

## SURVEYING

96/12/201

### (#) Introduction :-

Earth - earth is an oblate spheroid.

Diameter = 12740 km (Average)

Average Radius = 6370 km

At equator = 12756.75 km

At Poles = 12713.86 km

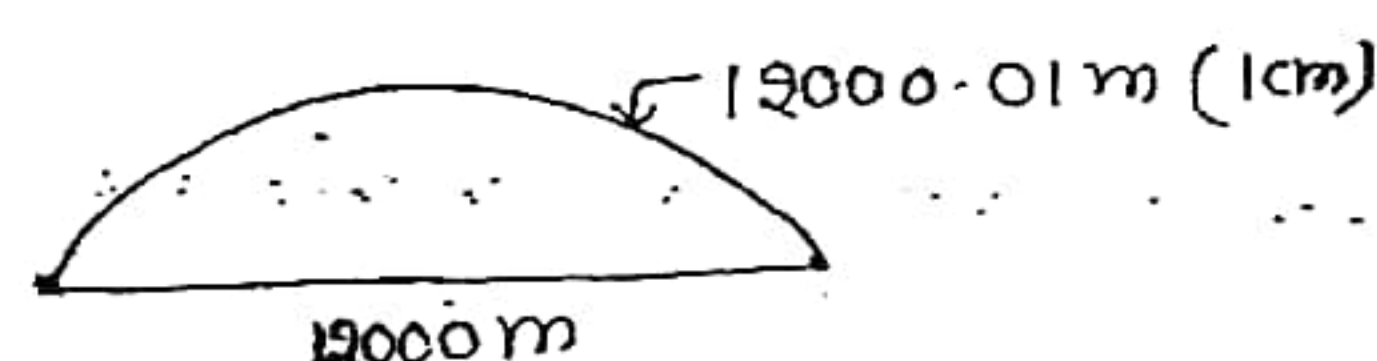
difference = 42.95 km

0.34%

### (#) Types of Surveying :-

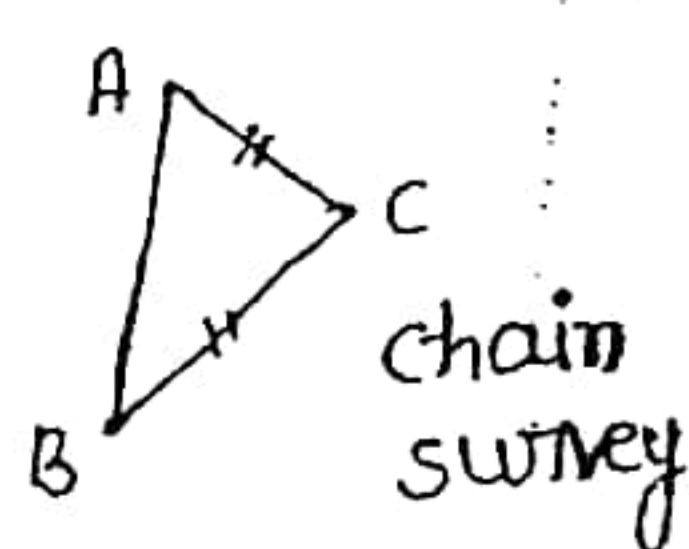
(1) Geodetic Survey :- If earth curvature is considered for survey work.

(2) Plane survey :- If earth curvature is not considered suitable for small distance.

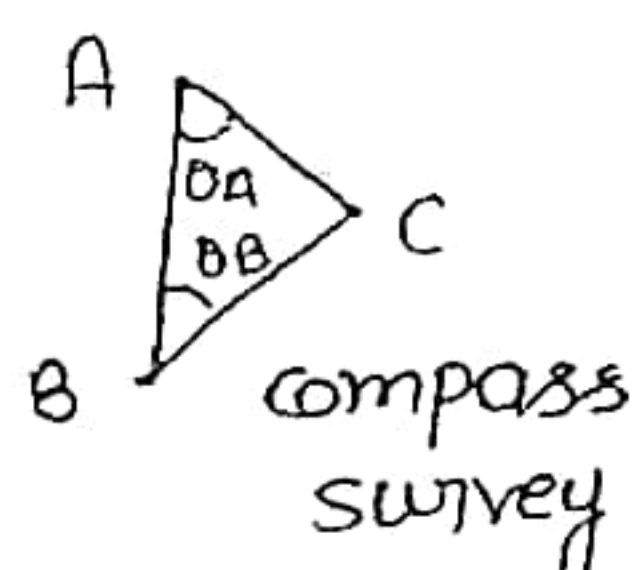


### (#) Principle of surveying :-

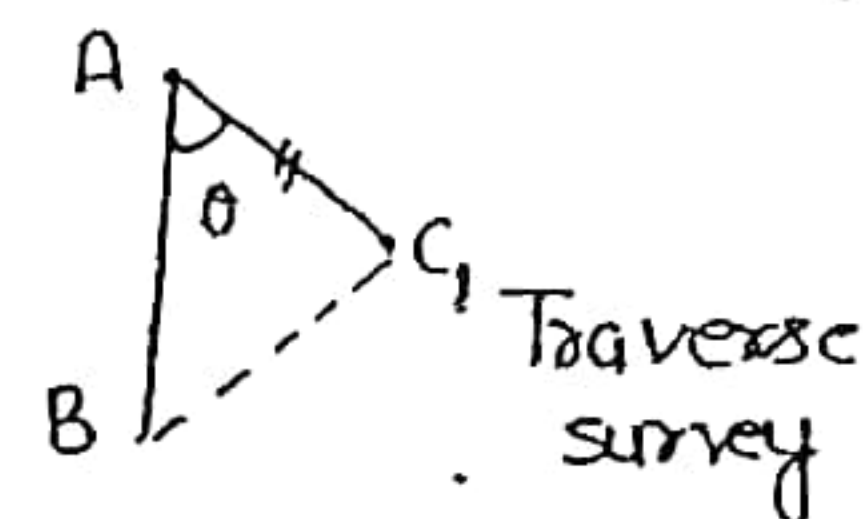
(1) Location of point is measured w.r.t. two reference point :-



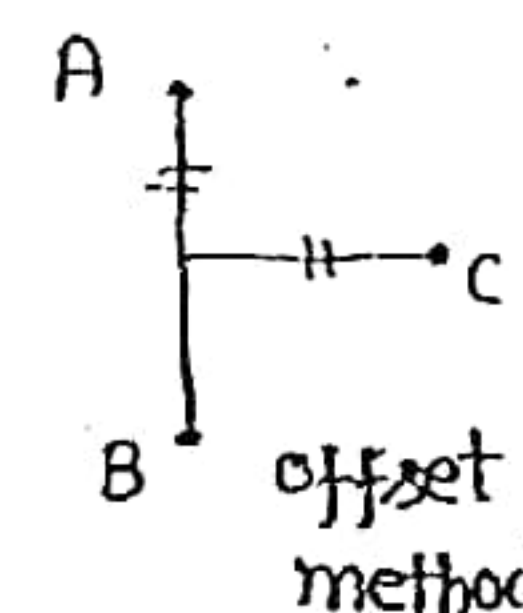
chain survey



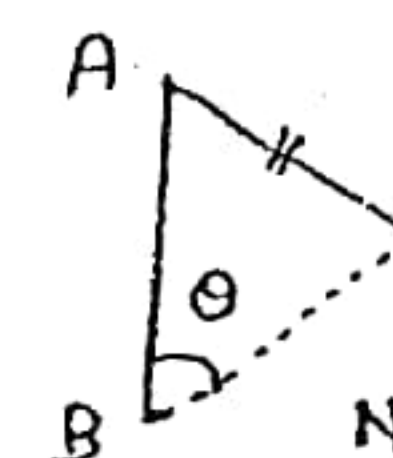
compass survey



traverse survey

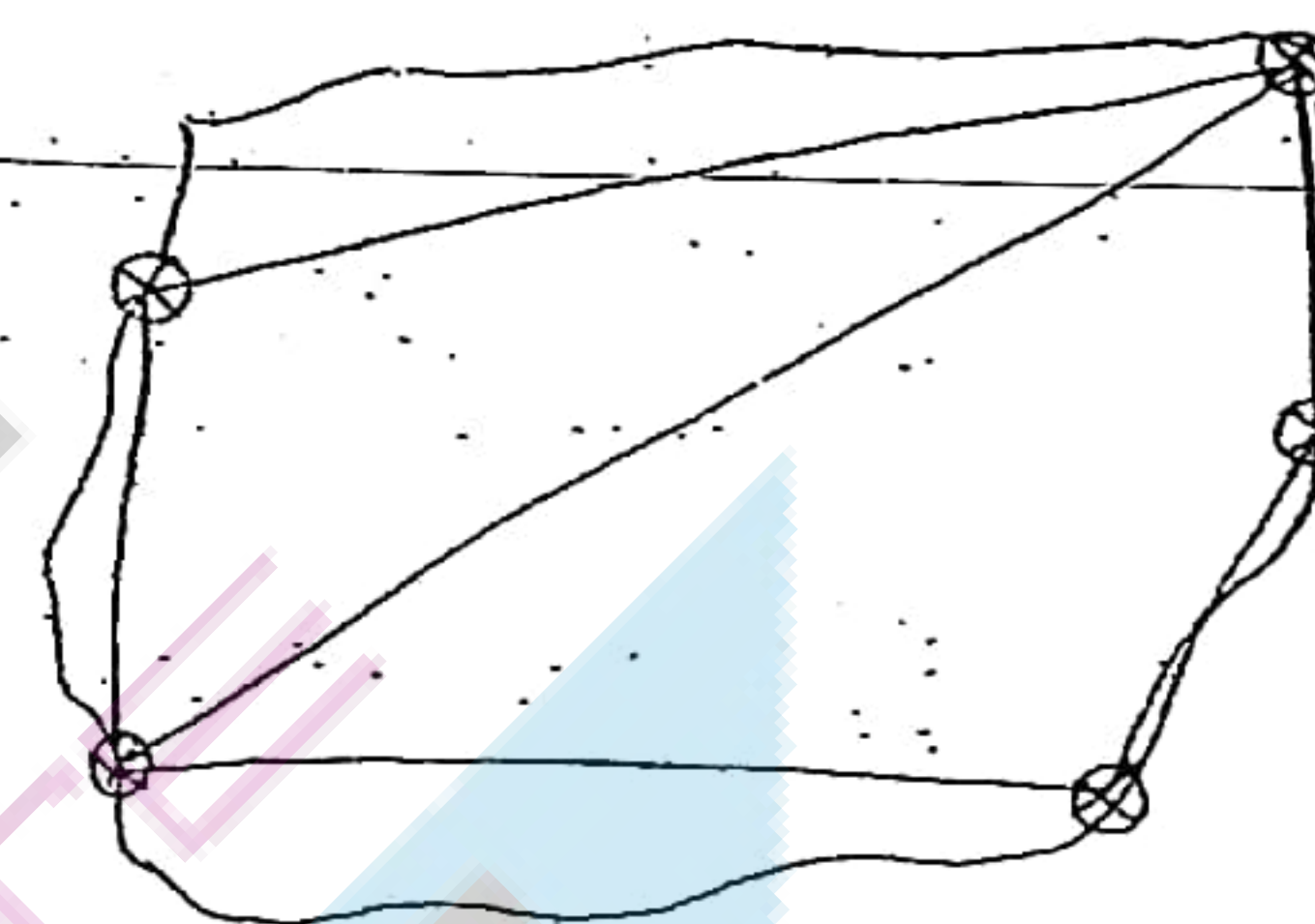


offset method



Not so common.

### (2) Working Whole to part :-



→ Major Control points are decided & measured accurately with high degree of precision. Minor details can be collected later to avoid the error to be accumulated.

### LINEAR MEASUREMENTS

1) Scale :- Scale is a ratio of map distance to ground distance.

If on the drawing.

Scale  $\Rightarrow$  1 cm = 100 m

1 cm on paper = 100 m on ground

Ratio :-  $\frac{1 \text{ cm}}{100 \text{ m}} = \frac{1 \text{ cm}}{10,000 \text{ cm}} = \frac{1}{10,000}$  (R.F.)

R.F = Representative fraction.

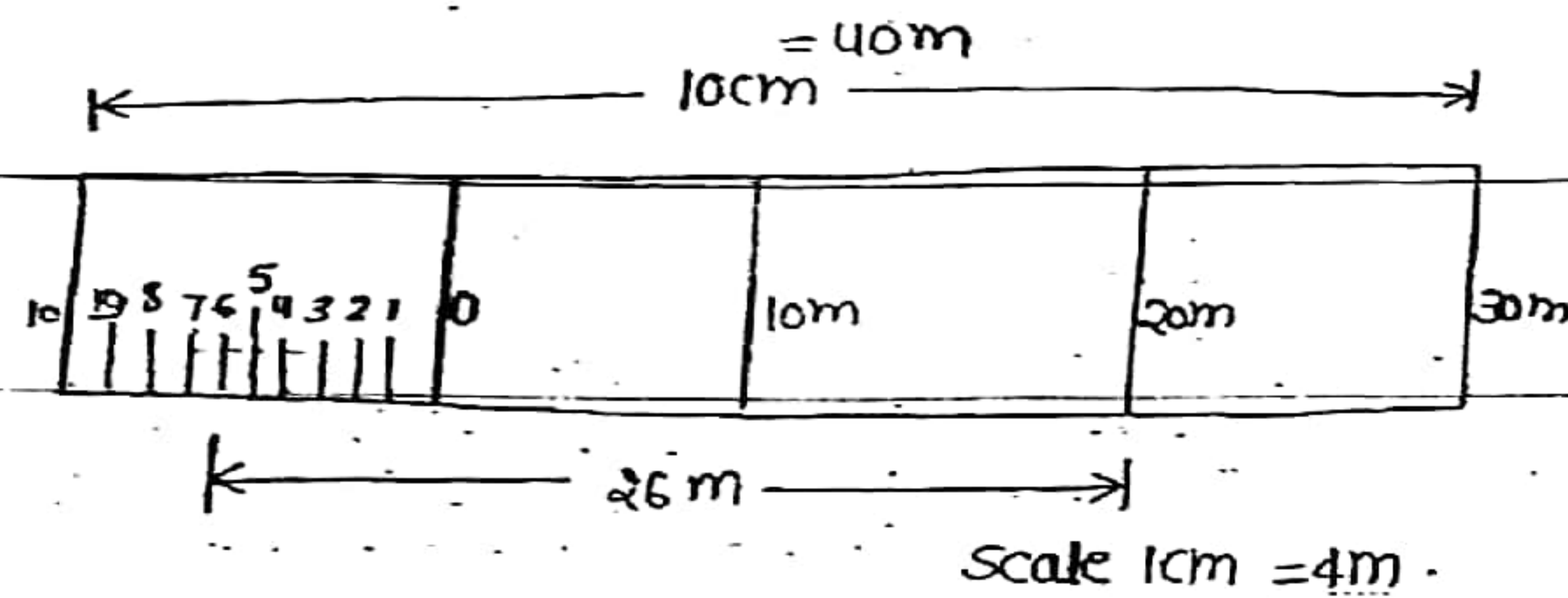


### Types of Scale :-

(1) Plane Scale :- Plane scale measure only two dimension.

$$1\text{cm} = 4\text{m}$$

##) How to make a scale  $1\text{cm} = 4\text{m}$  —



(1) Take a 10cm long line divide it in 4 equal parts. Each part is of 10m length.

