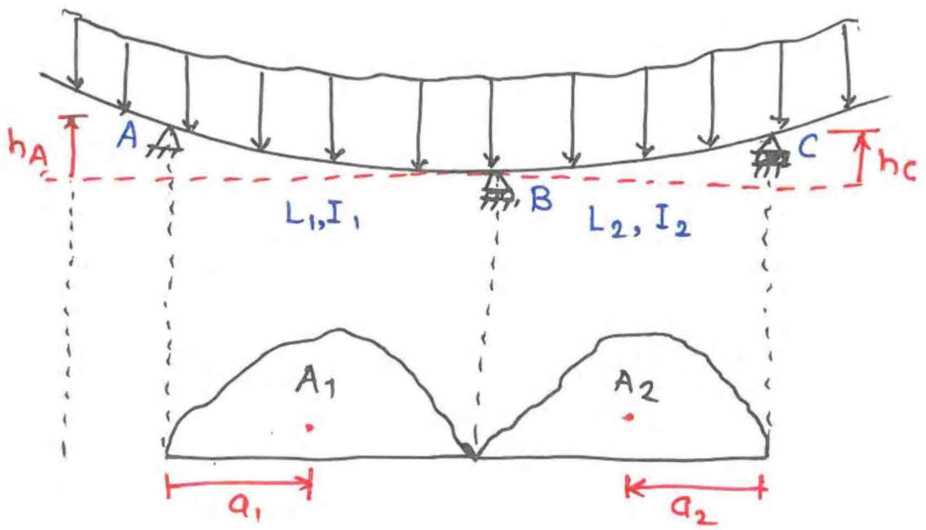
Step V: BMD :-

Draw BMD as discussed in section 1.16.



5.11. Three-Moment Equation.

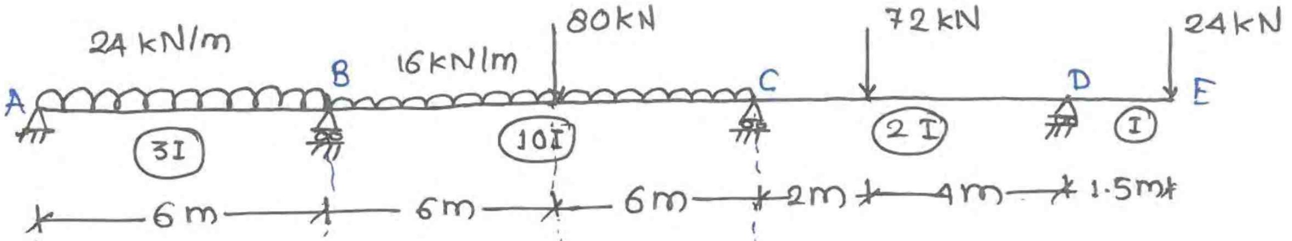
This method is used to analyse fixed and continuous beams



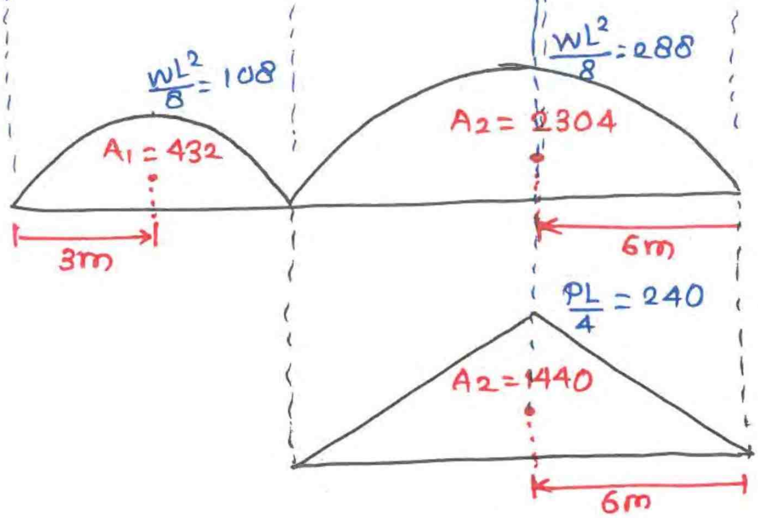
$$M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right) = \frac{-6A_1a_1}{L_1I_1} - \frac{6A_2a_2}{L_2I_2} + \frac{6Eh_A}{L_1} + \frac{6Eh_C}{L_2}$$

M_A, M_B, M_C = These are BM so +ve if sagging

Ex.



DSI = 2 so Two equations are required to analyze the structure.

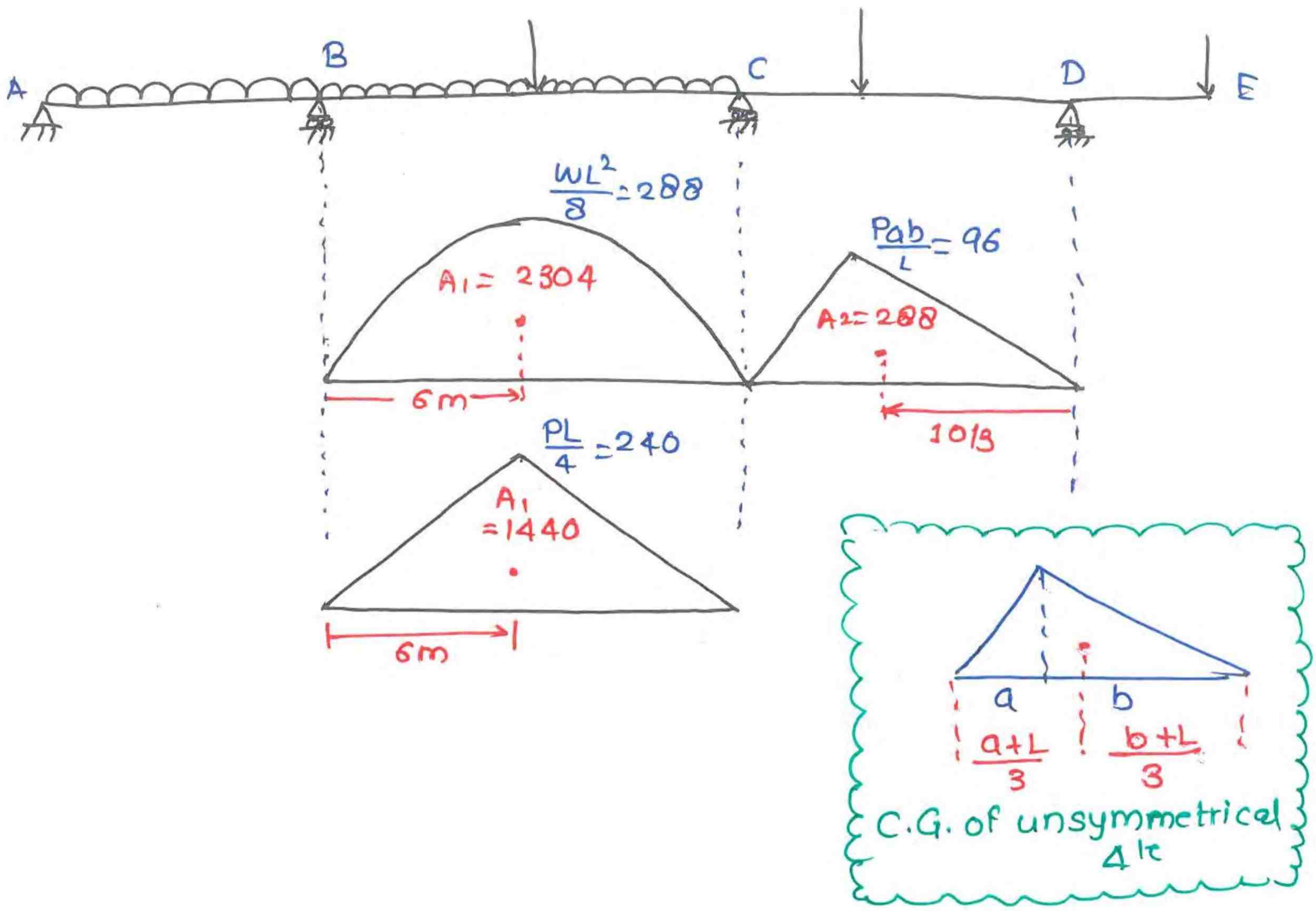


For AB and BC:

$$M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right) = \frac{-6A_1a_1}{L_1I_1} - \frac{6A_2a_2}{L_2I_2}$$

$$\Rightarrow 0\left(\frac{6}{3I}\right) + 2M_B\left(\frac{6}{3I} + \frac{12}{10I}\right) + M_C\left(\frac{12}{10I}\right) = \frac{-6\times432\times3}{6\times3I} - \left(\frac{6\times2304\times6}{12\times10I} + \frac{6\times1440\times6}{12\times10I}\right)$$

$$\Rightarrow 6.4M_B + 1.2M_C = 1555.2 \quad ------(1)$$



For BC & CD :-

$$M_B\left(\frac{L_1}{I_1}\right) + 2M_C\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_D\left(\frac{L_2}{I_2}\right) = \frac{-6A_1 a_1}{L_1 I_1} - \frac{6A_2 a_2}{L_2 I_2}$$

$$\Rightarrow \quad M_B\left(\frac{12}{10I}\right) + 2M_c\left(\frac{12}{10I} + \frac{6}{2I}\right) + (-36)\left(\frac{6}{2I}\right) = -\left(\frac{6\times 2304\times 6}{12\times 10I} + \frac{6\times 1440\times 6}{12\times 10I}\right) - \frac{6\times 288\times 10/3}{6\times 2I}$$

{ $M_D = -36$ kNm
-ve becoz hogging }

$$\Rightarrow \quad 1.2M_B + 8.4M_c = -1495.2 \quad \text{---(ii)}$$

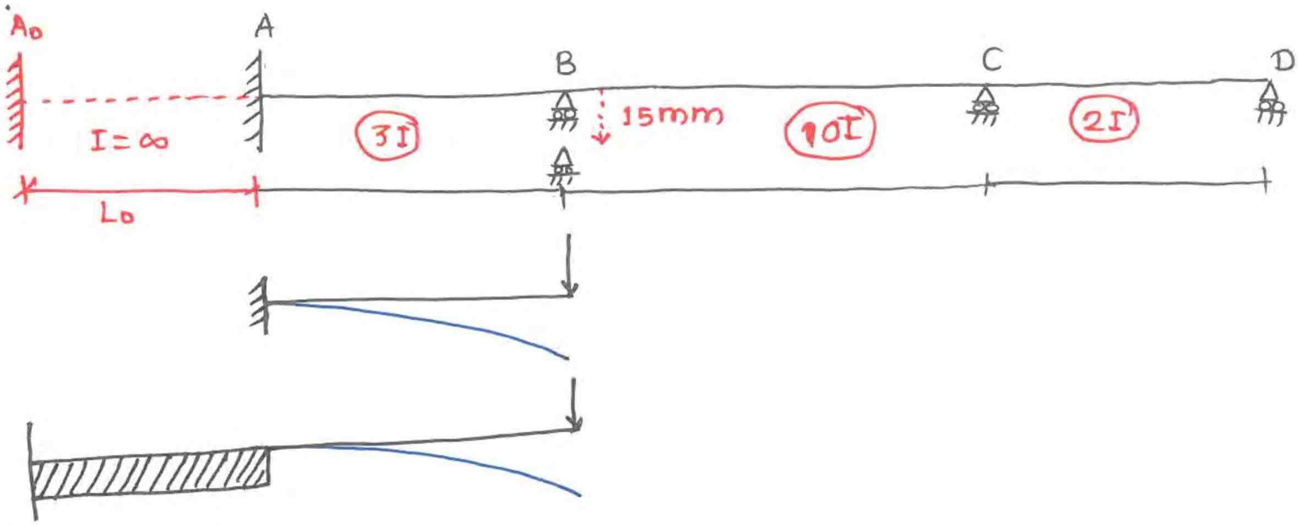
from eq^n (i) and (ii)

$M_B = -215.39$ kNm

$M_c = -147.22$ kNm

-ve sign indicates hogging BM.

Ex.2.



DSI = 3

so three equations are required to analyze the structure.

For A_0A and AB:-

$$M_{A_0}\left(\frac{L_1}{I_1}\right) + 2M_A\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_B\left(\frac{L_2}{I_2}\right) = \frac{6Eh_{A_0}}{L_1} + \frac{6Eh_B}{L_2}$$

$$\Rightarrow M_{A_0}\left(\frac{L_0}{\infty}\right) + 2M_A\left(\frac{L_0}{\infty} + \frac{6}{3I}\right) + M_B\left(\frac{6}{3I}\right) = \frac{6E \times 0}{L_0} + \frac{6E(-0.015)}{6}$$

$$\Rightarrow \quad 2M_A + M_B = -600 \quad \cdots (i)$$

For AB and BC

$$M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right) = \frac{6Eh_A}{L_1} + \frac{6Eh_C}{L_2}$$

$$\Rightarrow M_A\left(\frac{6}{3I}\right) + 2M_B\left(\frac{6}{3I} + \frac{12}{10I}\right) + M_C\left(\frac{12}{10I}\right) = \frac{6E(0.015)}{6} + \frac{6E(0.015)}{12}$$

$$\Rightarrow \quad M_A + 3.2M_B + 0.6M_C = 900 \quad \cdots (ii)$$

For BC and CD

$$M_B\left(\frac{L_1}{I_1}\right) + 2M_C\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_D\left(\frac{L_2}{I_2}\right) = \frac{6Eh_B}{L_1} + \frac{6Eh_D}{L_2}$$

$$\Rightarrow M_B\left(\frac{12}{10I}\right) + 2M_C\left(\frac{12}{10I} + \frac{6}{2I}\right) + M_D\left(\frac{6}{2I}\right) = \frac{6E(-0.015)}{12} + \frac{6E \times 0}{6}$$

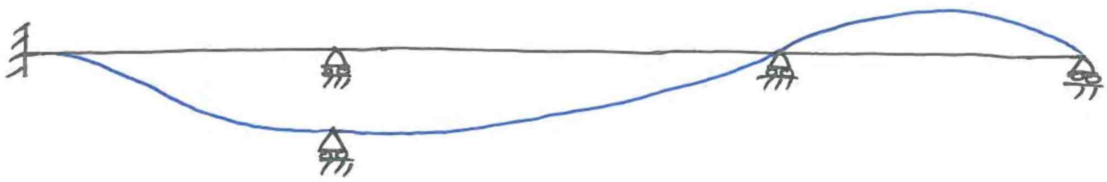
$$\Rightarrow \quad 0.6M_B + 4.2M_C = -300 \quad \cdots (iii)$$

from equation (i), (ii) and (iii)

$M_A = -537.7$ kNm

$M_B = 475.4$ kNm

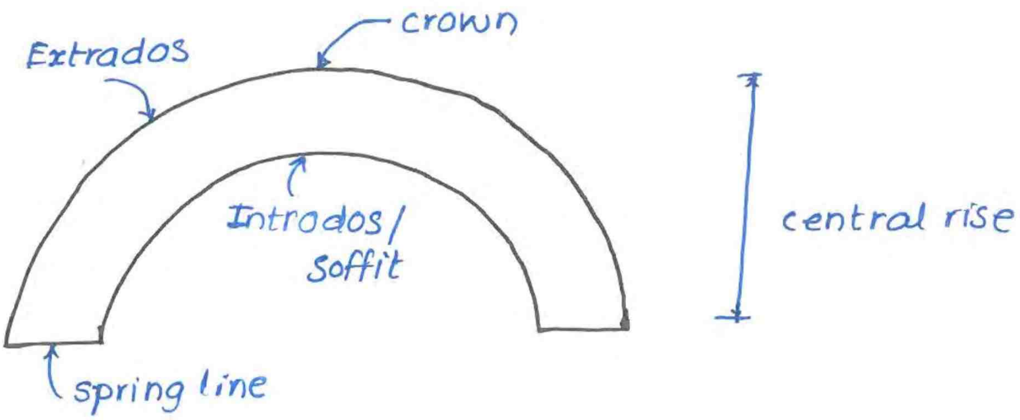
$M_C = -139.34$ kNm.



Arch

8.1 Introduction:

Arches are structural member which is predominantly subjected to axial compressive force and restricted for lateral movement at supports.



8.2 Reasons to construct Arch:

- Better use of materials which are strong in compression and weak in tension because arches are predominantly subjected to axial compression.
- For large span.
- For better aesthetic.
- For cultural representation.

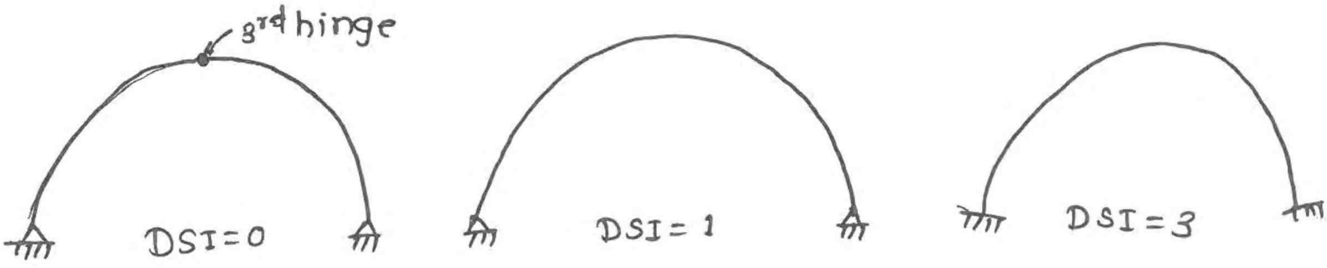
8.3 Classification of Arch:

A) Based on Architectural Point of View:-

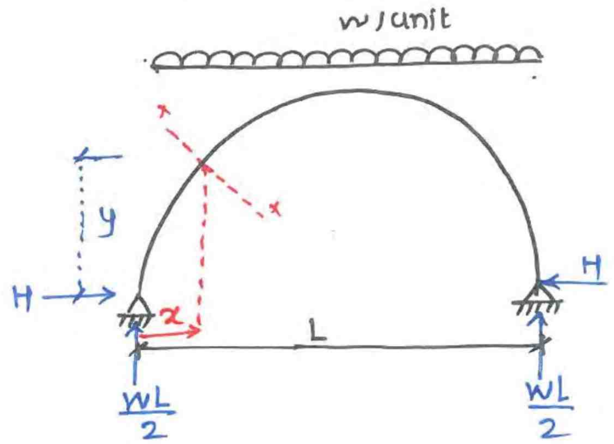
1) Parabolic
2) Circular
3) Elliptical
4) Any other shape.

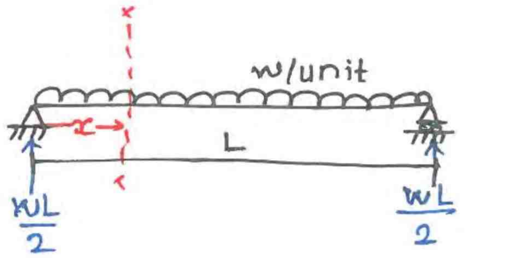
B) Based on Structural Design Point of View:-

1) 3-hinged Arch.
2) 2-hinged Arch.
3) Fixed Arch



8.4 Bending Moment in Arch:



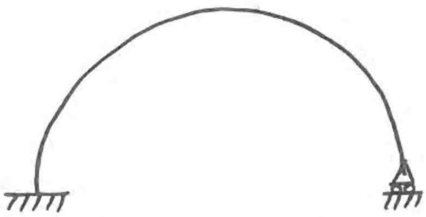


$$BM_{arch} = \underbrace{\frac{WL}{2}x - \frac{wx^2}{2}} - Hy \qquad BM_{ss.} = \frac{WL}{2} \cdot x - \frac{wx^2}{2}$$

$$BM_{arch} = BM_{ss} - Hy$$

• Conclusion:-

It means lateral reactions are mandatory for being an arch because it reduces net bending moment at any section.