(2) Now divide 1st part in another 10 parts. This smaller divisions will show 1m.

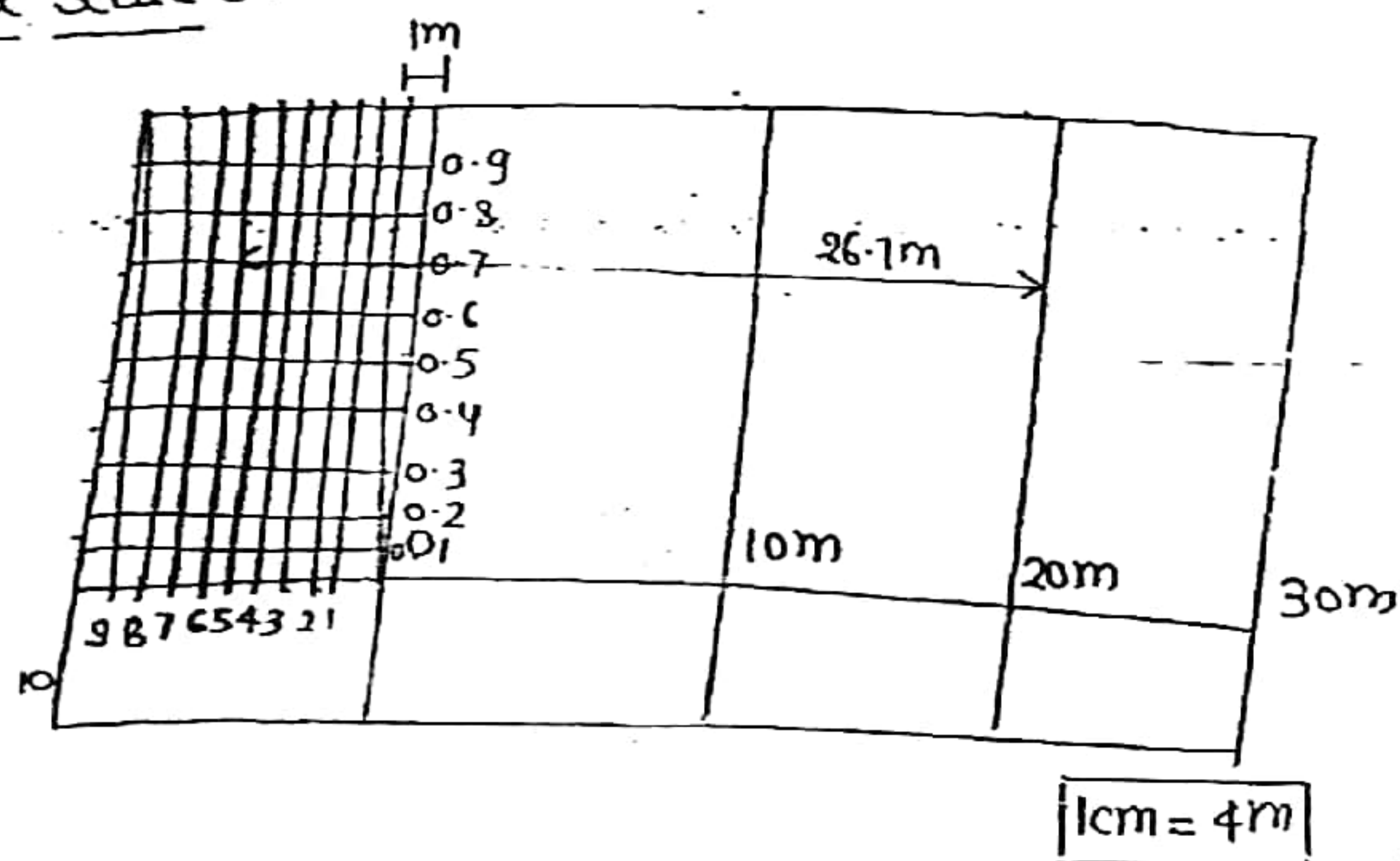
Two dimension that can be read

① 10m (Decimeter)

② 1m (meter)

(2) Diagonal Scale :-

this scale can read up to three dimension



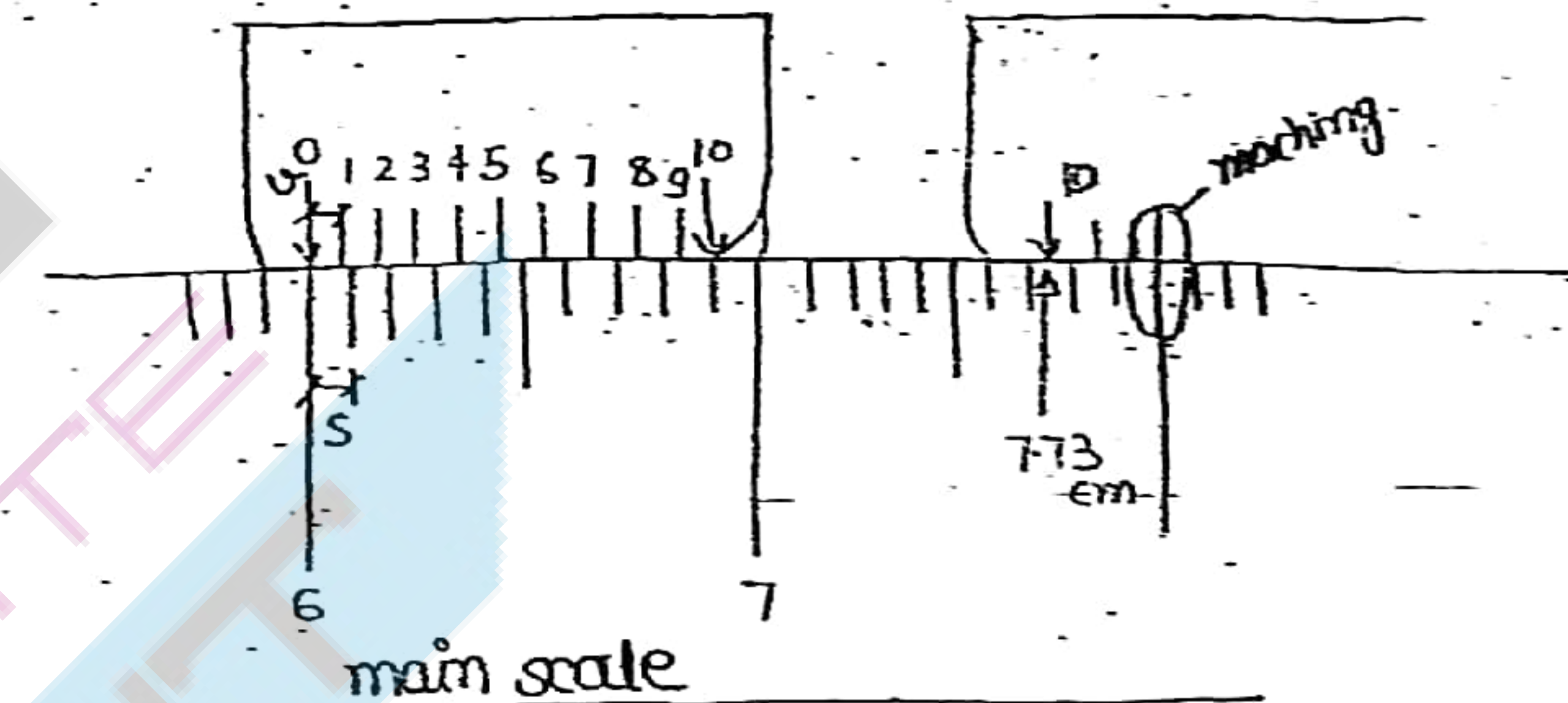
→ It works on the principle of similar triangle.

→ Three dimension

- ① 10m → Decimeter
- ② 1m → meter
- ③ 0.1m (10cm) → Decimeter

(3) Vernier Scale :-

(1) Direct Vernier :-



→ In case of direct vernier  $(n-1)$  division of main scale is divided into  $n$  divisions of vernier scale.

$$(n-1)S = n \times v$$

$$v = \frac{(n-1)}{n} \times S$$

##) Least Count :- least count is the minimum dimension that can be read by a scale.

$$\text{Least count} = S - v$$

$$= S - \frac{(n-1)}{n} \times S$$

$$= \frac{Sn - nS + S}{n} = \boxed{\frac{S}{n}}$$



For Example -

$$S = 1\text{m}$$

$$n = 10$$

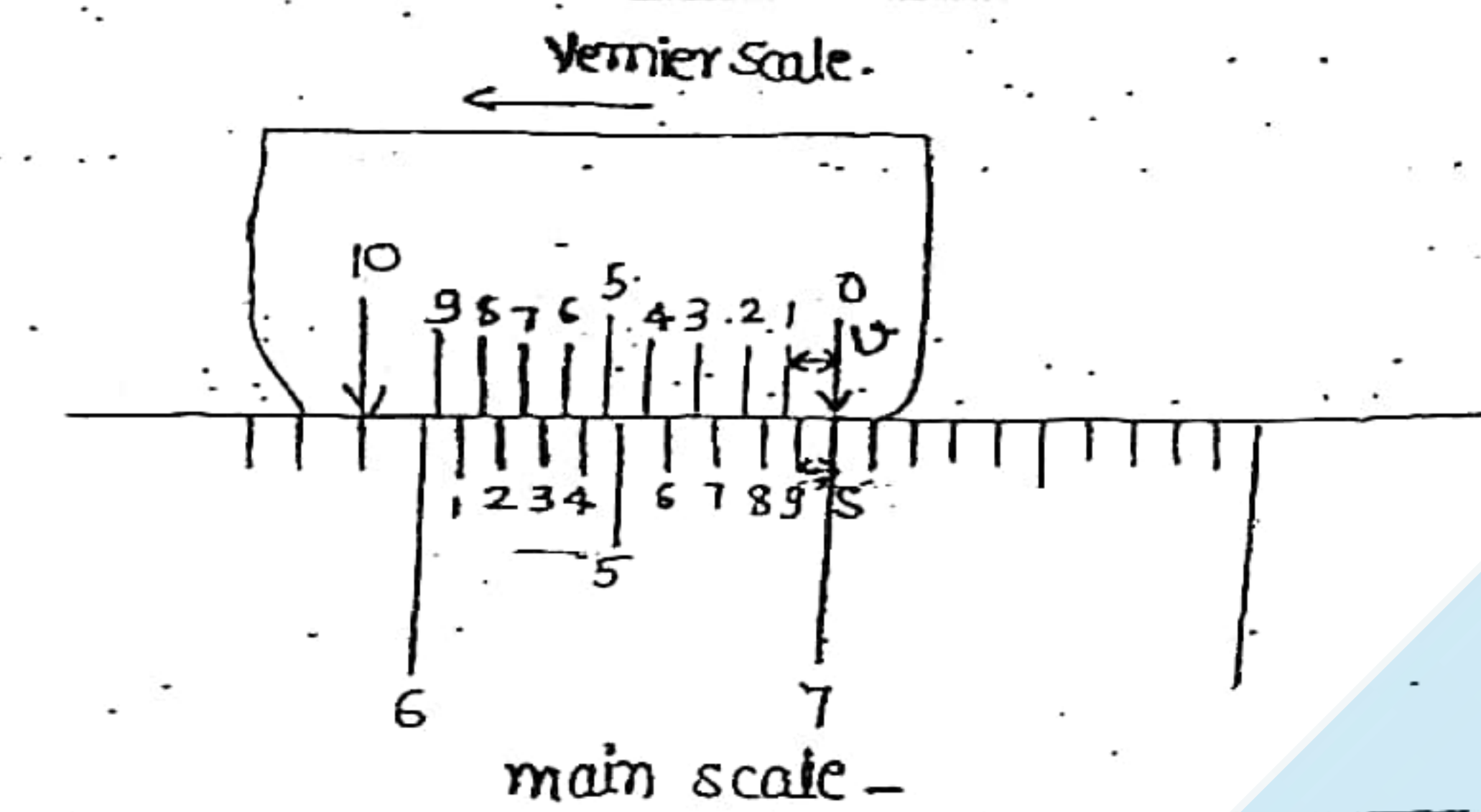
Least Count

$$= \frac{S}{n} = \frac{1\text{mm}}{10} = 0.1\text{mm}$$

⊕ Reading of Vernier is taken by the line of Vernier scale that will be exactly above of any one line of main scale.

In direct Vernier → Vernier scale moves in the same dir<sup>n</sup> of main scale.

(ii) Retrograde Verniers -



→ In this case

(n+1) division of main scale is divided into n division of Vernier scale.

$$(n+1)S = n \cdot v$$

$$v = \left( \frac{n+1}{n} \right) S \quad \text{--- ①}$$

Least Count →

$$= v - S = \left( \frac{n+1}{n} \right) S - S = \frac{nS + S - nS}{n} = \frac{S}{n}$$

$$= \frac{S}{n}$$

Example

$$S = 1\text{mm}$$

$$n = 10\text{mm}$$

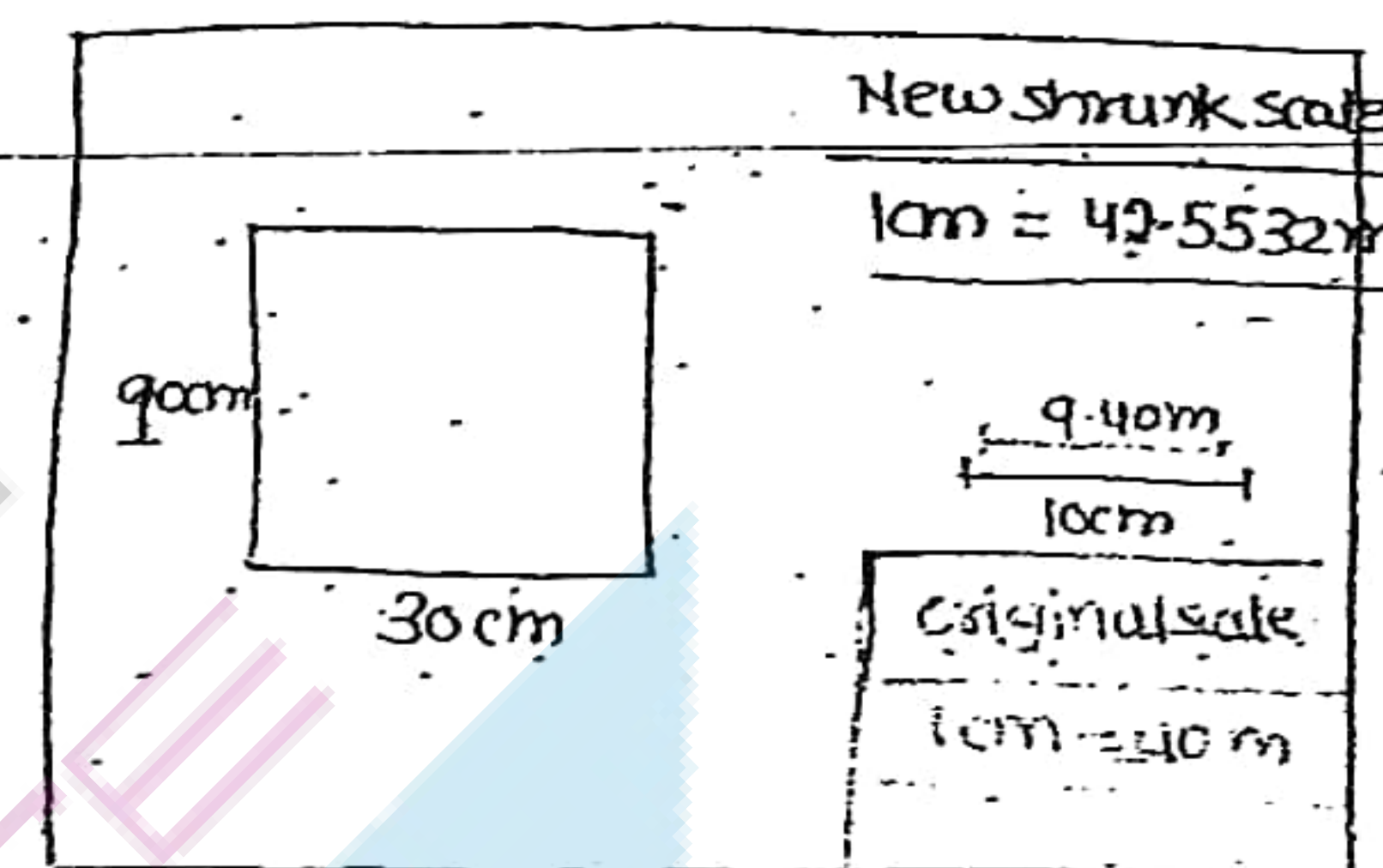
$$LC = \frac{1}{10} = 0.1\text{mm}$$

→ In this scale, Vernier scale moves in opposite dir<sup>n</sup> to the main scale.

(iii) Double Vernier -

The direct vernier (or Retrograde vernier), placed back to back with common zero value is called double vernier.

(4) Shrunk Scale -



Present dimension Read : 30cm x 10cm

9.40 line was 10cm long -

$$30\text{cm} \rightarrow \frac{10}{9.40} \times 30\text{cm}$$

$$= 31.915\text{cm}$$

$$\# \text{ Ground length} = 31.915 \times 40$$

$$= 1276.60\text{m}$$

$$10\text{cm} \rightarrow \frac{10}{9.4} \times 10 = 10.638\text{cm}$$

$$10.638 \times 40 = 425.53\text{m}$$

Actual area of plot

$$L \times B = 1276.6 \times 425.53 = 543234\text{m}^2$$



### # Shrink scale —

$$\text{Shrinkage Factor} = \frac{\text{shrink length}}{\text{original length}}$$

Shrink scale = shrinkage factor  $\times$  original scale.

### # shrinkage factor

$$\text{S.F.} = \frac{9.40}{10.0} = 0.94 \quad \left| \begin{array}{l} \text{original scale} \\ 1 \text{ cm} = 40 \text{ m} \\ = \frac{1}{4000} \end{array} \right.$$

$$\text{shrink scale} = 0.94 \times \frac{1}{4000} = \frac{1}{4255.319}$$

$$1 \text{ cm} = 42.5532 \text{ m}$$

$$\text{Area of the plot} = (30 \times 42.5532) \times (10 \times 42.5532)$$

$$A = 543232.23 \text{ m}^2$$

### # Correction due to Incorrect length of chain/tape :-

#### # gf

$L$  = Designated (True) length of a tape (should be) (say 30.0 m)

$L'$  = Wrong length of Tape (say 30.25 m)

$l'$  = wrong length of line measured.

$l$  = True length of line.

True  $\times$  True = wrong  $\times$  wrong

$$L \times l = L' \times l'$$

$$l = \left( \frac{L'}{L} \right) \times l' \quad \text{--- (A)}$$

example (1)  $L = 30 \text{ m}$   
 $L' = 30.25 \text{ m}$   
 $l' = 6500 \text{ m}$

$$l = \left( \frac{L'}{L} \right) \times l' = \frac{30.25}{30} \times 6500$$

$$l = 6554.167 \text{ m}$$

Ex: 2  $L = 30 \text{ m}$   
 $L' = 29.70 \text{ m}$   
 $l' = 6500 \text{ m}$

$$l = \left( \frac{L'}{L} \right) \times l' = \left( \frac{29.70}{30} \right) \times 6500$$

$$l = 6435 \text{ m}$$

Measured Value on ground.	Noted down value	Error	Correction
30.25 m (more)	30 m (less)	(-)ve (- 0.25)	(+)ve (+) 0.25 m
29.70 m (less)	30 m (more)	(+)ve (+ 0.3)	(-)ve (-) 0.30 m



### # Tape Correction :-

#### (1) Correction due to standardization :-

If length of Tape/chain is not correct.

#### # Correction per chain length :-

$$C = (L' - L) \left[ \begin{array}{l} +ve \rightarrow (+) \\ -ve \rightarrow (-) \end{array} \right] \begin{array}{l} \text{Correction} \\ \text{value} \end{array}$$

#### # Total Correction required :-

$$C_{\text{Total}} = (\text{No. of chains}) \times C$$

$$C_{\text{Total}} = \left( \frac{L'}{L} \right) (L' - L)$$

#### Case: (1)

$$L = 30 \text{ m}$$

$$L' = 30.10 \text{ m}$$

$$l' = 7200 \text{ m}$$

$$l = \left( \frac{L'}{L} \right) \times l'$$

$$= \frac{30.10}{30} \times 7200$$

$$l = 7224 \text{ m}$$

Correction per chain length

$$C = (L' - L) = (30.10 - 30.0)$$

$$C = 0.10 \text{ m}$$

$$\text{No. of chains} = \frac{7200}{30} = 240 \text{ chains}$$

$$\text{Total correction} = 240 \times 0.10 = +24 \text{ m}$$

$$\text{Corrected length} = 7200 + 24 = 7224 \text{ m}$$

# This correction may be either '+ve' or '-ve'

#### # In case of Area :-

$$A = \left( \frac{L'}{L} \right)^2 \times A' \quad \text{--- (2)}$$

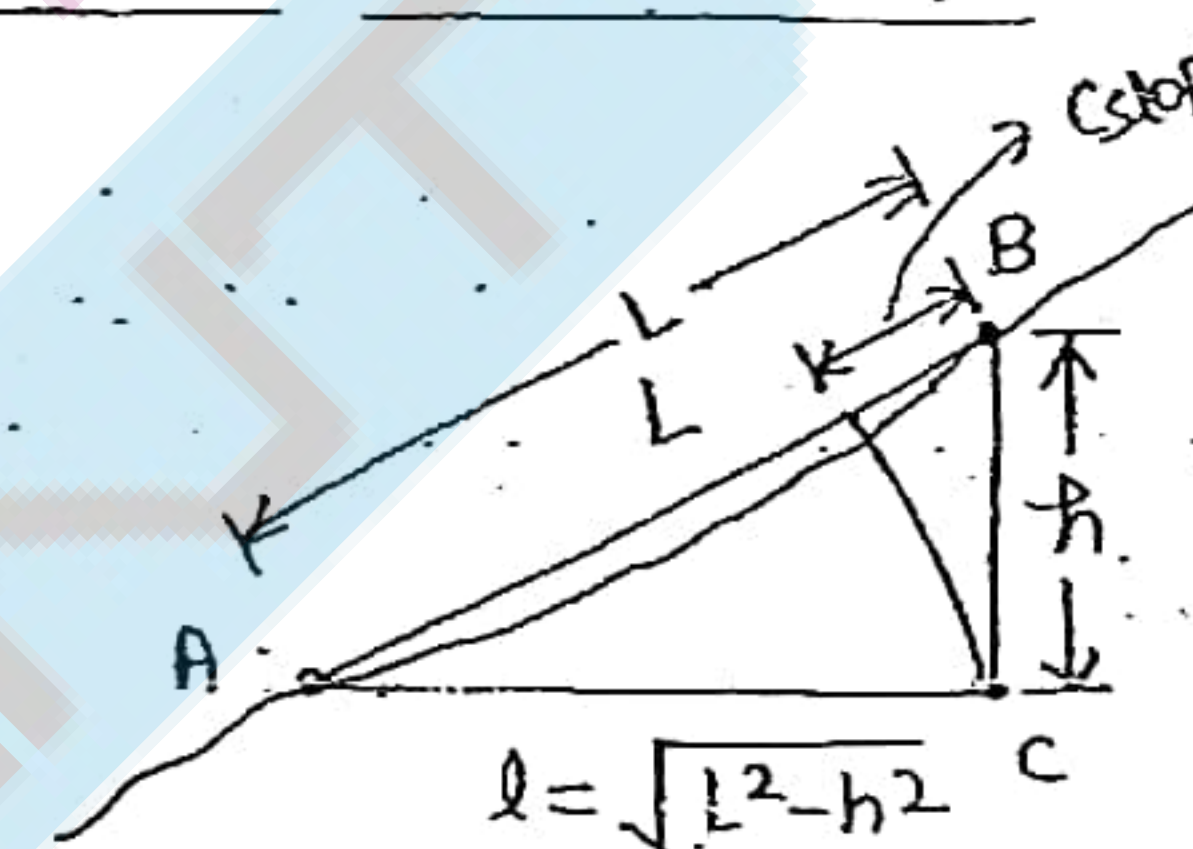
#### # In case of Volume :-

$$V = \left( \frac{L'}{L} \right)^3 \times V' \quad \text{--- (3)}$$

#### # For length :-

$$l = \left( \frac{L'}{L} \right) \times l' \quad \text{--- (1)}$$

#### # (2) Correction due to slope :-



Correction: For slope

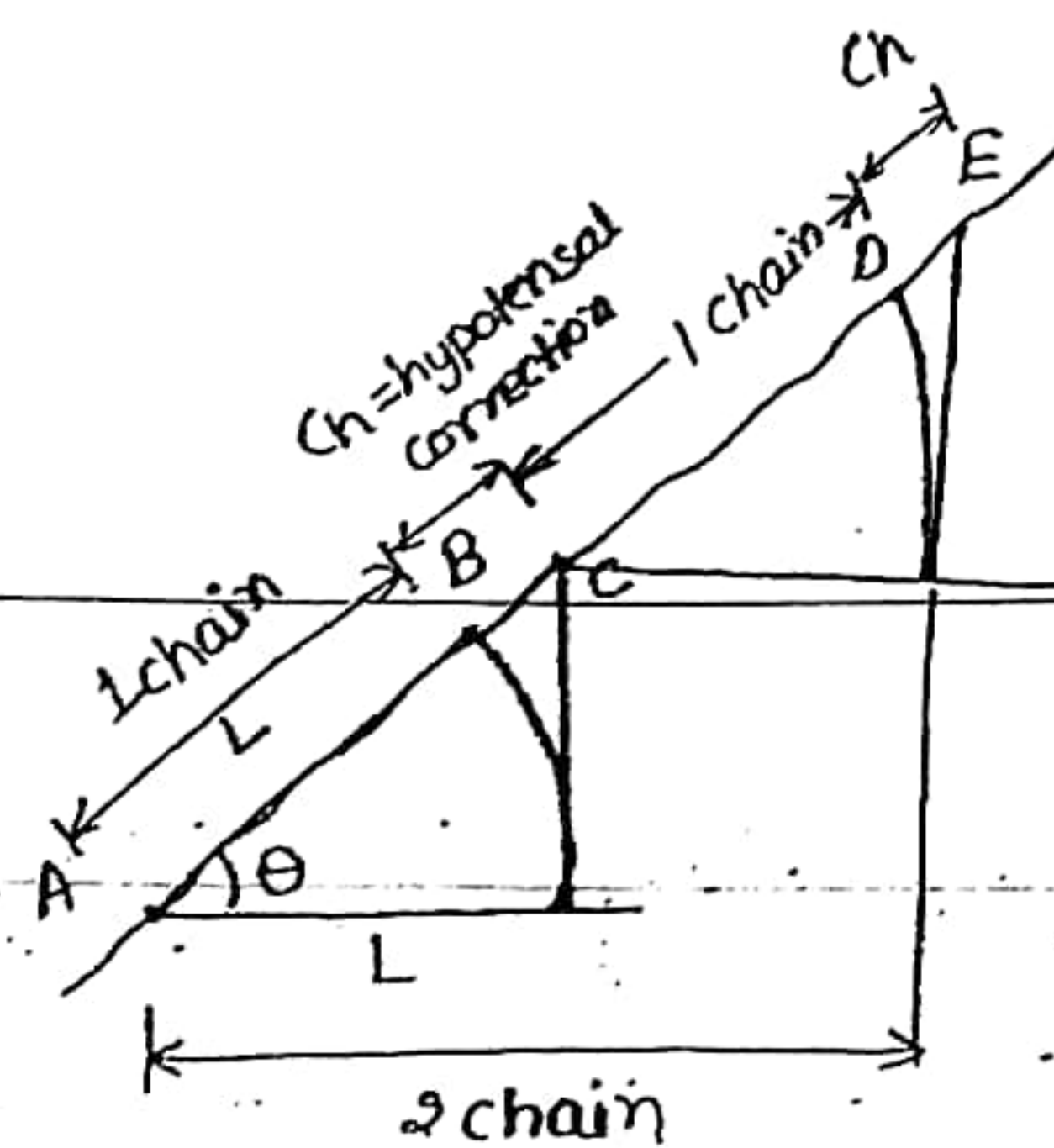
$$C_{\text{slope}} = L - l = L - \sqrt{L^2 - h^2} \rightarrow \text{exact difference.}$$

$$C_{\text{slope}} = -\frac{h^2}{2L} \rightarrow \text{approximate formula.}$$

This correction is always (-)ve.