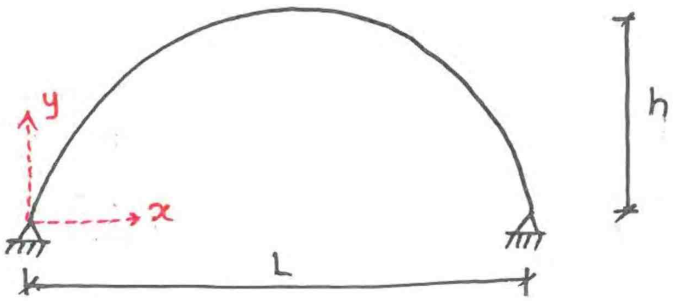


Beam curved in Elevation

Arch.

8.5 Analysis of 3-Hinged Arch:

8.5.1 3-Hinged Parabolic Arch:-

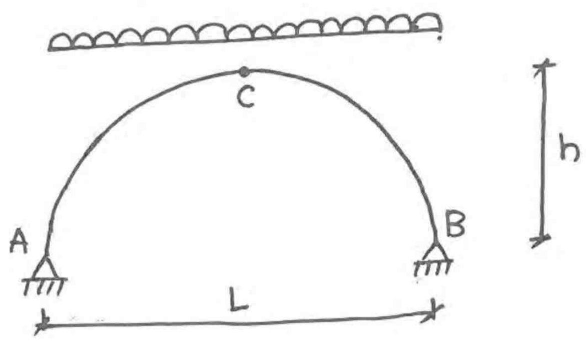


Equation of parabolic profile

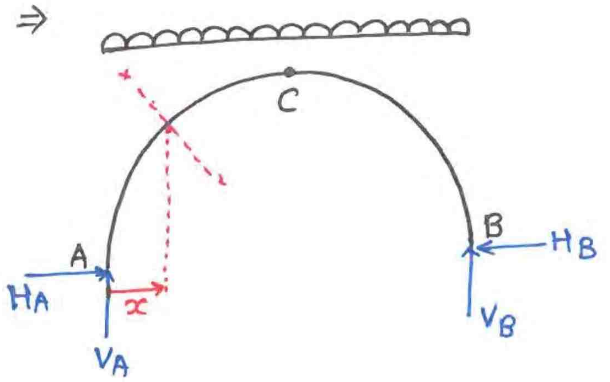
$$y = \frac{4h}{L^2} x (L - x)$$

- Origin is at left support
- Y-axis is upward.
- L is span of equal level supports.

Ex. Calculate horizontal thrust at supports and BM at any section in a symmetrical 3-hinged parabolic arch subjected to udl on horizontal span



$\Rightarrow$



$\Sigma F_x = 0$

$\Rightarrow H_A - H_B = 0 \quad \ldots (i)$

$\Sigma F_y = 0$

$\Rightarrow V_A + V_B - w \times L = 0 \quad \ldots (ii)$

$\Sigma M_z = 0$

$\Rightarrow \Sigma M_A = 0$

$\Rightarrow w \times L \times \frac{L}{2} - V_B \times L = 0 \quad \ldots (iii)$

$M_c = 0$ (LHS)

$\Rightarrow V_A \times \frac{L}{2} - w \times \frac{L}{2} \times \frac{L}{4} - Hh = 0 \quad \ldots (iv)$

from eq^n (i) to (iv)

$V_A = V_B = \frac{wL}{2}$

$H_A = H_B = \frac{wL^2}{8h}$

BM at any section

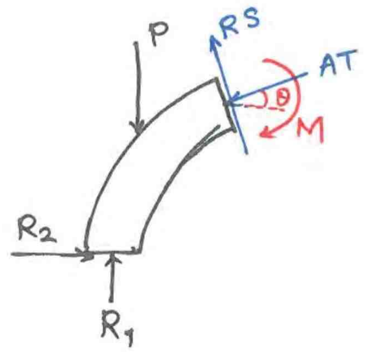
$$BM_x = V_A \cdot x - \frac{wx^2}{2} - Hy$$

$$= \frac{wL}{2}x - \frac{wx^2}{2} - \frac{wL^2}{8h}\left\{\frac{4hx}{L^2}(L-x)\right\}$$

$$= 0$$

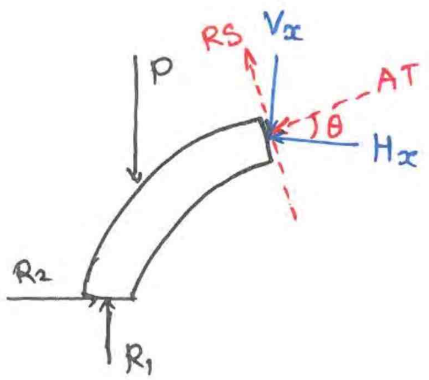
8.5.2 Normal / Axial Thrust and Radial Shear:

Method I:-



By using $\Sigma F_x = 0$ and $\Sigma F_y = 0$, AT and RS can be calculated.

Method II:



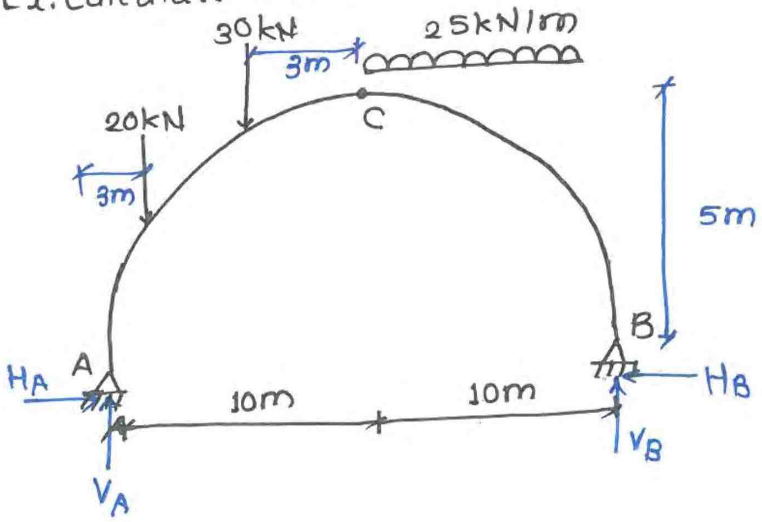
$$AT = H_x \cos\theta + V_x \sin\theta$$

$$RS = H_x \sin\theta - V_x \cos\theta$$

H_x = Net horizontal force on segment towards right

V_x = Net vertical force on segment upward.

Ex. Calculate horizontal thrust at support, Axial thrust and Radial shear at 5m from support A. Arch is parabolic.



⇒ $\Sigma F_x = 0$

$H_A - H_B = 0$ .....(i)

$\Sigma F_y = 0$

$V_A + V_B - 20 - 30 - 25 \times 10 = 0$ ...(ii)

$\Sigma M_z = 0$

$\Rightarrow \quad \Sigma M_A = 0$

$\Rightarrow \quad 20\times3 + 30\times7 + 25\times10\times15 - V_B\times20 = 0 \quad \cdots$ (iii)

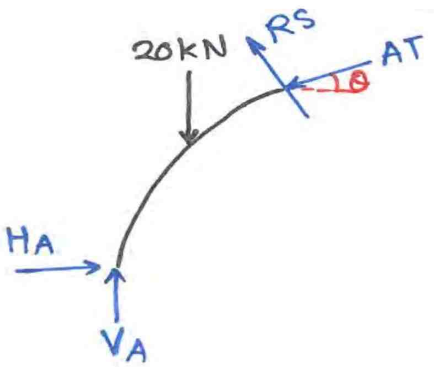
$M_c = 0$ RHS

$\Rightarrow \quad -V_B\times10 + H_B\times5 + 25\times10\times5 = 0 \quad \cdots\cdots$ (iv)

from equation (i) to (iv)

$$\boxed{\begin{aligned} H_A = H_B &= 152\,kN \\ V_A &= 99\,kN \\ V_B &= 201\,kN. \end{aligned}}$$

For RS and AT:-



$y = \frac{4h}{L^2}x(L-x)$

$\Rightarrow \frac{dy}{dx} = \frac{4h}{L^2}(L-2x)$

at $x=5$

$\Rightarrow \tan\theta = \frac{4\times5}{20^2}(20-2\times5)$

$= 0.5$

$\sin\theta = 0.45$

$\cos\theta = 0.89$

Now, $\Sigma F_x = 0$

$H_A - AT\cos\theta - RS\sin\theta = 0$

$\Rightarrow 152 - AT\times0.89 - RS\times0.45 = 0 \quad \cdots$ (v)

$\Sigma F_y = 0$

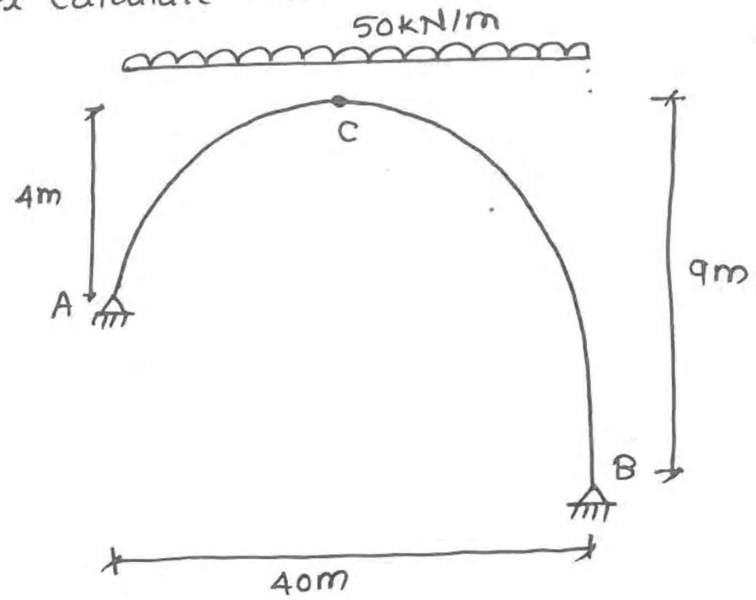
$V_A - 20 - AT\sin\theta + RS\cos\theta = 0$

$\Rightarrow 99 - 20 - AT\times0.45 + RS\times0.89 = 0 \quad \cdots$ (vi)