### Hypotenusal Correction



$$C_h = L(\sec\theta - 1)$$

$$= 100 \text{ links} \times \frac{\theta^2}{2}$$

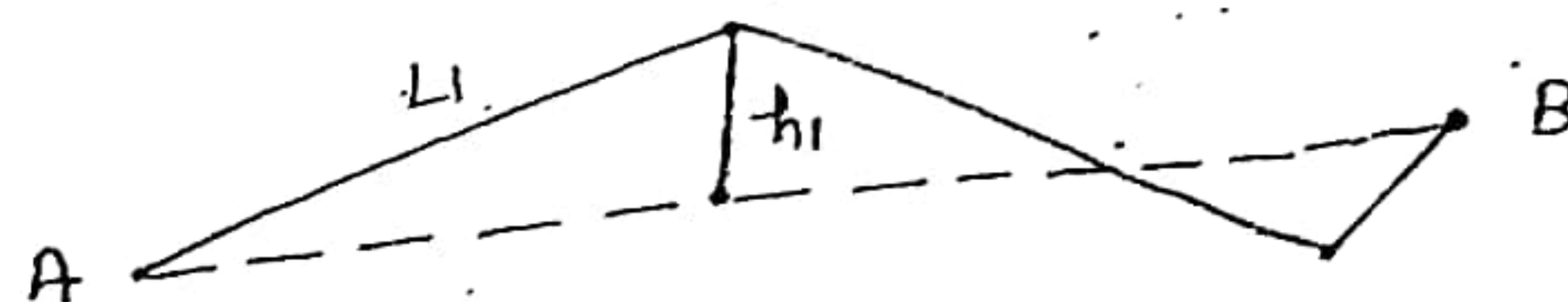
$$= 50 \theta^2 \text{ links}$$

$$AC = L \sec\theta$$

$$BC = AC - AB = L \sec\theta - L$$

$C_h = L(\sec\theta - 1) \rightarrow$  Hypotenusal Correction applied (added) after every chain length

### (3) Correction due to Alignment :-



Correction due to alignment -

$$C_{al} = L - \sqrt{L^2 - h^2}$$

$$C_{al} = \frac{h^2}{2L} \rightarrow \text{This correction is also } (-) \text{ve. (always)}$$

### (4) Correction due to temperature :-

Correction:

$$C_T = (T_m - T_0) \cdot \alpha \cdot L$$

$T_m$  = Temp. at the time of measurement.

$T_0$  = " " " Standard.

$T_m > T_0$	Length of Tape more	Error (-)ve	Correction (+)ve
$T_m < T_0$	Tape length less	+ve	(-)ve

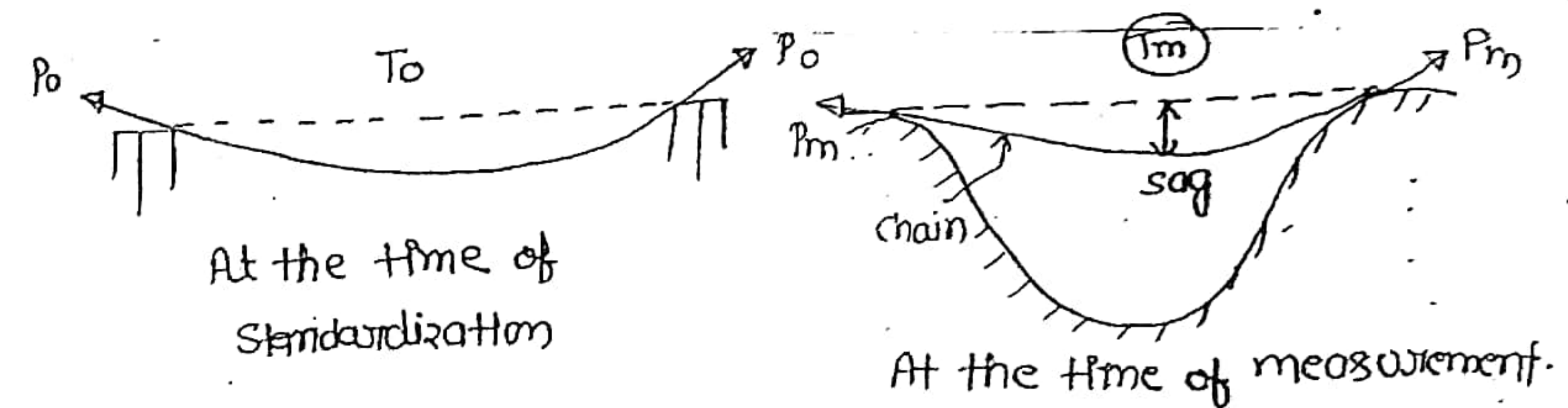
### (5) Correction due to pull applied :-

$$C_{pull} = \frac{(P_m - P_0) \cdot L}{AE}$$

$P_m$  = Pull applied at the time of measurement.

$P_0$  = " " " " Standardization.

$P_m > P_0$	Error (-)ve	Correction (+)ve
$P_m < P_0$	(+)ve	(-)ve



At the time of Standardization

At the time of measurement.



(6) Due to Sag:-

$$C_{sag} = \frac{(w \cdot L)^2 \cdot L}{24 P_m^2}$$

The correction is always (-ve)

$w =$  wt. of tape/unit length.

# Normal tension :-

It is the value of pull ( $P_m$ ) applied so that (+ve) pull correction is same as (-ve) sag correction and they neutralize each other.

$$C_{pull} = C_{sag}$$

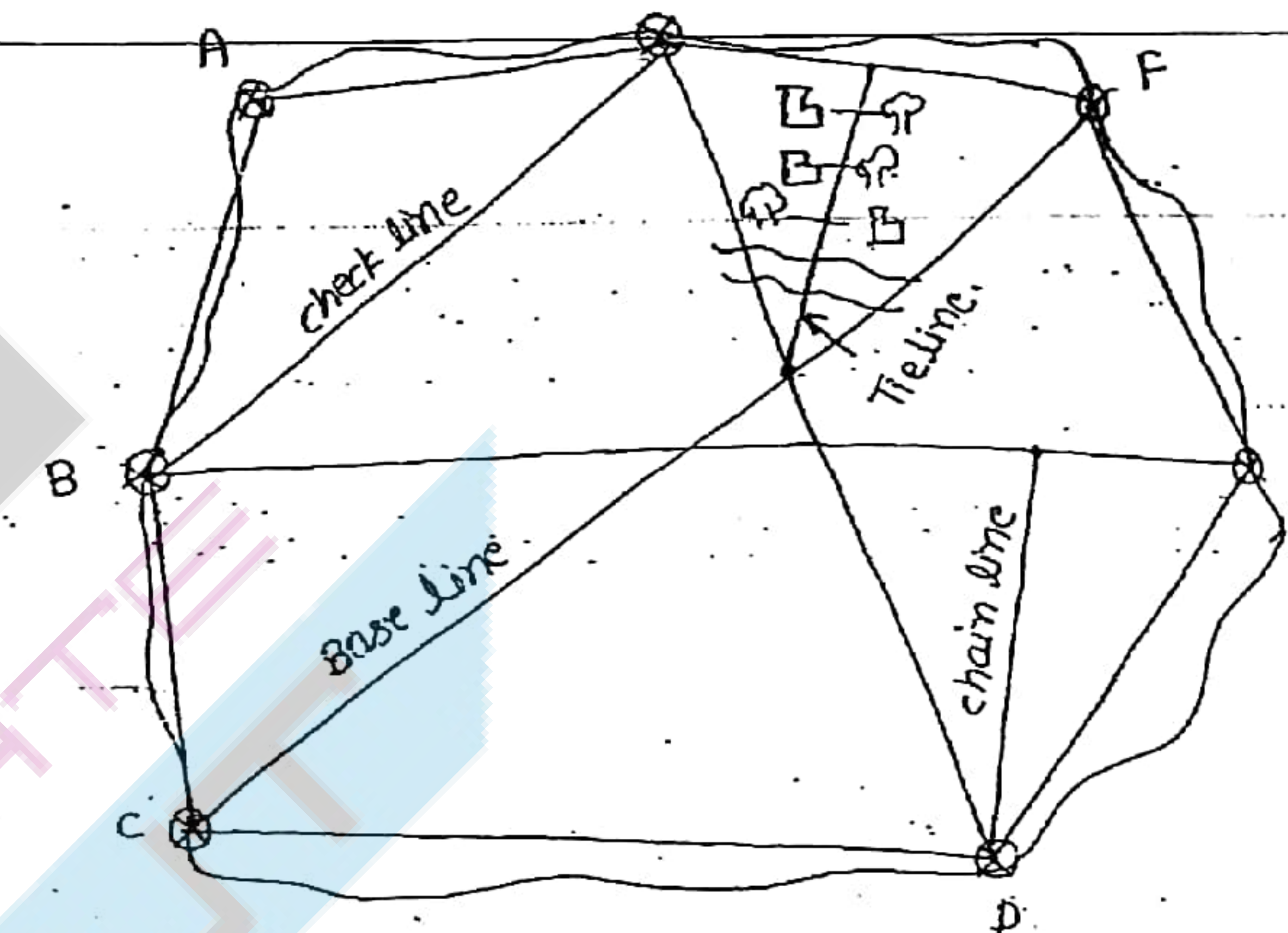
$$\frac{(P_m - P_0) L}{AE} = \frac{(wL)^2 L}{24 P_m^2}$$

Solve by trial & error

## CHAIN SURVEY

# Limiting length of offset :-

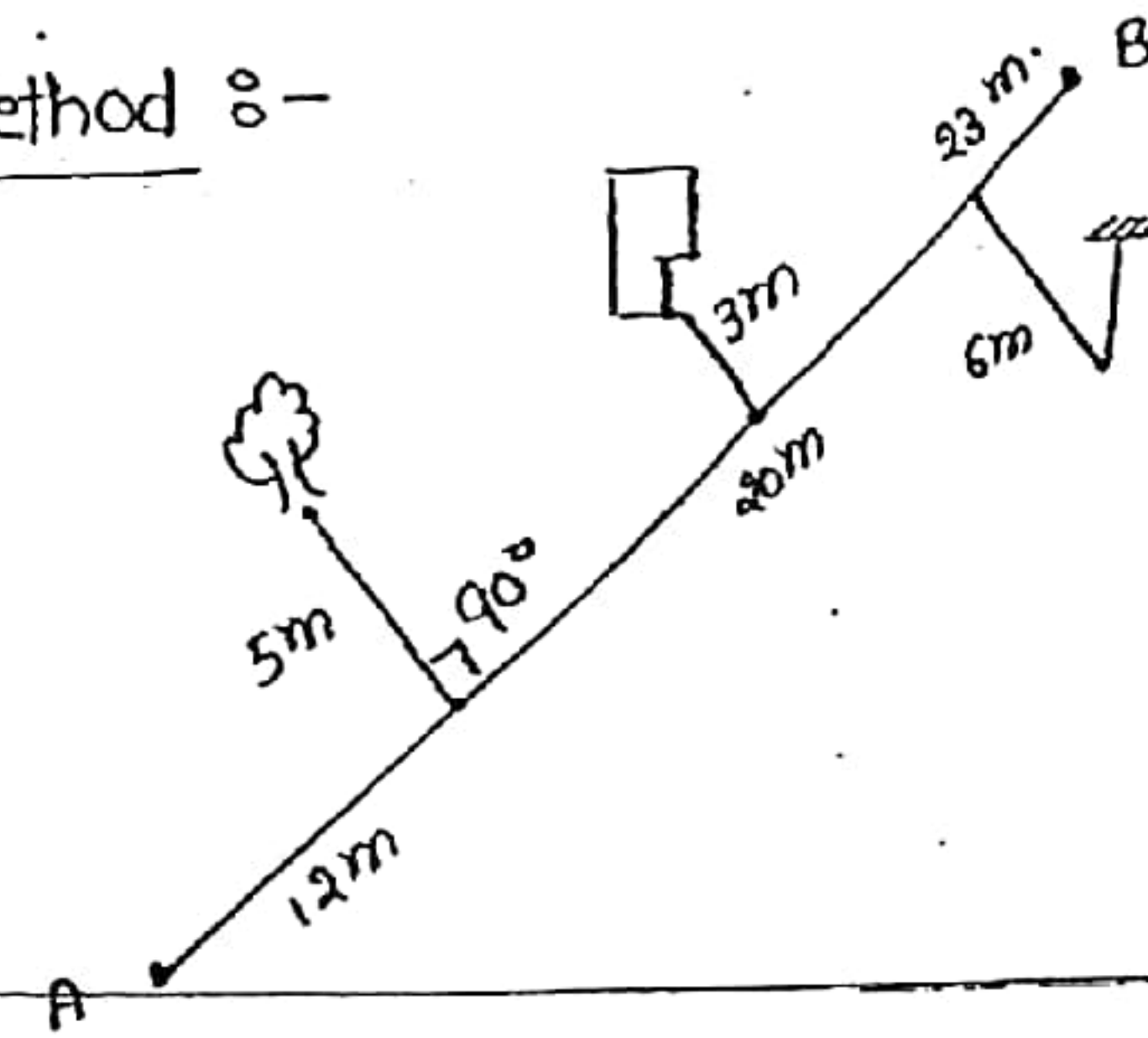
Important Points of chain survey :-



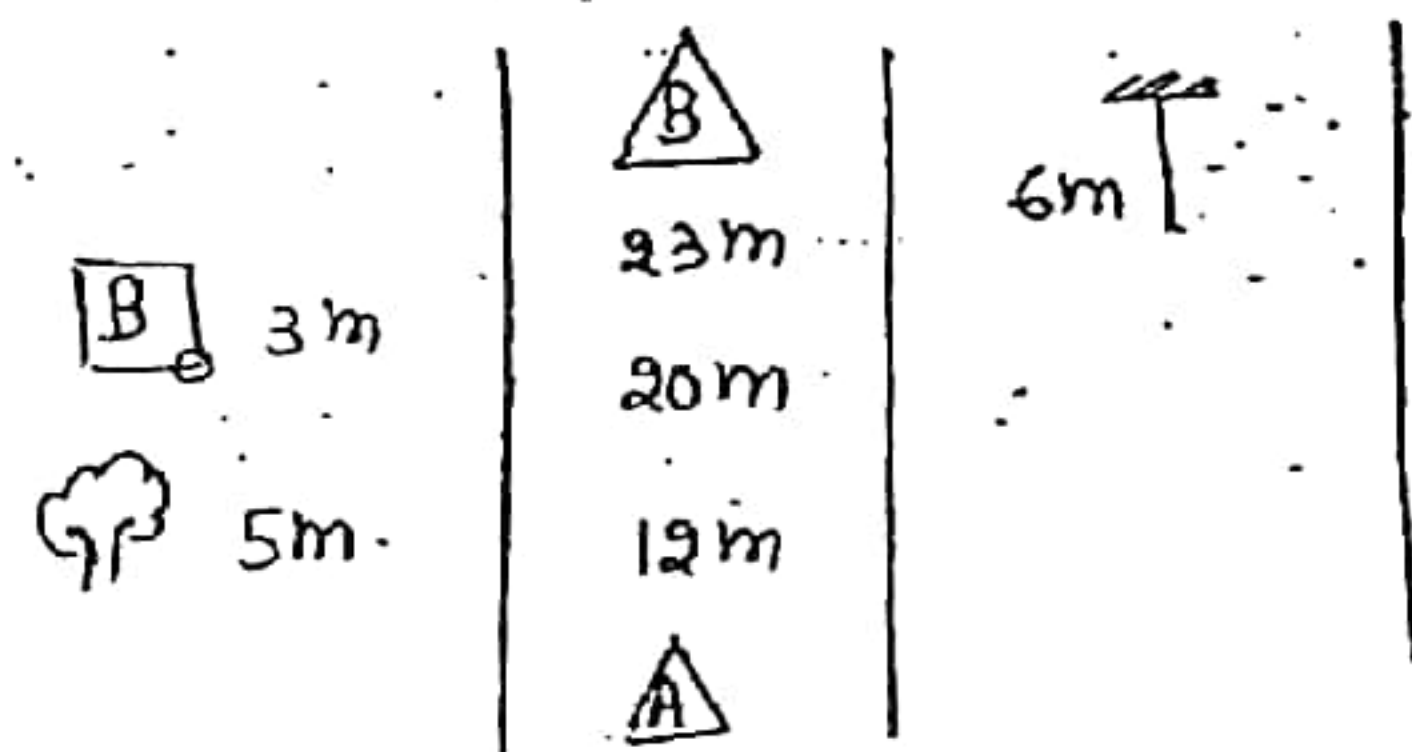
- (1) Main stations :- Major control points to divide the area are called main stations.
- (2) Main lines :- Lines joining main stations.
- (3) Base lines :- The longest line in the area that divide the total area almost in two parts.
- (4) check line :- Check lines are measured to check the accuracy of survey work done.
- (5) Tie line :- Any line drawn to collect more information about different objects in area (for collecting details.)