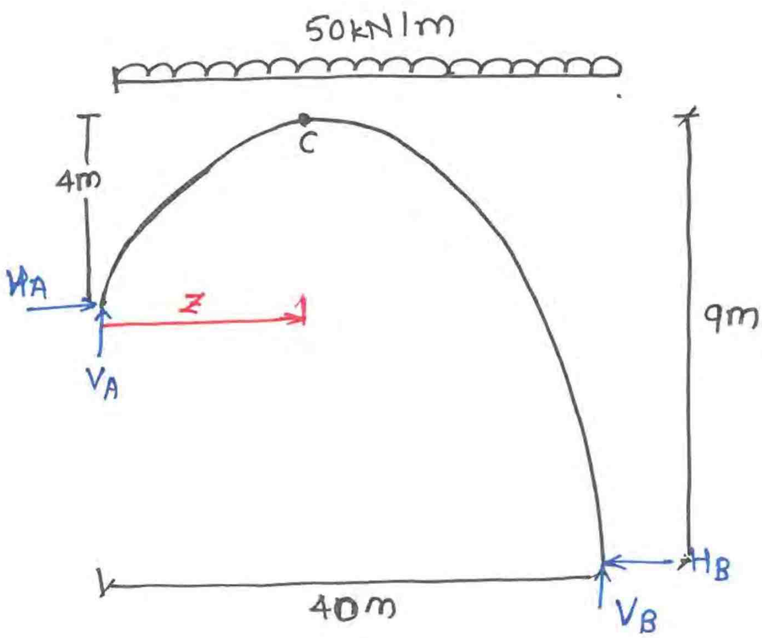
from eq^n (v) and (vi)

$$\boxed{\begin{aligned} AT &= 171.98\,kN \\ RS &= -2.68\,kN. \end{aligned}}$$

Ex Calculate horizontal thrust for parabolic arch given below. Hinge is at crown.



$\Rightarrow$



$$\Sigma F_x = 0$$

$$\Rightarrow H_A - H_B = 0 \quad \text{---(i)}$$

$$\Sigma F_y = 0$$

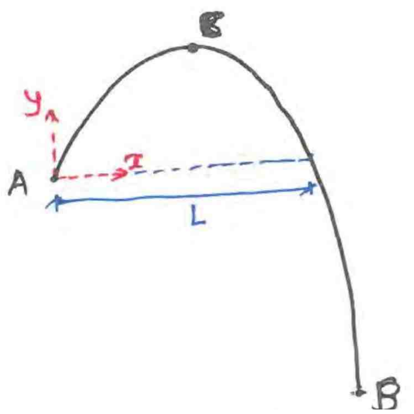
$$\Rightarrow V_A + V_B - 50 \times 40 = 0 \quad \text{---(ii)}$$

$$\Sigma M_z = 0$$

$$\Rightarrow \quad \Sigma M_A = 0$$

$$\Rightarrow$$

In above 4 equations, 5 unknowns (H_A, V_A, H_B, V_B and z) are present so one more equation is required. 5th equation can be formulated using equation of parabola



$$y = \frac{4h}{L^2} x (L - x)$$

$$\Rightarrow y = \frac{4 \times 4}{L^2} x (L - x)$$

At $x = 0, y = 0$

$$\Rightarrow 0 = \frac{4 \times 4}{L^2} \times 0 \times (L - 0)$$

At $x = \frac{L}{2}$, $y = 4m$

$$\Rightarrow 4 = \frac{4 \times 4}{L^2} \frac{L}{2}\left(L - \frac{L}{2}\right)$$

$$\Rightarrow 4 = 4 \qquad \text{(useless)}$$

At $x = 40$, $y = -5m$

$$\Rightarrow -5 = \frac{4 \times 4}{L^2} \times 40\,(L - 40)$$

$$\Rightarrow L = 32m$$

So $z = \frac{L}{2} = \frac{32}{2} = 16m$ $\quad \cdots$ (v)

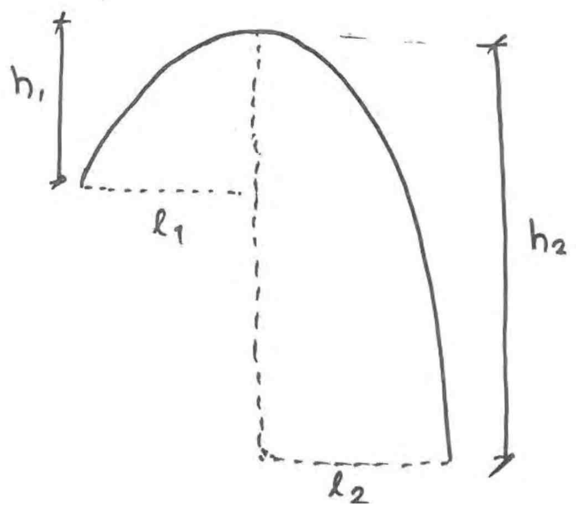
from (i) to (v)

$H_A = H_B = 1600$ kN

$V_A = 800$ kN

$V_B = 1200$ kN.

Alternatively:-



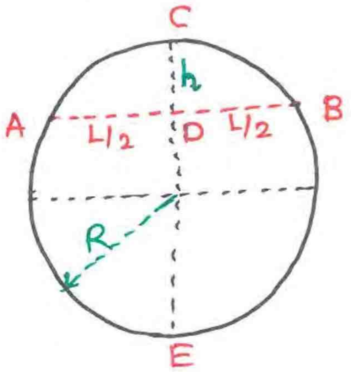
$$\boxed{\sqrt{\frac{h_1}{h_2}} = \frac{l_1}{l_2}} \; ^{**}$$

$$\Rightarrow \sqrt{\frac{4}{9}} = \frac{z}{40 - z}$$

$$\Rightarrow z = 16m$$

8.5.3 Circular 3-Hinged Arch:

Property of circle:



$$AD \times DB = CD \times DE$$

$$\Rightarrow L/2 \times L/2 = h \times (2R-h)$$

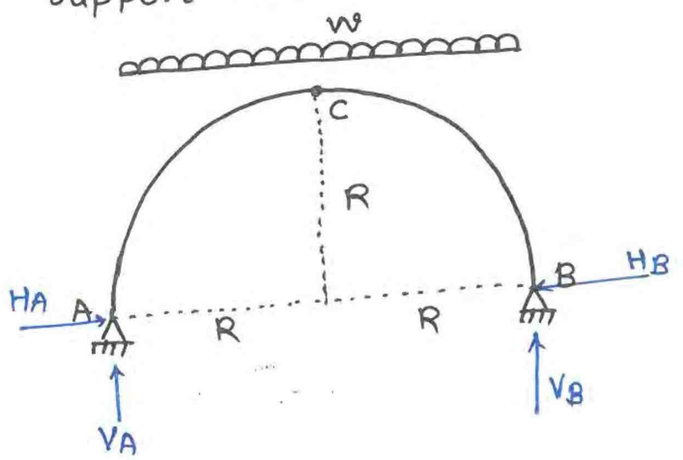
Ex. What is the central rise of a segmental circular arch of radius 250 m and span 80m.

$$\frac{L}{2} \times \frac{L}{2} = h \times (2R-h)$$

$$\frac{80}{2} \times \frac{80}{2} = h \times (2 \times 250 - h)$$

$$h = 3.22\,m$$

Ex. A 3-hinged symmetrical semicircular arch is subjected to udl 'w' on horizontal span. calculate horizontal thrust at support and BM at any section.



$$\Sigma F_x = 0$$

$$\Rightarrow H_A - H_B = 0 \quad \text{---(i)}$$

$$\Sigma F_y = 0$$

$$V_A + V_B - w(2R) = 0 \quad \text{---(ii)}$$

$$\Sigma M_z = 0$$

$$\Rightarrow \Sigma M_A = 0$$

$$\Rightarrow w \times (2R) \times R - V_B \times (2R) = 0 \quad \text{---(iii)}$$

$M_c = 0$ (LHS)

$$\Rightarrow V_A \times R - H_A \times R - \frac{wR^2}{2} = 0 \quad \text{---(iv)}$$

From eq^n (i) to (iv)

$$V_A = V_B = wR$$

$$H_A = H_B = \frac{wR}{2}$$

- **Note:**

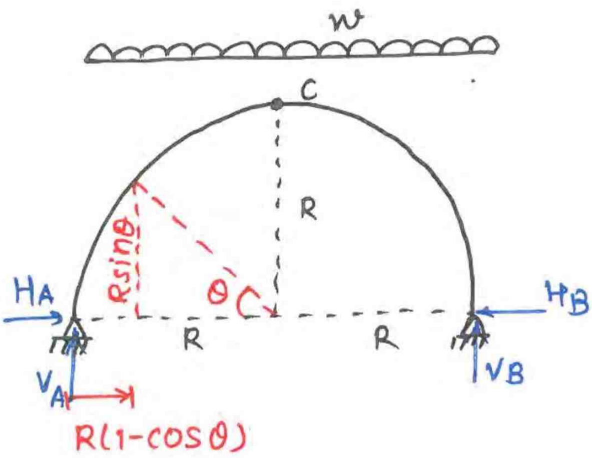
Expression of H of 3-Hinged parabolic arch subjected to UDL.

$$H = \frac{wL^2}{8h}$$

putting $L = 2R$ & $h = R$

$$H = \frac{wR}{2}$$

It means horizontal thrust of 3-hinged arch does not depend on shape of arch.