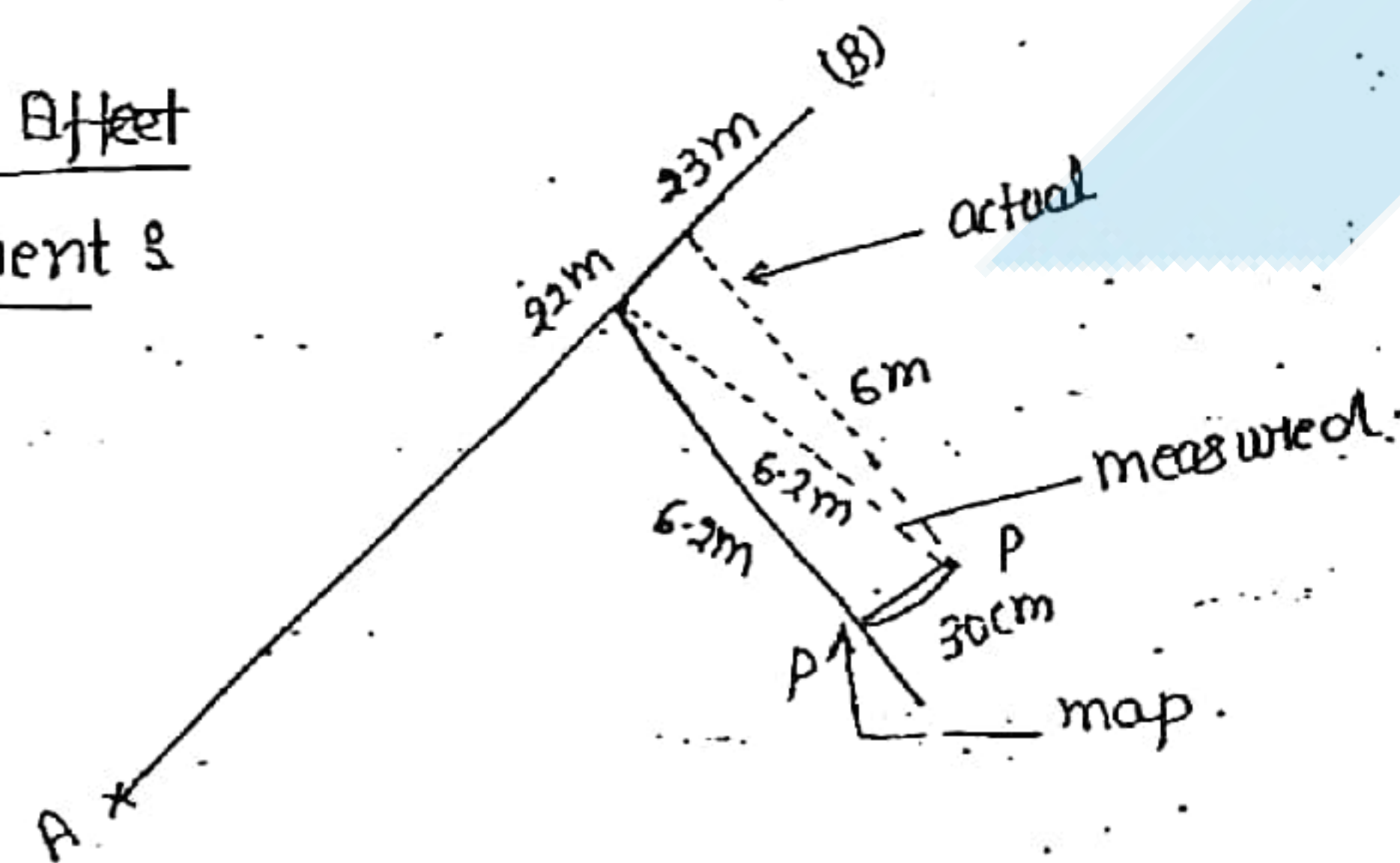
Offset Method :-



Field Book



Error In Offset Measurement :-



If error  
 $\pm 0.025 \text{ cm}$   
 $\pm 0.25 \text{ mm}$

If scale  $1 \text{ cm} = 20 \text{ m}$

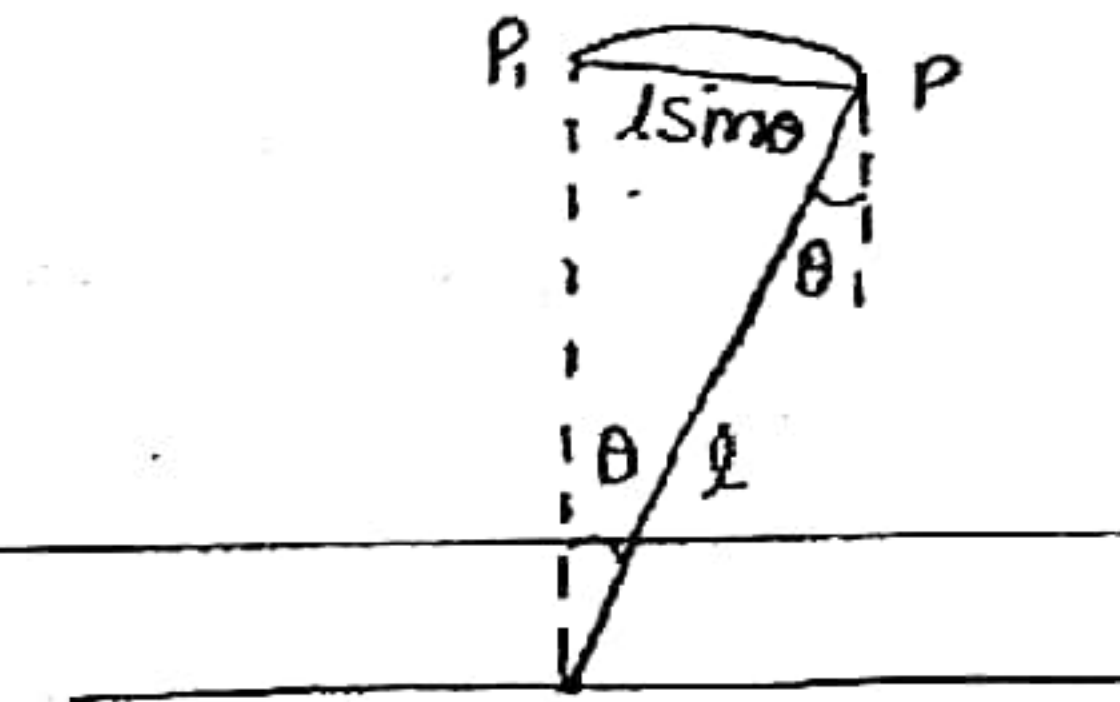
$$30 \text{ cm} \Rightarrow \frac{30 \text{ m}}{20 \text{ m}} = 0.015 \text{ cm} = 0.15 \text{ mm}$$

If scale  $1 \text{ cm} = 2 \text{ m}$

$$30 \text{ cm} \Rightarrow \frac{0.36}{2} = 0.15 \text{ cm} = 1.5 \text{ mm}$$

Limiting length of offset :-

Case (1) If the error is in laying dir<sup>n</sup> only :-



P = actual location of point on the ground.

P' = Plotted position of point on the drawing.

$\theta$  = error in laying direction.

Length of error on-ground

$$= l \sin \theta \text{ (meter)}$$

If scale of drawing  $1 \text{ cm} = S \text{ meter}$

Length of error on drawing :-

$$= \frac{l \sin \theta}{S} \text{ cm}$$

Max<sup>m</sup> length of error allowed on the drawing =  $0.25 \text{ mm}$

$$= 0.025 \text{ cm}$$

$$\text{So, } \frac{l \sin \theta}{S} \text{ cm} = 0.025 \text{ cm}$$

$$l = \frac{0.025 S}{\sin \theta}$$

Limiting length of offset



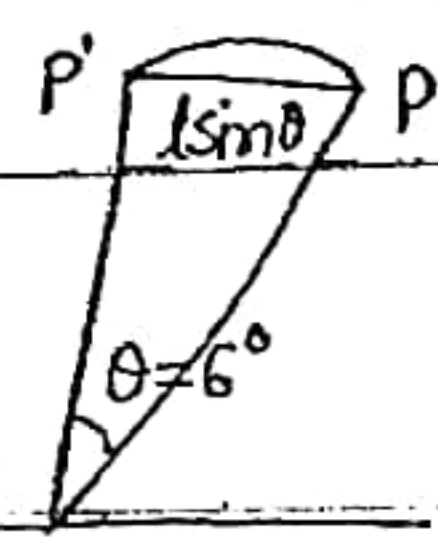
Ques: (1) If scale of drawing is

(i)  $1\text{cm} = 60\text{m}$

(ii)  $1\text{cm} = 5\text{m}$

For above cases, find out the limiting length of offset. If max<sup>m</sup> error allowed in laying dir<sup>n</sup> of the offset is  $6^\circ$ .

Solution:



max<sup>m</sup> error in dir<sup>n</sup>  $\theta = 6^\circ$

If limiting length of offset =  $l\text{m}$ .

max<sup>m</sup> error =  $l \sin \theta\text{m}$ .

Case (1) scale  $\Rightarrow 1\text{cm} = 60\text{m}$

Length of error on the drawing =  $\frac{l \sin \theta}{60}\text{cm} \neq 0.025\text{cm}$

$$l = \frac{0.025 \times 60}{\sin 6^\circ}\text{meter}$$

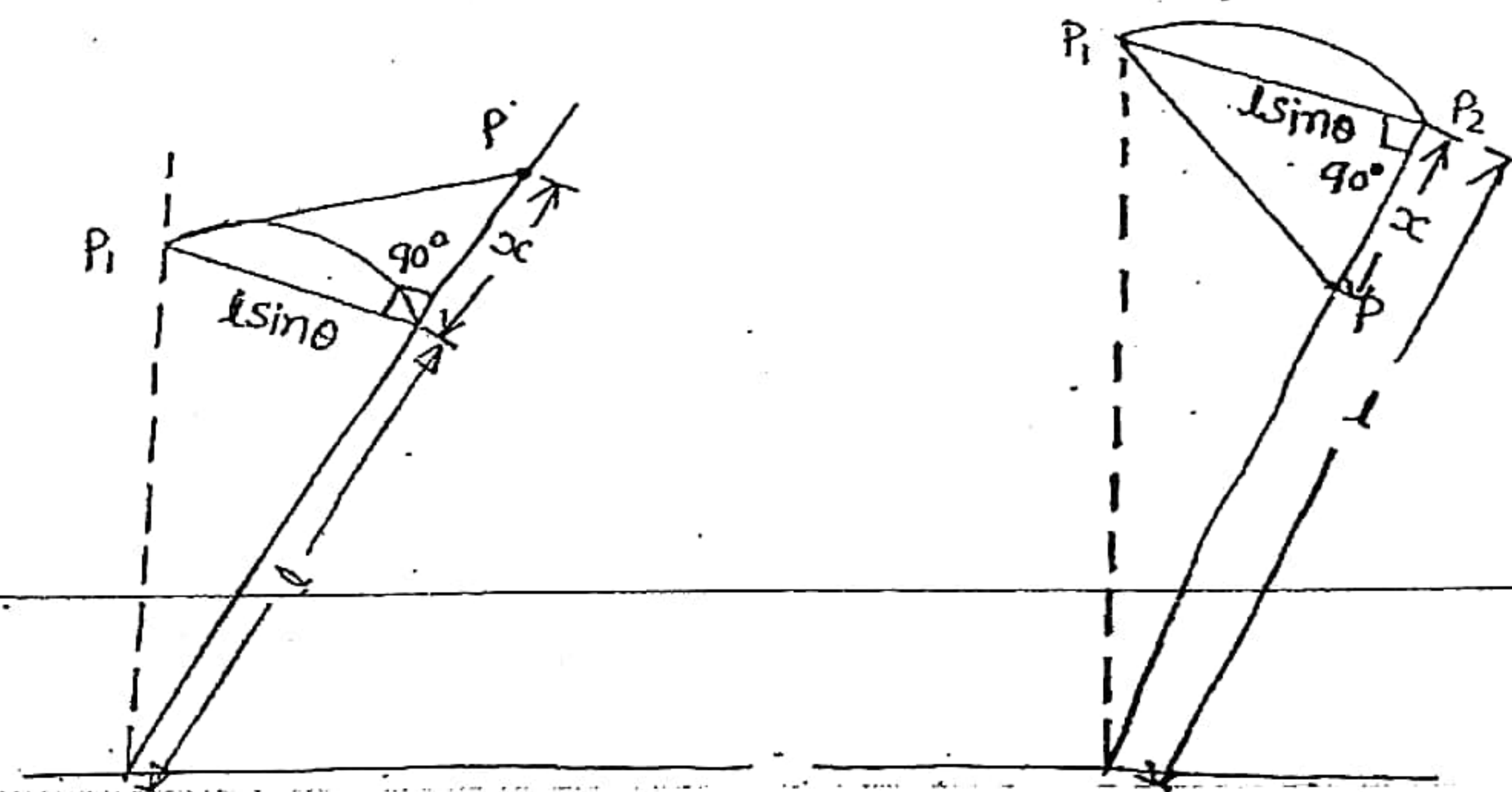
$l = 14.35\text{m}$

Case (2) scale  $\Rightarrow 1\text{cm} = 5\text{m}$

$l = \frac{0.025 \times 5}{\sin 6^\circ}\text{meter}$

$l = 1.19\text{m}$

Case (2) When the error in laying dir<sup>n</sup> as well as in length measurement also :-



P = Actual position of point on ground.

P<sub>1</sub> = Plotted position of point on drawing.

PB<sub>2</sub> =  $x$  = Error in length measurement.

PB<sub>2</sub> =  $l \sin \theta$  = Error due to wrong direction.

Total length of error on the ground

$PP_1 = \sqrt{PP_2^2 + PB_2^2} = \sqrt{(l \sin \theta)^2 + x^2}$

Length of error on the drawing (if scale is  $1\text{cm} = 5\text{meter}$ )

$= \frac{\sqrt{(l \sin \theta)^2 + x^2}}{5}\text{cm} \neq 0.025\text{cm}$

$(l \sin \theta)^2 + x^2 = (0.025 \times 5)^2$  — (A)

Ques (2) If max<sup>m</sup> length of an offset allowed is  $20\text{m}$  & max<sup>m</sup> error allowed in length measurement is  $25\text{cm}$ , scale of drawing is  $1\text{cm} = 50\text{m}$ . Find out max<sup>m</sup> error that can be allowed in laying dir<sup>n</sup> of offset. Max<sup>m</sup> error on drawing =  $0.025\text{cm}$ .



Sol<sup>n</sup>:

$$x = 25 \text{ cm}$$

$$= 0.25 \text{ m}$$

$$\theta = ?$$

$$l = 20 \text{ m}$$

Scale of drawing is  $1 \text{ cm} = 50 \text{ m}$

$$(l \sin \theta)^2 + x^2 = (0.025 \times 50)^2$$

$$(20 \sin \theta)^2 + (0.25)^2 = (0.025 \times 50)^2$$

$$\sin \theta = 0.061$$

$$\theta = 3.51$$

$$\theta = 3^\circ 30' 39''$$

$$\text{If } \theta = 5^\circ$$

$$l = ?$$

$$x = 25 \text{ cm} = 0.25 \text{ m}$$

$$1 \text{ cm} = 50 \text{ m} \text{ — scale}$$

$$(l \sin 5^\circ)^2 + (0.25)^2 = (0.025 \times 50)^2$$

$$l = 14.05 \text{ m}$$

## COMPASS SURVEY

### ⊕ System of angle measurement :-

#### (1) Most commonly used :-

$$1 \text{ Circum} = 360 \text{ degree}$$

$$1 \text{ degree} = 60 \text{ min.}$$

$$1 \text{ min} = 60 \text{ sec.}$$

#### (2) Centesimal system :-

$$1 \text{ Circum} = 400 \text{ grades}$$

$$1 \text{ grade} = 100 \text{ centigrade}$$

$$1 \text{ centigrade} = 100 \text{ Centi-Centigrade}$$

#### (3) Hour system :-

$$1 \text{ Circum} = 24 \text{ hours}$$

$$1 \text{ hour} = 60 \text{ min.}$$

$$1 \text{ min} = 60 \text{ sec.}$$

### ⊕ Relation :

$$24 \text{ hour} = 360 \text{ degree}$$

$$1 \text{ hour} = 15^\circ \text{ degree}$$

$$60 \text{ minute} = 15 \times 60 \text{ minute}$$

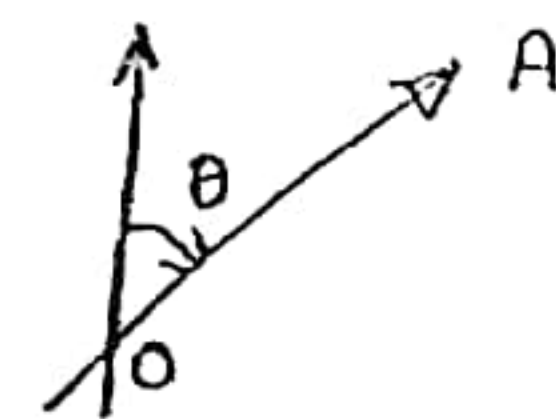
$$1 \text{ minute of time} = 15 \text{ minute of angle}$$

$$1 \text{ Sec. of time} = 15 \text{ Sec. of angle}$$



(2) Meridian :- A fixed line w.r.t. which bearing of line can be measured.

Bearing :- Angle measured w.r.t. a fixed meridian.



(3) True Meridian :- The line joining true north & true south on earth surface is called true meridian.

⇒ Axis of rotation of earth meets earth surface at two points, these are true north and true south.

True Bearing :- True Bearing is the bearing measured w.r.t. true meridian is called true bearing.

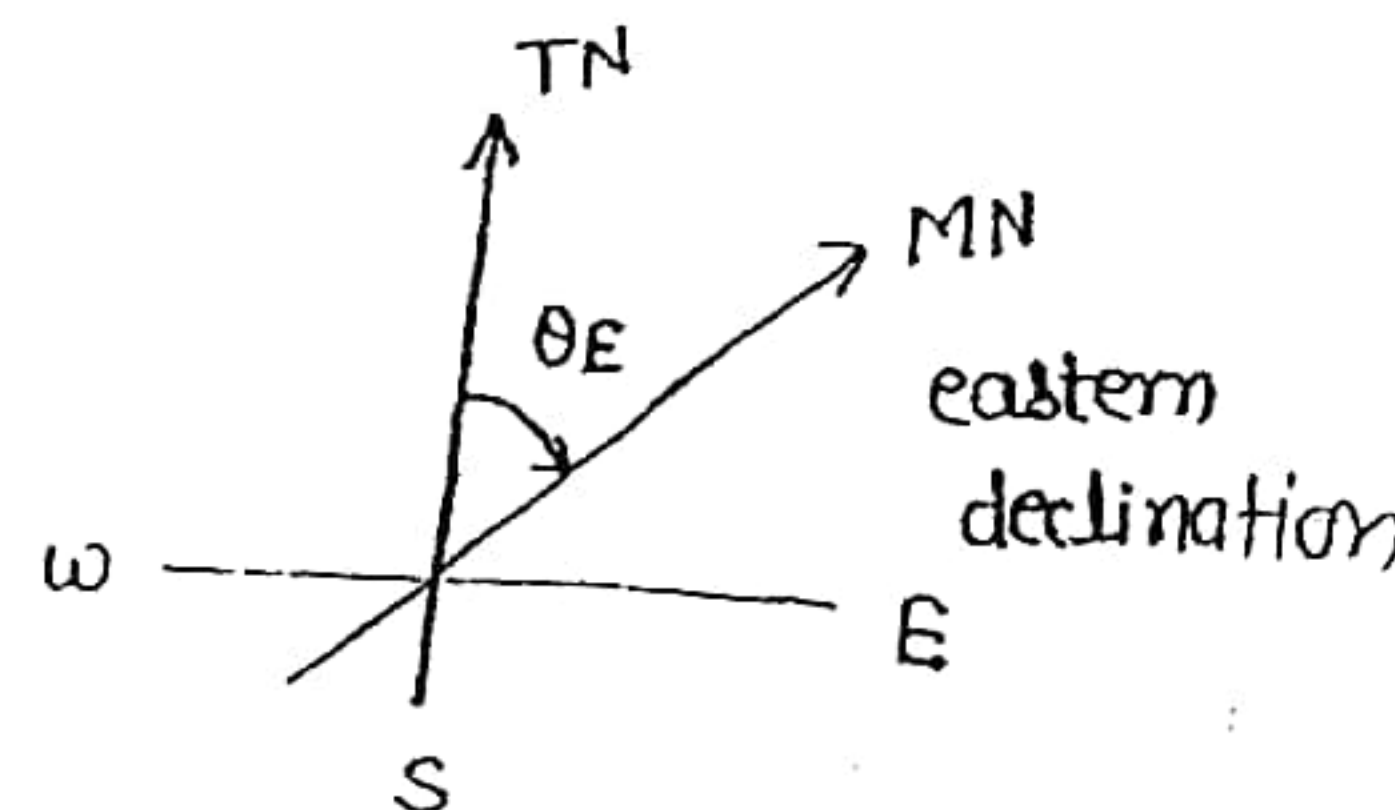
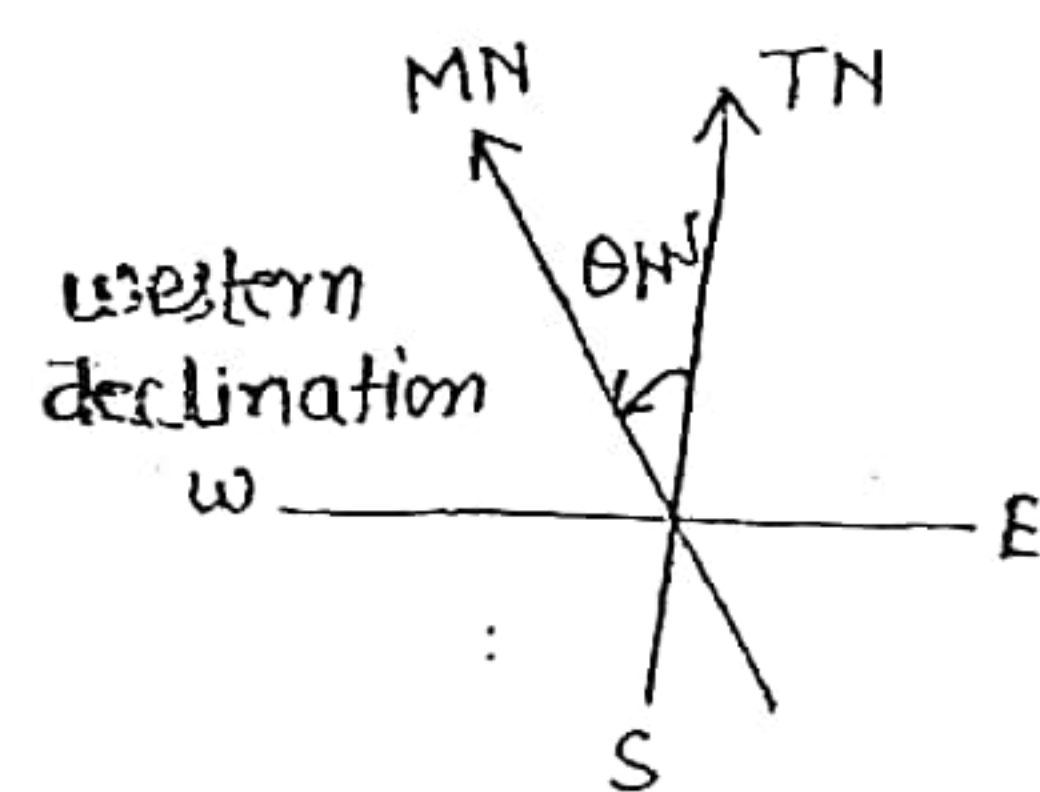
(4) Magnetic Meridian :-

Dir<sup>n</sup> of magnetic flux in the area show the magnetic meridian.

⇒ A magnet will show the dir<sup>n</sup> of magnetic north & south.

Magnetic Bearing :- The bearing measured w.r.t. magnetic meridian.

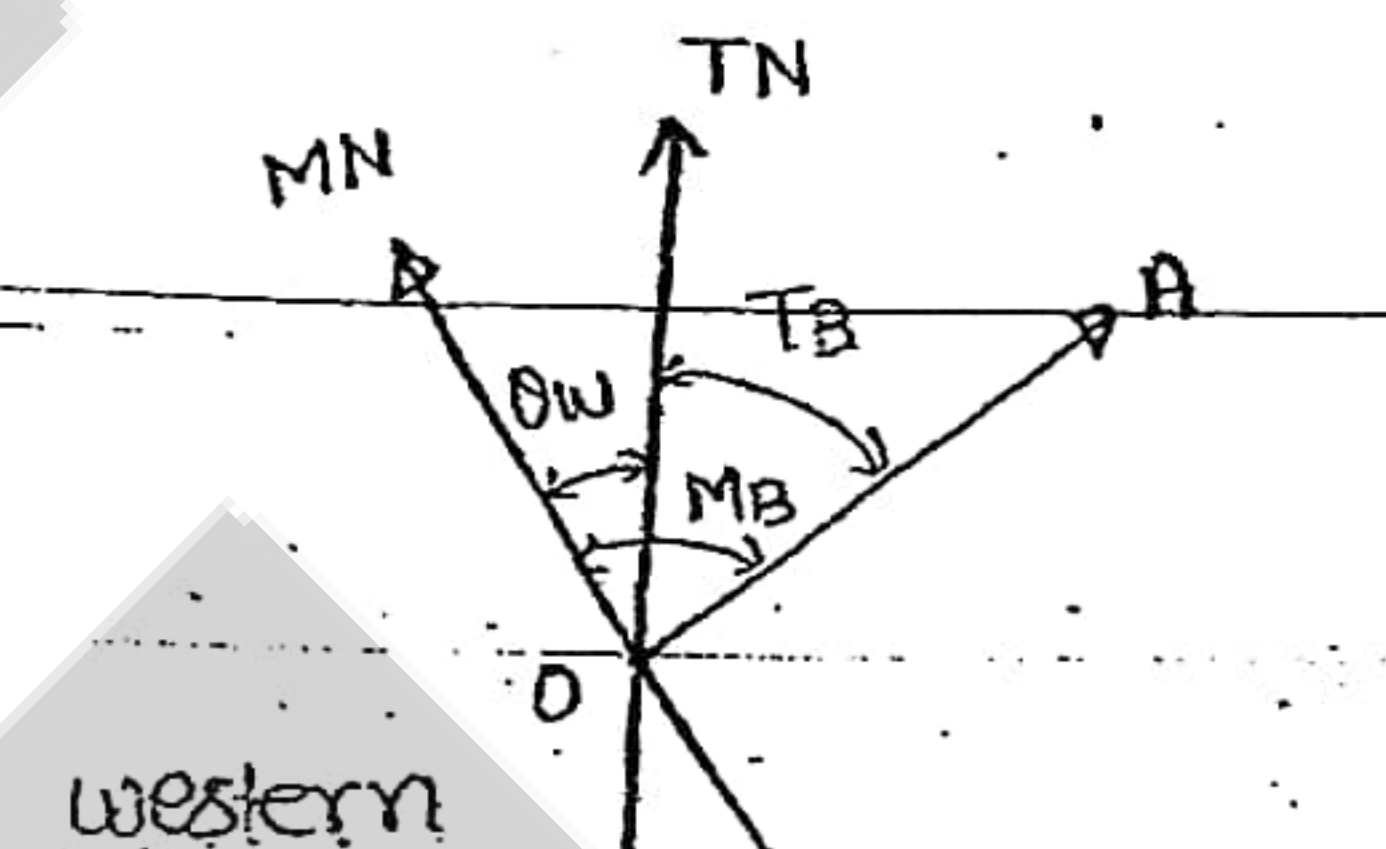
(5) Declination :- The difference of angle (horizontal angle) b/w true meridian & magnetic meridian is called declination.