BM at any section:-

$$BM_\theta = V_A \times R(1-\cos\theta) - H_A \cdot R\sin\theta - \frac{w\{R(1-\cos\theta)\}^2}{2} \cancel{=0}$$

$$= wR \times R(1-\cos\theta) - \frac{wR}{2} \times R\sin\theta - \frac{wR^2}{2}(1 - 2\cos\theta + \cos^2\theta)$$

$$= wR^2 - \cancel{wR^2\cos\theta} - \frac{wR^2}{2}\sin\theta - \frac{wR^2}{2} + \cancel{wR^2\cos\theta} - \frac{wR^2}{2}\cos^2\theta$$

$$\Rightarrow BM_\theta = \frac{wR^2}{2} - \frac{wR^2}{2}\cos^2\theta - \frac{wR^2}{2}\sin\theta$$

$$= \frac{wR^2}{2}(1 - \cos^2\theta - \sin\theta)$$

$$= \frac{wR^2}{2}(\sin^2\theta - \sin\theta)$$

for maximum BM :-

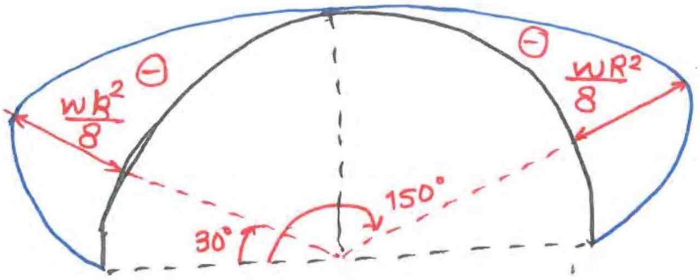
$$\frac{d(BM_\theta)}{d\theta} = 0$$

$$\Rightarrow \frac{wR^2}{2}(2\sin\theta\cos\theta - \cos\theta) = 0$$

$$\Rightarrow \cos\theta(2\sin\theta - 1) = 0$$

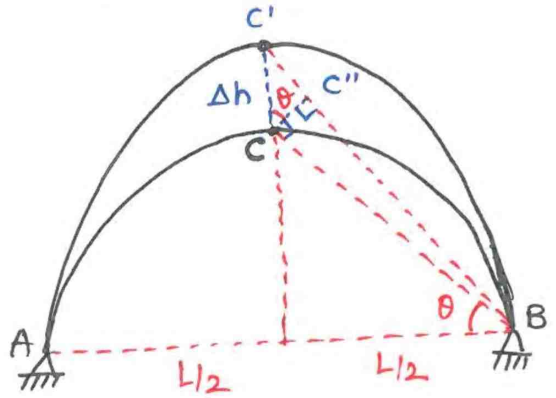
Now, $2\sin\theta - 1 = 0 \Rightarrow \sin\theta = 1/2 \Rightarrow \theta = 30^\circ$ & 150°

At $\theta = 30°$ and $150°$ $\quad BM = \frac{-wR^2}{8}$



BMD.

8.5.4 Effect of Temperature of 3-Hinged Arch:-



$$C'C'' = BC \cdot \alpha \cdot T$$

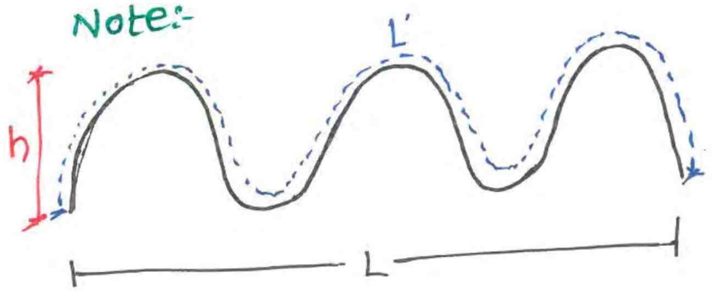
$$= \sqrt{h^2 + (L/2)^2} \cdot \alpha \cdot T$$

$$\Delta h = CC' = \frac{C'C''}{\sin\theta}$$

$$= \frac{\sqrt{h^2 + (L/2)^2} \cdot \alpha \cdot T}{h / \sqrt{h^2 + (L/2)^2}}$$

$$\boxed{\Delta h = \frac{(4h^2 + L^2)\,\alpha T}{4h}}$$

Note:-



horizontal expansion $= L\alpha T$

Vertical expansion $= h\alpha T$

Ex. A 3-hinged arch of span 20 m and central rise 4 m is subjected to udl 25 kN/m. This arch is also subjected to rise in temperature of 40°C. What is the change in horizontal thrust due to increase of temperature? $\alpha = 12 \times 10^{-6}/°C$.

$\Rightarrow$

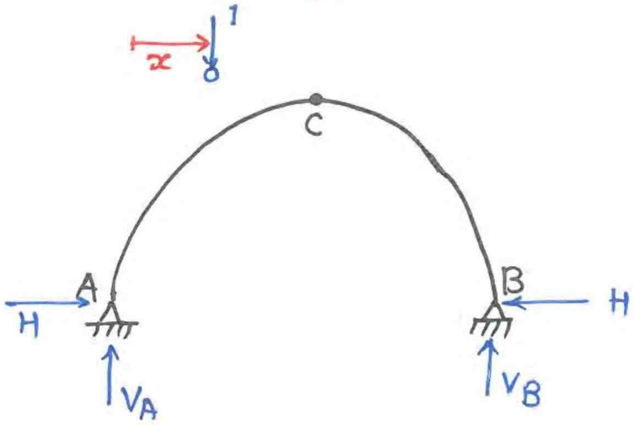
$$H = \frac{wL^2}{8h}$$

$$\frac{dH}{dh} = -\frac{wL^2}{8h^2} \Rightarrow dH = -\frac{wL^2}{8h^2}\,dh = -\frac{wL^2}{8h^2}\left(\frac{(4h^2+L^2)\cdot\alpha T}{4h}\right)$$

$$= -1.08\,kN.$$

with increase in 'h'

8.5.5 ILD of 3-Hinged Arch:-



For V_A:-

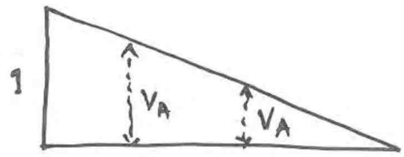
$\Sigma M_B = 0$

$\Rightarrow V_A \times L - 1 \times (L - x) = 0$

$\Rightarrow V_A = \frac{L-x}{L}$

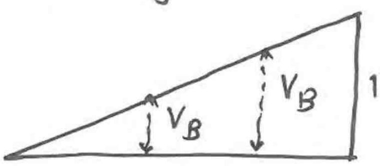
At $x = 0$, $V_A = 1$

$x = L$, $V_A = 0$



ILD of V_A

Similarly,



ILD of V_B.

For H:-

Case I:- $0 \leq x \leq \frac{L}{2}$

$M_c = 0$ (LHS)

$\Rightarrow V_A \times \frac{L}{2} - H \times h - 1 \times (\frac{L}{2} - x) = 0$

$\Rightarrow \left(\frac{L-x}{L}\right) \frac{L}{2} - H \times h - \left(\frac{L}{2} - x\right) = 0$

$\Rightarrow H = \frac{x}{2h}$

At $x = 0$, $H = 0$

$x = \frac{L}{2}$, $H = \frac{L}{4h}$

Case II:- $\frac{L}{2} \leq x \leq L$

$M_c = 0$ (LHS)

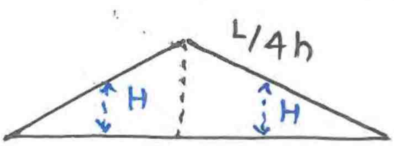
$\Rightarrow V_A \times \frac{L}{2} - H \times h = 0$

$\Rightarrow \left(\frac{L-x}{L}\right) \times \frac{L}{2} - H \times h = 0$

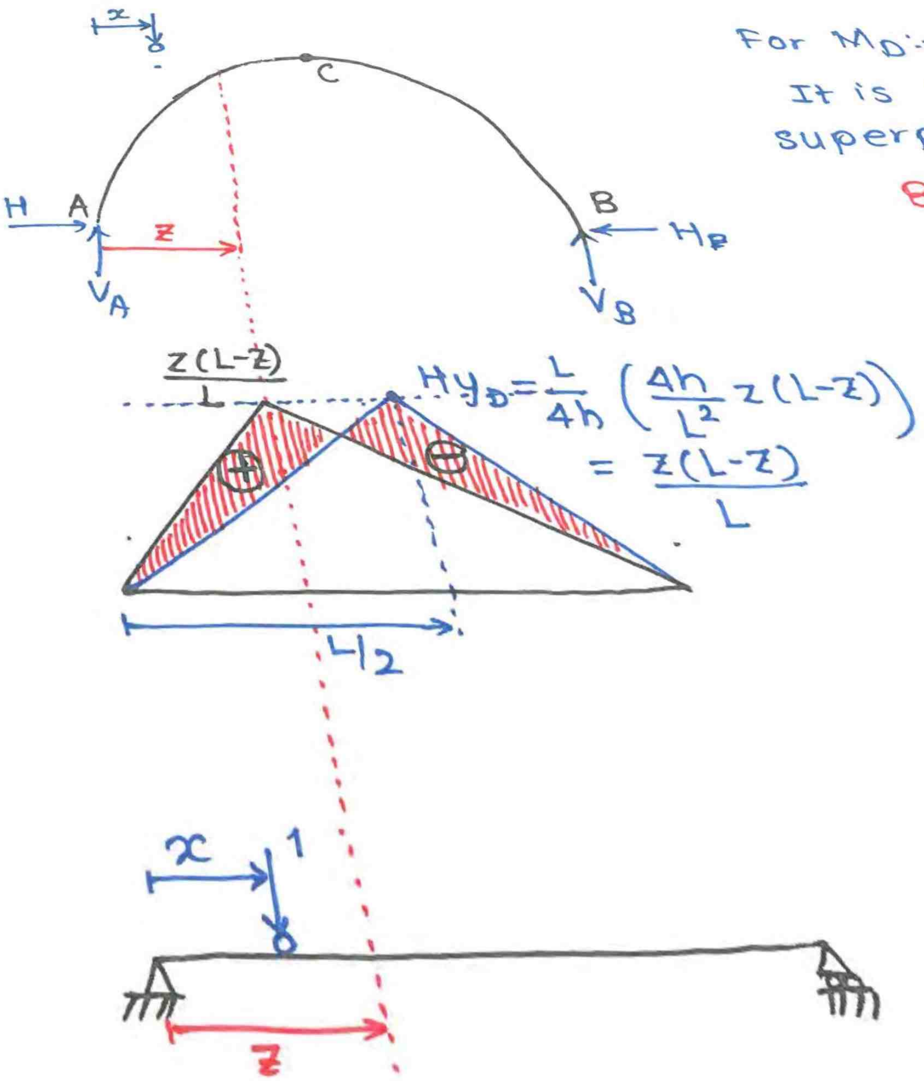
$\Rightarrow H = \frac{L-x}{2h}$

At $x = \frac{L}{2}$, $H = \frac{L}{4h}$

$x = L$, $H = 0$



ILD of H



For M_D:-

It is plotted by method of superposition.

$BM_{arch} = BM_{ss} - Hy$

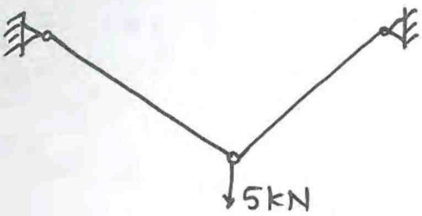
$$Hy_D = \frac{L}{4h}\left(\frac{4h}{L^2} z(L-z)\right)$$
$$= \frac{z(L-z)}{L}$$

9. Cable Structure.

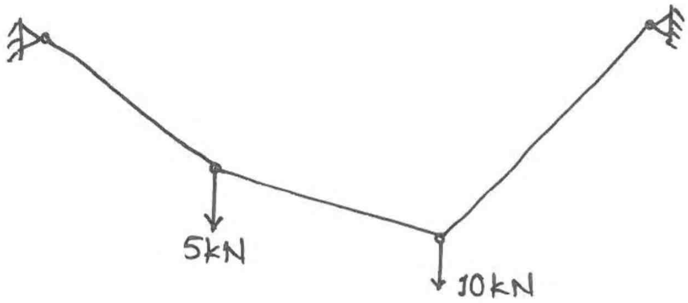
9.1 Assumptions:-

1) Cables are axially inextensible.
2) Cables are perfectly flexible so shear force and bending moment is zero everywhere.

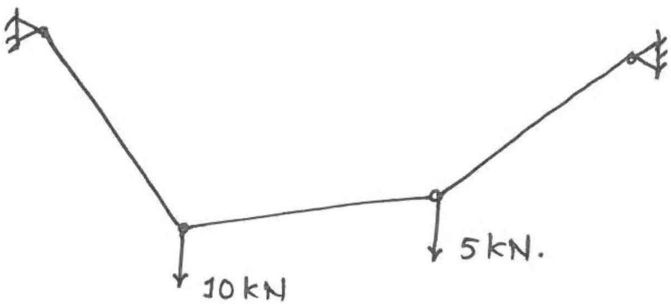
9.2 Stability of Cable Structure:-



DSI=0, Stable



DSI = -1



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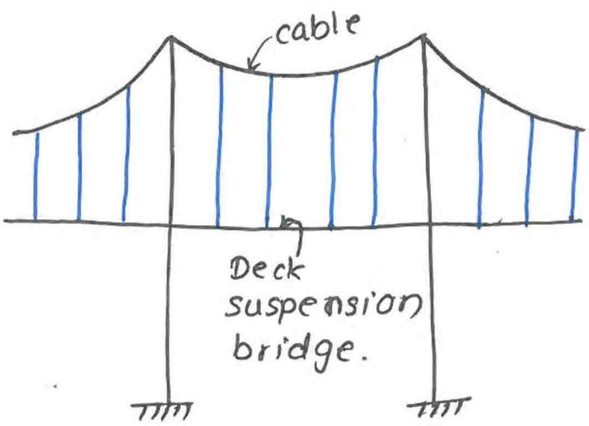
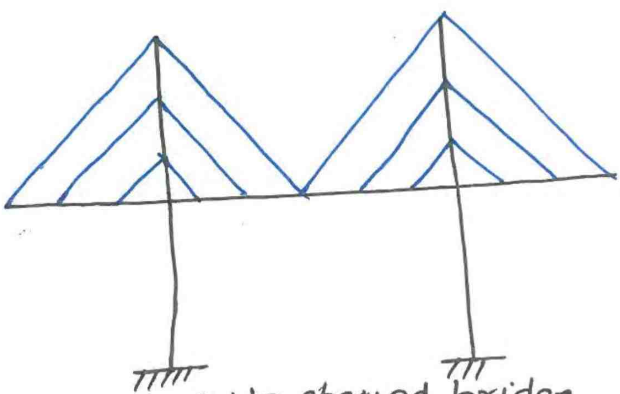
Cable structures with DSI<0 may also be stable for particular load combination. If loading pattern of structure changes then shape of structure also changes. This makes a structure unstable.

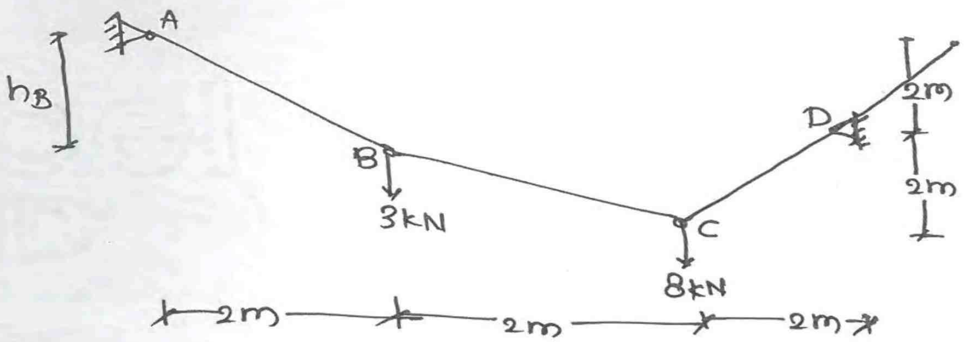
9.3 Two-Hinged and Three Hinged Stiffned suspension Bridge:-

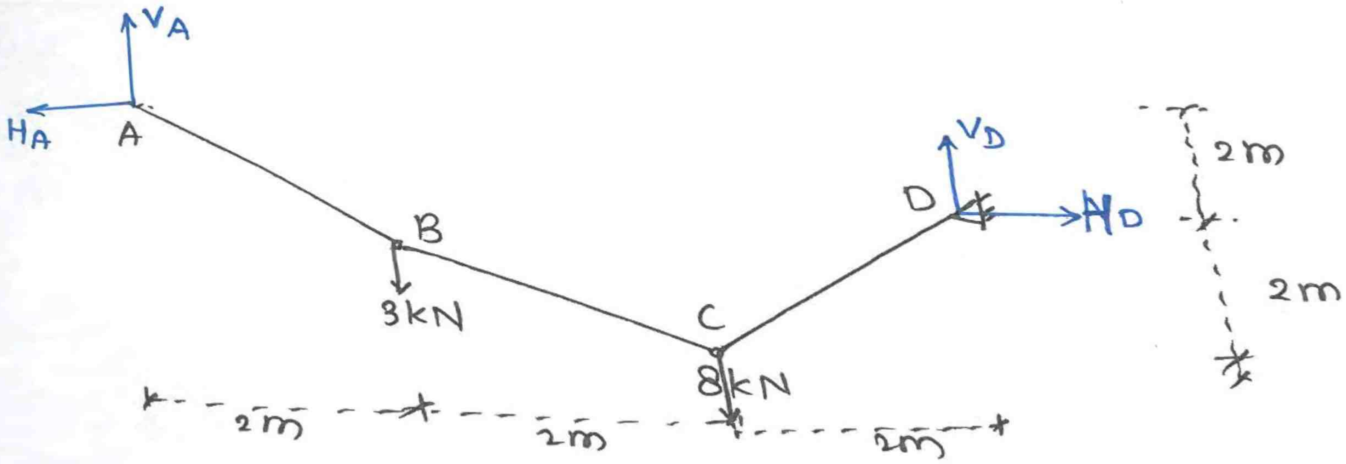




Cable stayed bridge.

Ex. Calculate tension in each segment of cable and vertical sag at B.





$$\Sigma F_x = 0$$

$$\Rightarrow -H_A + H_D = 0 \quad \text{---(i)}$$

$$\Sigma F_y = 0$$

$$V_A + V_D - 3 - 8 = 0 \quad \text{--(ii)}$$

$$\Sigma M_z = 0$$

$$\Sigma M_A = 0$$

$$-V_D \times 6 - H_D \times 2 + 3 \times 2 + 8 \times 4 = 0 \quad \text{---(iii)}$$

$$\Sigma M_C = 0 \text{ (RHS)}$$

$$H_D \times 2 - V_D \times 2 = 0 \quad \text{--(iv)}$$

from eq^n (i) to (iv)

$$H_A = H_D = 4.75\,kN$$

$$V_A = 6.25\,kN$$

$$V_D = 4.75\,kN.$$

$$T_{AB} = \sqrt{V_A^2 + H_A^2}$$

$$= \sqrt{6.25^2 + 4.75^2}$$

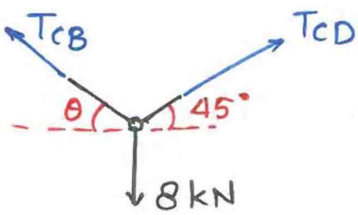
$$= 7.85\,kN$$

$$T_{CD} = \sqrt{V_D^2 + H_D^2}$$

$$= \sqrt{4.75^2 + 4.75^2}$$

$$= 6.72\,kN$$

Joint C :-



$$\Sigma F_x = 0$$

$$\Rightarrow -T_{CB}\cos\theta + T_{CD}\cos 45° = 0 \quad \cdots (v)$$

$$\Sigma F_y = 0$$

$$T_{CB}\sin\theta + T_{CD}\sin 45° - 8 = 0 \quad \cdots (vi)$$

from (v) and (vi)

$$T_{CB} = 5.75\,kN$$

For vertical sag at B.

$$M_B = 0 \quad (LHS)$$

$$\Rightarrow V_A \times 2 - H_A \times h_B = 0$$

$$\Rightarrow \quad h_B = 2.63\,m$$

From equation (i) to (v)

$H_A = H_E = 4\,kN$

$V_A = 2\,kN$

$V_E = 1\,kN$

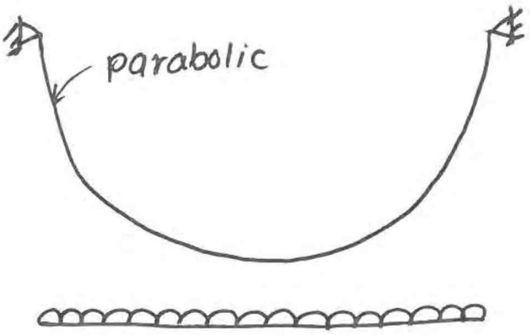
for h_B :-

$M_B = 0$ (L.H.S.)