Declination may be -

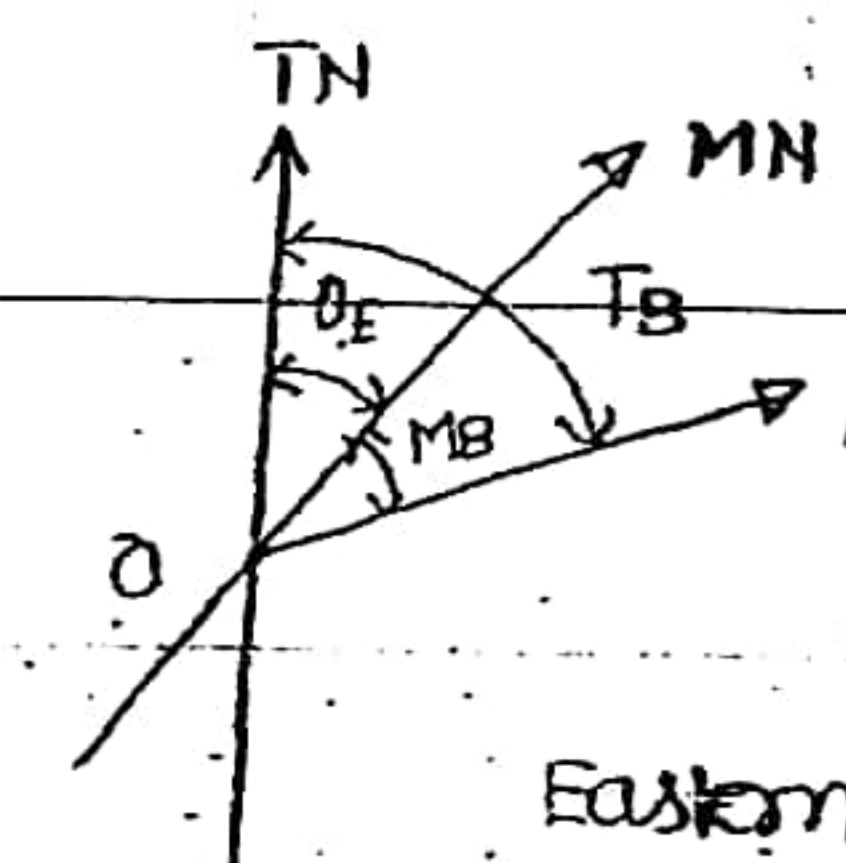
(i) Western → If MM (magnetic meridian) is towards west from T.M.

(ii) Eastern → " " " " " east from T.M.



$$T.B. = MB - \theta W$$

western declination



$$T.B. = MB + \theta E$$

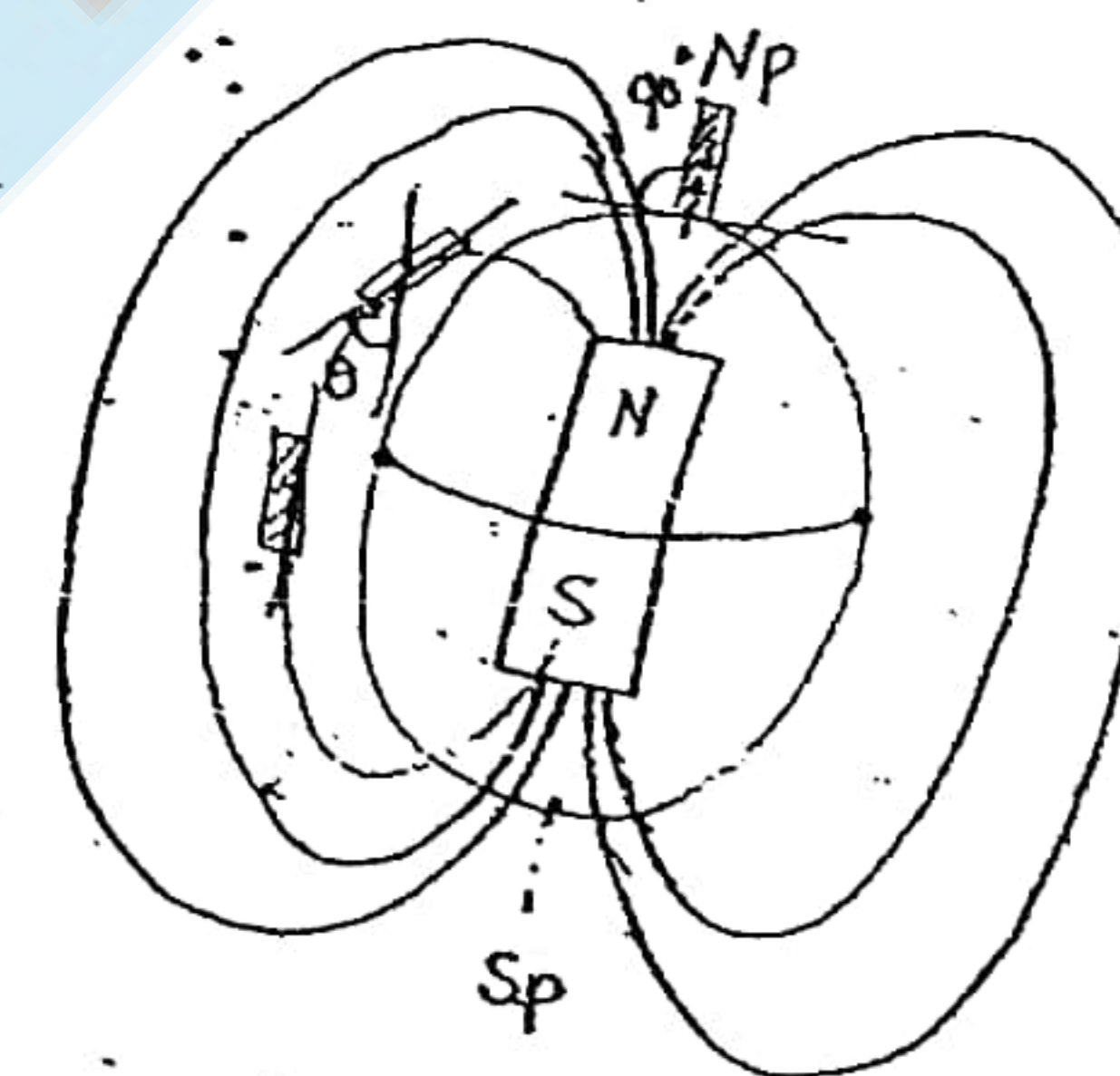
eastern declination.

(6) Angle of dip :-

Angle of dip -

at equator = 0

At poles = 90°



If a magnetic is hanged freely from its c.g it will align itself in the direction of magnetic flux. It will make some angle with horizontal. The vertical angle of magnetic flux direction w.r.t. horizontal at that area is called angle of dip.