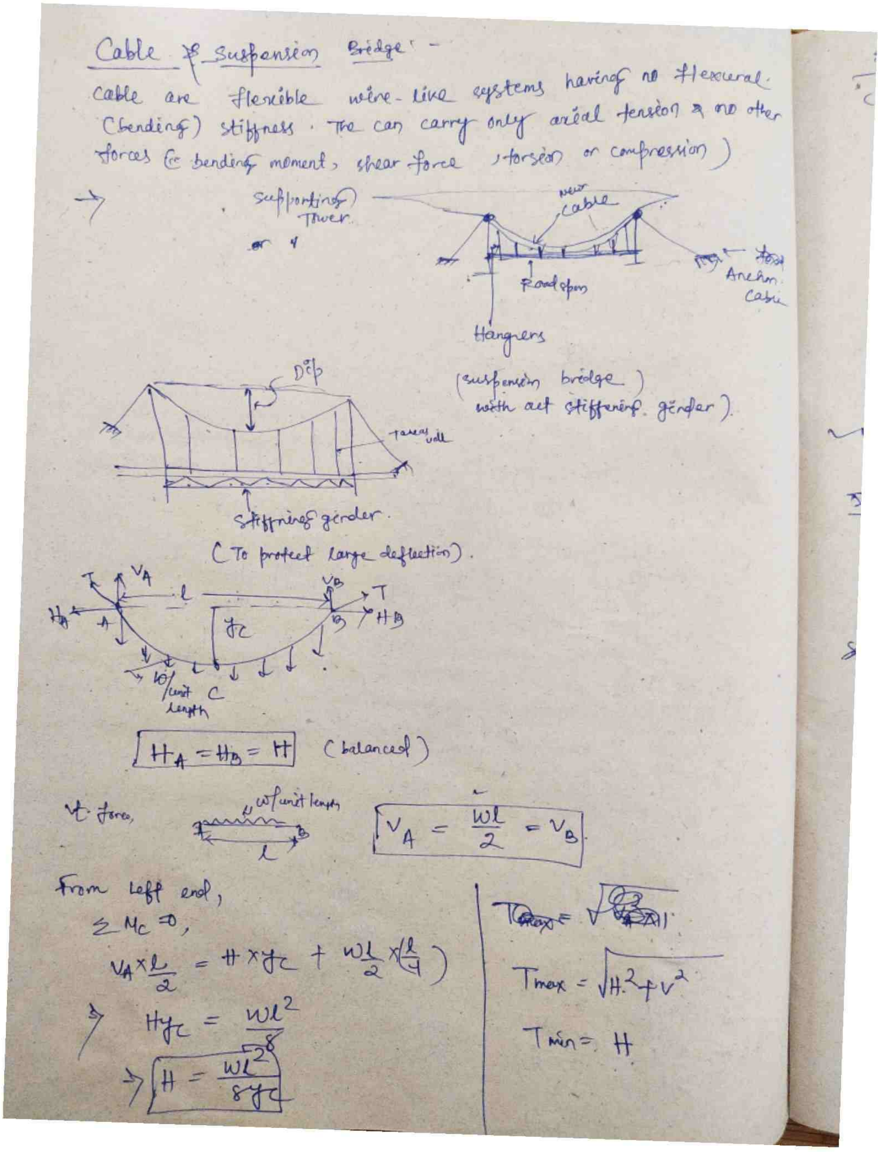
$\Rightarrow V_A \times 2 - H_A \times h_B = 0$

$\Rightarrow \boxed{h_B = 2.63\,m}$

9.4 Cable subjected to udl:-



UDL on horizontal span (Bridge)



Cable & Suspension Bridge :-

Cable are flexible wire-like systems having no flexural (bending) stiffness. The can carry only axial tension & no other forces (i.e. bending moment, shear force, torsion or compression)

$\rightarrow$ supporting Tower

main cable

Anchor Cable

Road span

Hangers

(suspension bridge with out stiffening girder)

Dip

Taking udl

stiffening girder.

(To protect large deflection).

T, V_A, l, V_B, T, H_A, A, y_C, B, H_B

w/unit length, C

$\boxed{H_A = H_B = H}$ (balanced)

Vt. force,

w/unit length

l

$\boxed{V_A = \frac{wl}{2} = V_B}$

From left end,

$\Sigma M_C = 0$,

$V_A \times \frac{l}{2} = H \times y_C + \frac{wl}{2} \times \left(\frac{l}{4}\right)$

$\Rightarrow H y_C = \frac{wl^2}{8}$

$\Rightarrow \boxed{H = \frac{wl^2}{8 y_C}}$

$T_{max} = \sqrt{H^2 + V^2}$

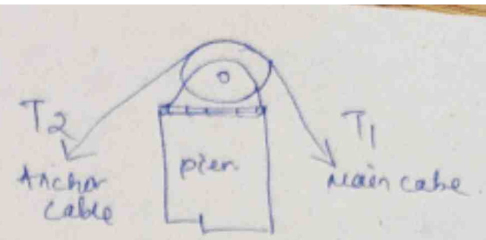
$T_{min} = H$

- **Note:-**
  If vertical sag is small as compared to span then parabolic and caternary profiles are considered as same for practical purposes.

Considering a cable subjected to downward UDL on horizontal span and supported at unequal level as given below.

Supporting tower :-

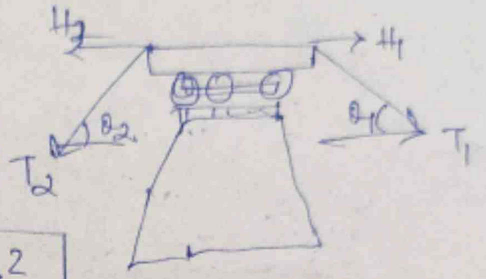
(i) frictionless pulley :-



$\boxed{T_1 = T_2}$

Tension in main cable = Tension in Anchor cable.

ii) Saddle on frictionless Roller :-



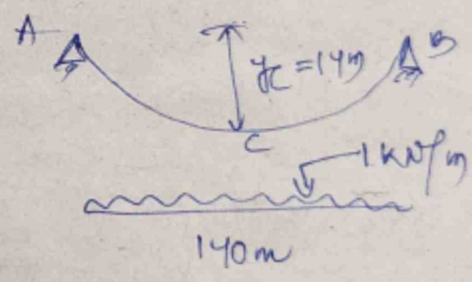
$T_2 \neq T_1$

$H_1 = H_2$.

Length of cable $\boxed{S = L + \dfrac{8y_c^2}{3L}}$

Q:- A suspension cable 140m span & 14m central dip carries a udl of 1 kN/m. calculate the max^m & min^m Tension in cable :

Sol



$$V_A = \frac{wl}{2} = \frac{1 \times 140}{2} = 70\ kN = V_B$$

$$H = H_A = H_B = \frac{wl^2}{8y_c} = \frac{1 \times 140^2}{8 \times 14} = 175\ kN.$$

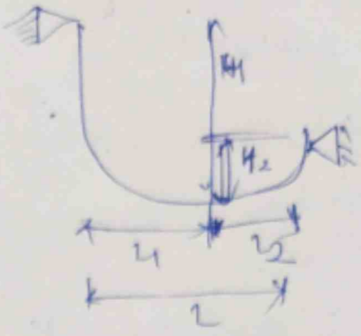
$$T_{max^m} = \sqrt{175^2 + 70^2} = 188.48\ kN \quad (\text{At the support})$$

$$T_{min} = H = 175\ kN \quad (\text{at Lowest point of cable i.e. at C point})$$

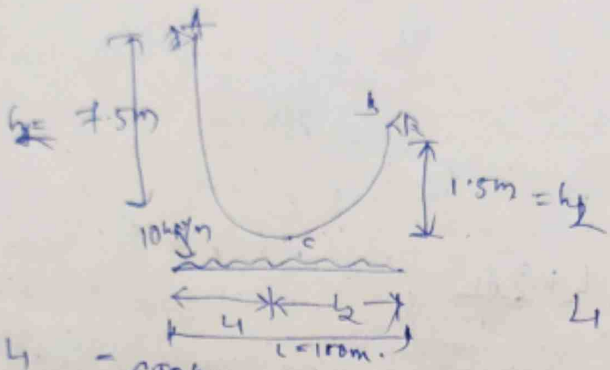
Case 1 - Unequal length :-

$$L_1 = \frac{L\sqrt{H_1}}{\sqrt{H_1}+\sqrt{H_2}}$$

$$L_2 = \frac{L\sqrt{H_2}}{\sqrt{H_1}+\sqrt{H_2}}$$



Ex



$$L_1 = \frac{100\sqrt{7.5}}{\sqrt{7.5}+\sqrt{1.5}} = 69m$$

$L_2 = L - L_1 = 100 - 69$
$= 31m$

$V_A = wL_1 = 690kN$

& $V_B = wL_2 = 310kN$

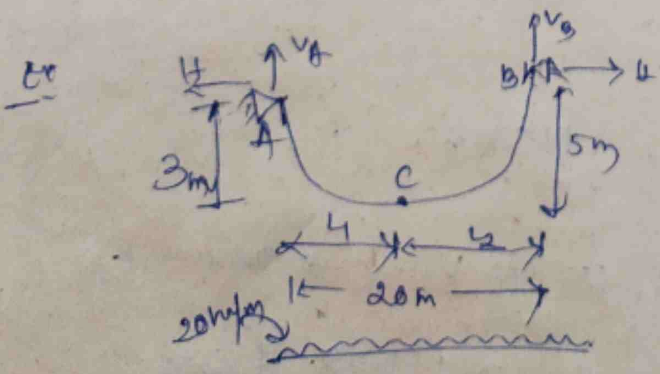
$$H = \frac{wL_1^2}{2h_1} = \frac{10 \times 69^2}{2 \times 7.5} = 3174 kN$$

$T_{min} = H = 3174 kN$

$T_{max} = \sqrt{H^2 + V_A^2}$ ($\underline{V_A} > V_B$)
Take V_A (which is greater)

$= 3248.13 kN.$

Ex



Sol:
$$L_1 = \frac{L\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}} = \frac{20\sqrt{3}}{\sqrt{3}+\sqrt{5}} = 8.73\,m$$

$$L_2 = 20 - L_1 = 11.27\,m.$$

$$V_A = wL_1 = 20 \times 8.73 = 174.648\,kN.$$

$$V_B = wL_2 = 20 \times 11.27 = 225.4\,kN.$$

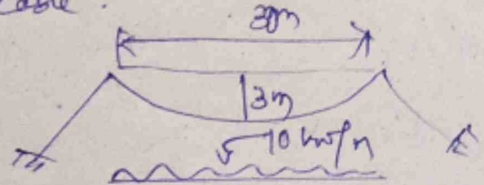
$V_B = Max^m$.

$$H = \frac{wL_2^2}{2h_2} = \frac{20 \times 11.27^2}{2 \times 5} = 254\,0\,kN.$$

$$Max^m \text{ Tension} = \sqrt{H^2 + V_B^2} = 339.74\,kN.$$

$$T_{min} = H$$

Q. A suspension cable having 30m span & 3m dip loaded with UDL of 10kN/m. Find the max^m tension & length of cable.



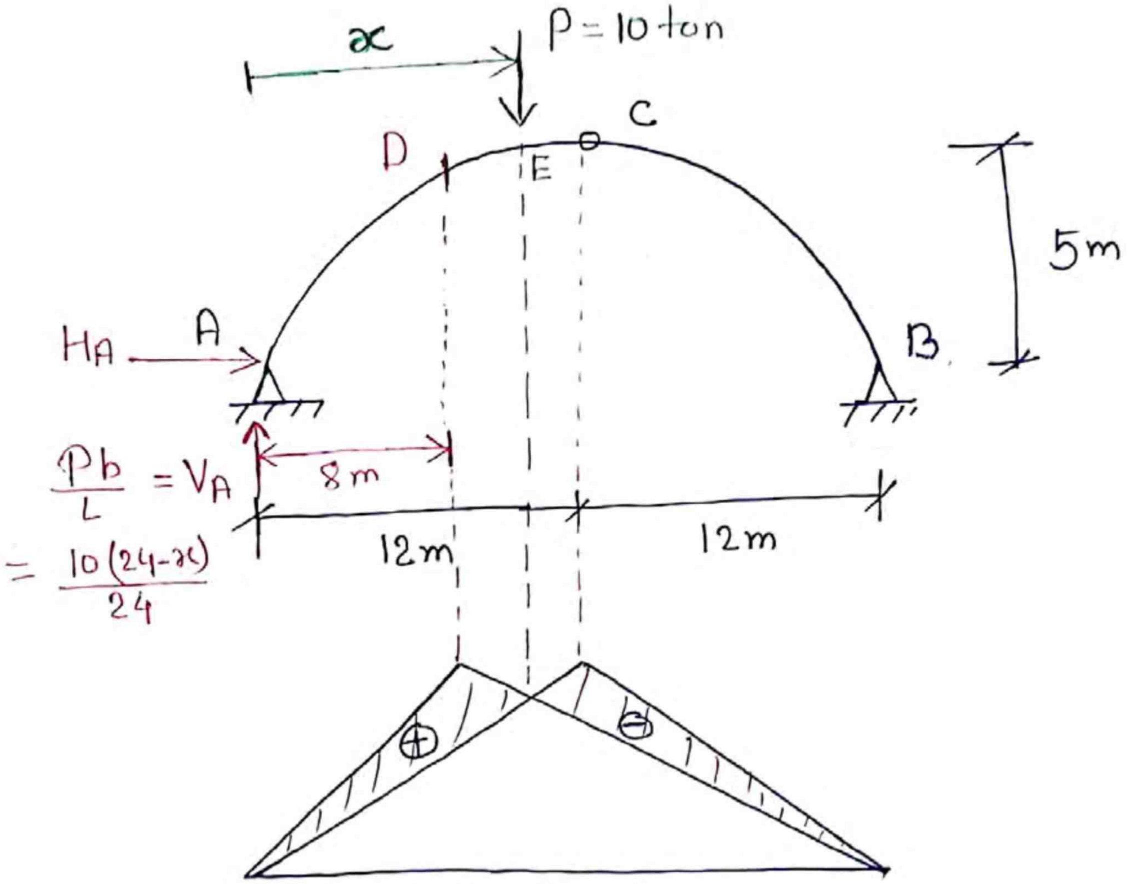
Sol.
$$V_A = V_B = \frac{wL}{2} = \frac{10 \times 30}{2} = 150\,kN.$$

$$H = \frac{wL^2}{8y_c} = \frac{10 \times 30^2}{8 \times 3} = 375\,kN.$$

$$T_{max} = \sqrt{375^2 + 150^2} = 403.877\,kN.$$

Length of cable,
$$S = L + \frac{8y_c^2}{3L} = 30 + \frac{8 \times 3^2}{3 \times 30}$$
$$= 30.8\,m \text{ Ans}$$

Ex A symmetrical 3-hinged arch (Parabolic) of span 24m & central rise 5m carries a single vertical point load of 10 ton. Locate the position of load on arch in order that the bending moment is zero at section 8m from left hinge. For this position of load, calculate the bending moment under the load.



ILD of BM_D

From ILD, It is clear that load placed at E will produce zero BM at D.

$$M_C = 0 \quad (LHS)$$

$$\Rightarrow V_A \times \frac{L}{2} - H_A \times 5 - P\left(\frac{L}{2} - x\right) = 0.$$

$$\Rightarrow \quad H_A = ??$$

$$BM_D = 0$$

$$\Rightarrow V_A \times 8 - H_A \times y_D = 0$$

$$\Rightarrow \frac{10(24-x)}{24} \times 8 - H_A \left\{ \frac{4 \times 5}{24^2} \, 8(24-8) \right\} = 0.$$

$$\Rightarrow \quad x = \boxed{10.29\,m.}$$

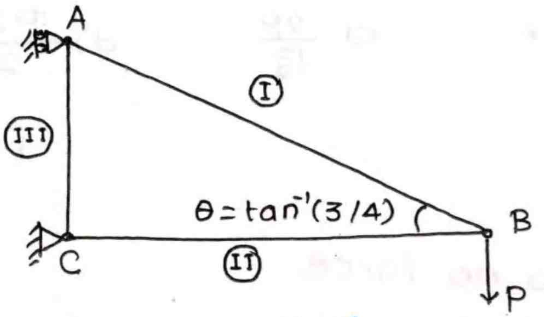
Now,

$$BM_E = V_A \times x - H_A \, y_x$$

$$= \frac{10(24-10.29)}{24} \times 10.29 - H_A \left\{ \frac{4 \times 5}{24} \, 10.29(24-10.29) \right.$$

$$= \boxed{45.6 \text{ kNm.}}$$

Q.6. A cantilever truss carries a concentrated load P as shown in the figure below:



What are the magnitudes of axial forces in the members I, II and III, respectively?

a) 1.00 P, 1.33 P and 1.67 P

b) 1.67 P, 1.33 P and 1.00 P

c) 1.33 P, 0.75 P and 0.60 P

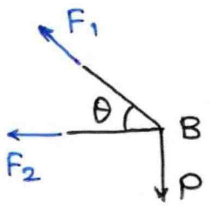
d) 0.60 P, 0.75 P and 1.00 P

[ESE : 2006]

Solution: $\theta = \tan^{-1}(3/4) \Rightarrow \tan\theta = \frac{3}{4} \Longrightarrow \sin\theta = \frac{3}{5};\ \cos\theta = \frac{4}{5}$

.... (3, 4, 5 ⇒ pythagorean triple)

Joint B:



$\Sigma F_y = 0$

$F_1 \sin\theta - P = 0$

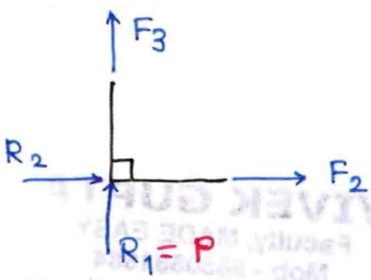
$$F_1 = \frac{P}{\sin\theta} = \frac{P}{3/5} \Rightarrow F_1 = 1.67P \quad \text{.....(i)}$$

$\Sigma F_x = 0$

$-F_2 - F_1\cos\theta = 0$

$$F_2 = -F_1\cos\theta = -\frac{P}{\tan\theta} = -\frac{P}{3/4} \Rightarrow F_2 = 1.33P \quad \text{...(ii)}$$

Joint C:



$\Sigma F_y = 0$

$F_3 + R_1 = 0$

$F_3 = -R_1 = -P$ .....(iii)

From (i), (ii) and (iii)

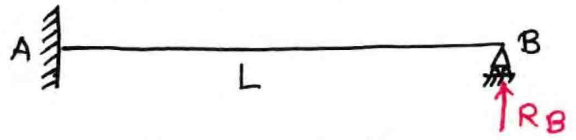
Ans: b) 1.67P, 1.33P, 1.00P

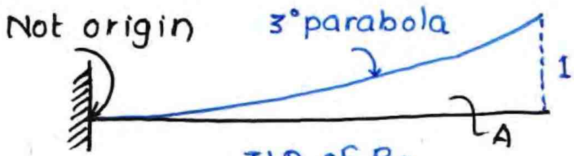
(F_1, F_2, F_3)

Q.12. What is the area of influence line diagram for the reaction at the hinged end of uniform propped cantilever beam of span L?

a) $\frac{L}{8}$ b) $\frac{L}{2}$ c) $\frac{L}{4}$ d) $\frac{3L}{8}$

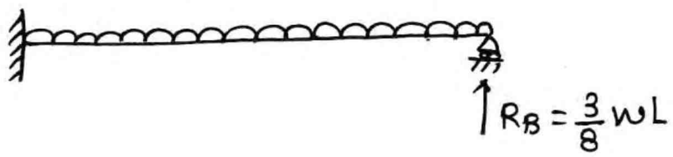
[ESE: 2009]

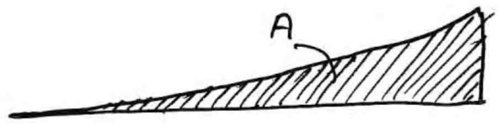




ILD of R_B

Assuming beam is subjected to udl





ILD of R_B

$$w \times \text{Area of ILD of } R_B = R_B$$

$$w \times A = \frac{3}{8} wL$$

$$\boxed{A = \frac{3}{8} L}$$