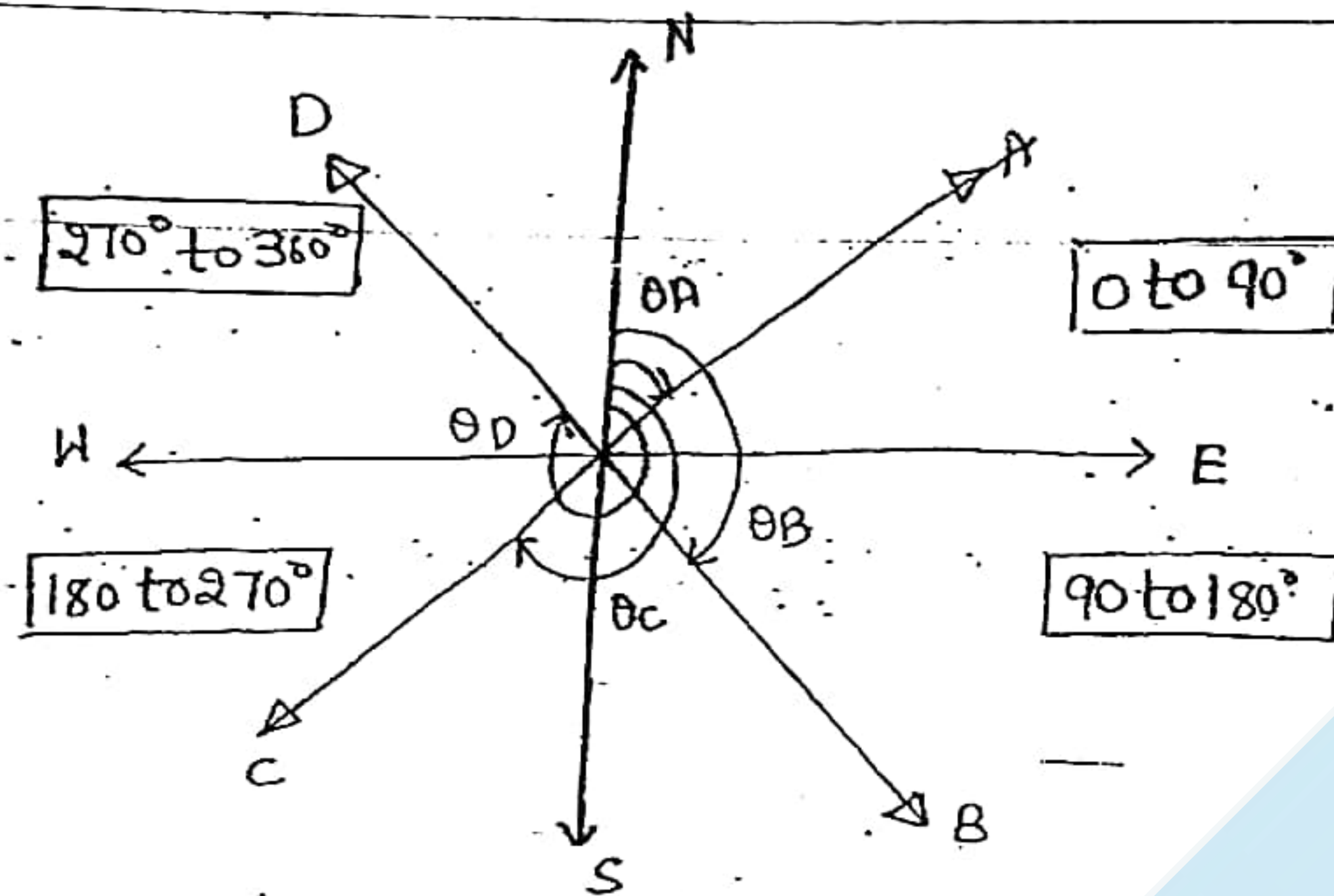
Note 8- At equator = dip angle = 0

At pole = 90°

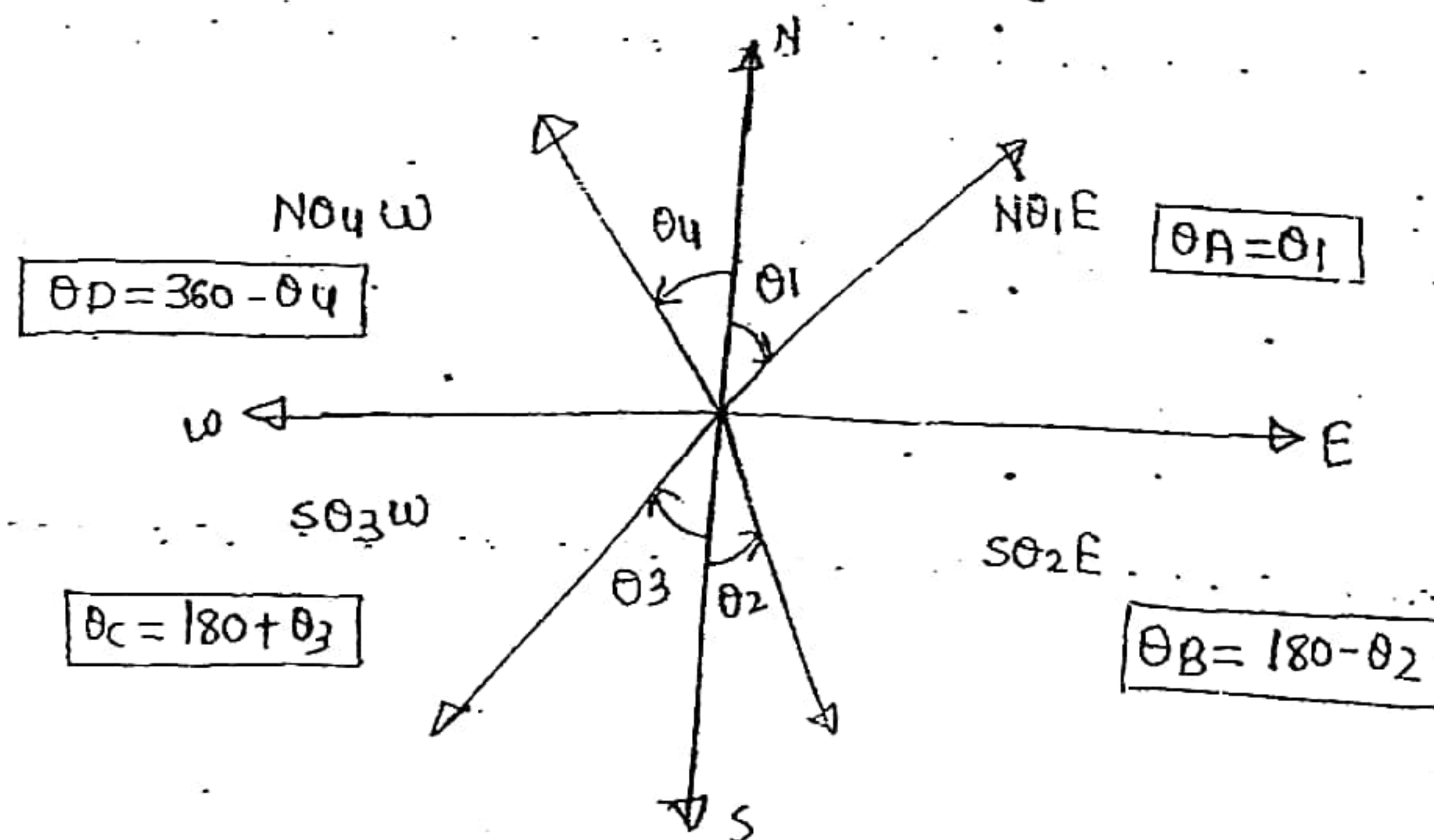
at other place = 0 to 90°

### System of Bearing Measurements-

#### (1) WCB Method (Whole circle Bearing method)



#### (2) QSB Method (Quadrantal system of Bearing) Also called Reduced bearing.



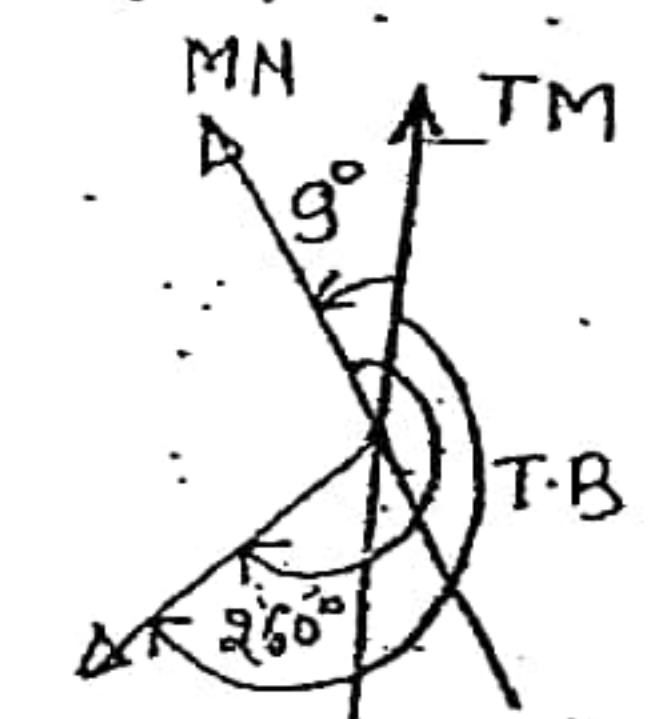
WCB		QSB	
$\theta_A$	0 to 90°	N $\theta_1$ E	$\theta_1 = \theta_A$
$\theta_B$	90° to 180°	S $\theta_2$ E	$\theta_2 = \theta_B - 180^\circ$
$\theta_C$	180° to 270°	S $\theta_3$ W	$\theta_3 = \theta_C - 180^\circ$
$\theta_D$	270° to 360°	N $\theta_4$ W	$\theta_4 = \theta_D - 360^\circ$

Ques: (1) If magnetic bearing of a line is 260° and declination is (i) 9° W (ii) 6° E calculate T.B. of the line -

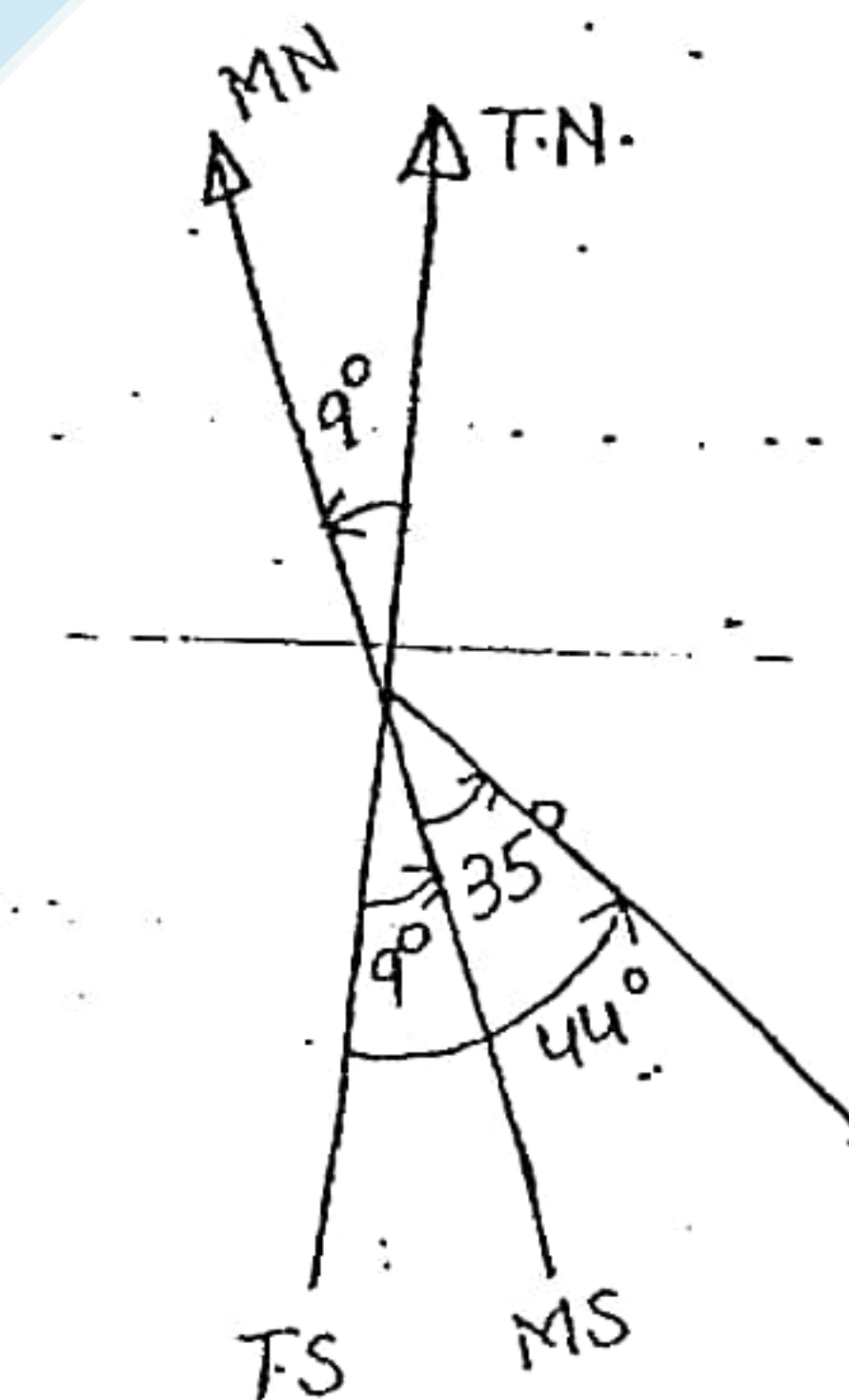
$$T.B. = M.B. \pm \theta$$

(i) 9° W  $\Rightarrow T.B. = 260^\circ - 9^\circ = 251^\circ$

(ii) 6° E  $\Rightarrow T.B. = 260^\circ + 6^\circ = 266^\circ$



Sol: (2) If magnetic bearing = S 35° E declination (i) 9° W (ii) 5° E calculate T.B.



W.C.B

$$180 - 35 = 145^\circ$$

$$M.B. = 145^\circ$$

(i) 9° W

$$T.B. = 145^\circ - 9^\circ = 136^\circ$$

$$S (180 - 136^\circ) E$$

$$T.B. = S 44^\circ E$$



(ii)  $5^\circ E$ 

$$T.B = 145^\circ + 5^\circ$$

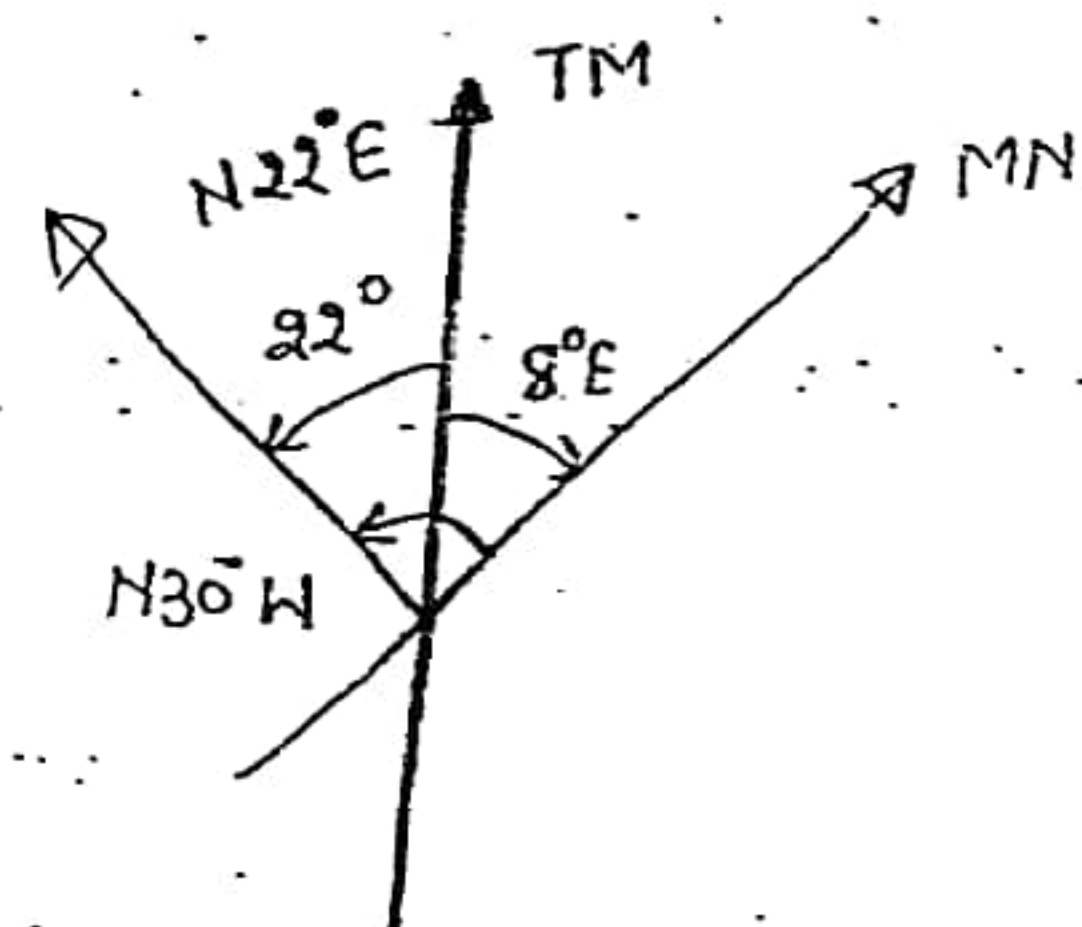
$$= 150^\circ$$

$$= S(180 - 150)E$$

$$= \underline{S 30^\circ E}$$

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Ques 8(2) If magnetic bearing of a line is  $N 30^\circ W$ , and declination is  $8^\circ E$ . Calculate the true bearing of line.



Mag. Bearing of line =  $N 30^\circ W$ .

$$= 360^\circ - 30^\circ$$

$$= 330^\circ \text{ (MCB)}$$

(ii) True Bearing of line

$$T.B = MB + \theta E$$

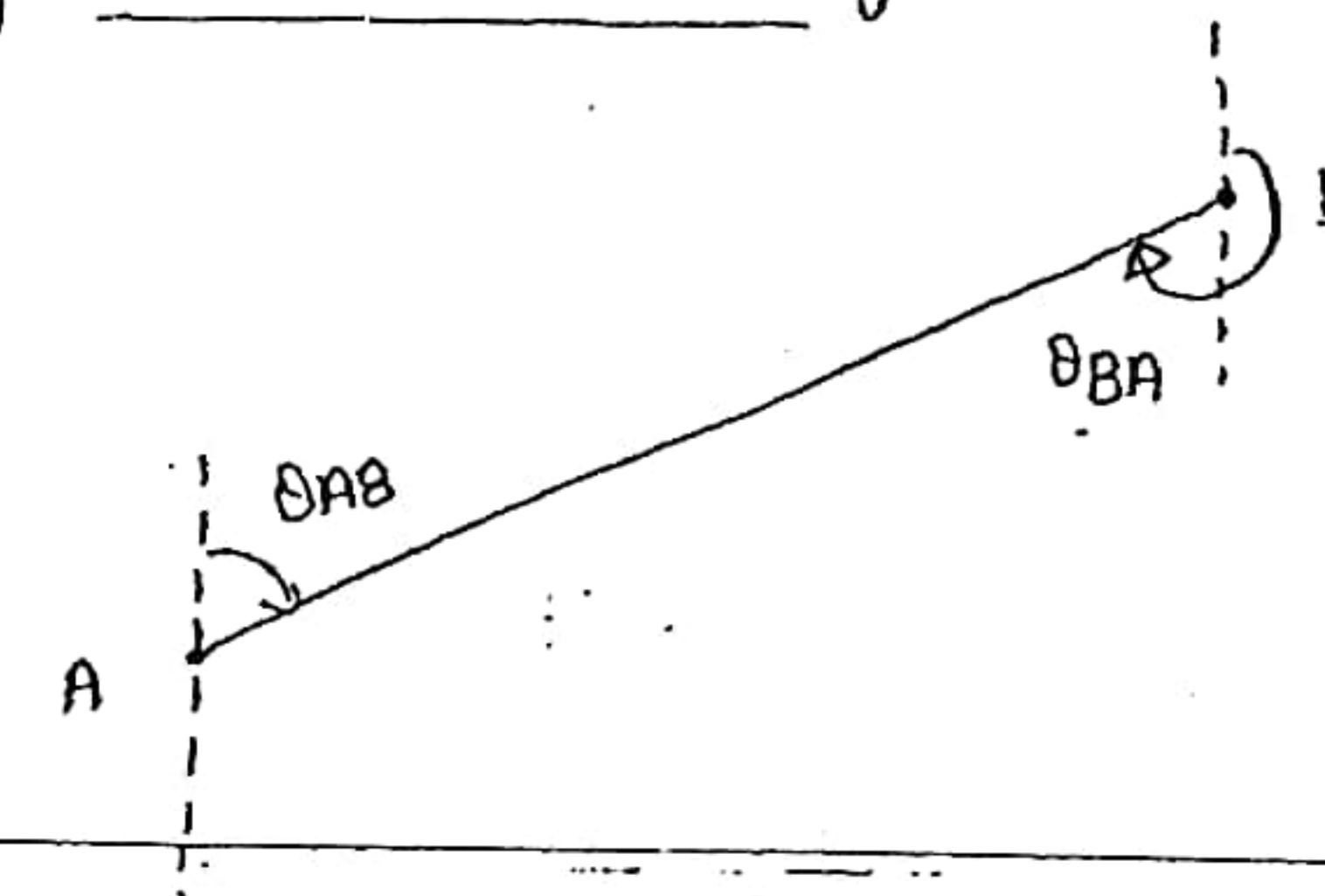
$$= 330^\circ + 8^\circ$$

$$= 338^\circ$$

$$= N(360 - 338)W$$

$$= \underline{N 22^\circ W}$$

Fore Bearing and Back Bearing :-



For line AB

$$F.B = \theta_{AB}$$

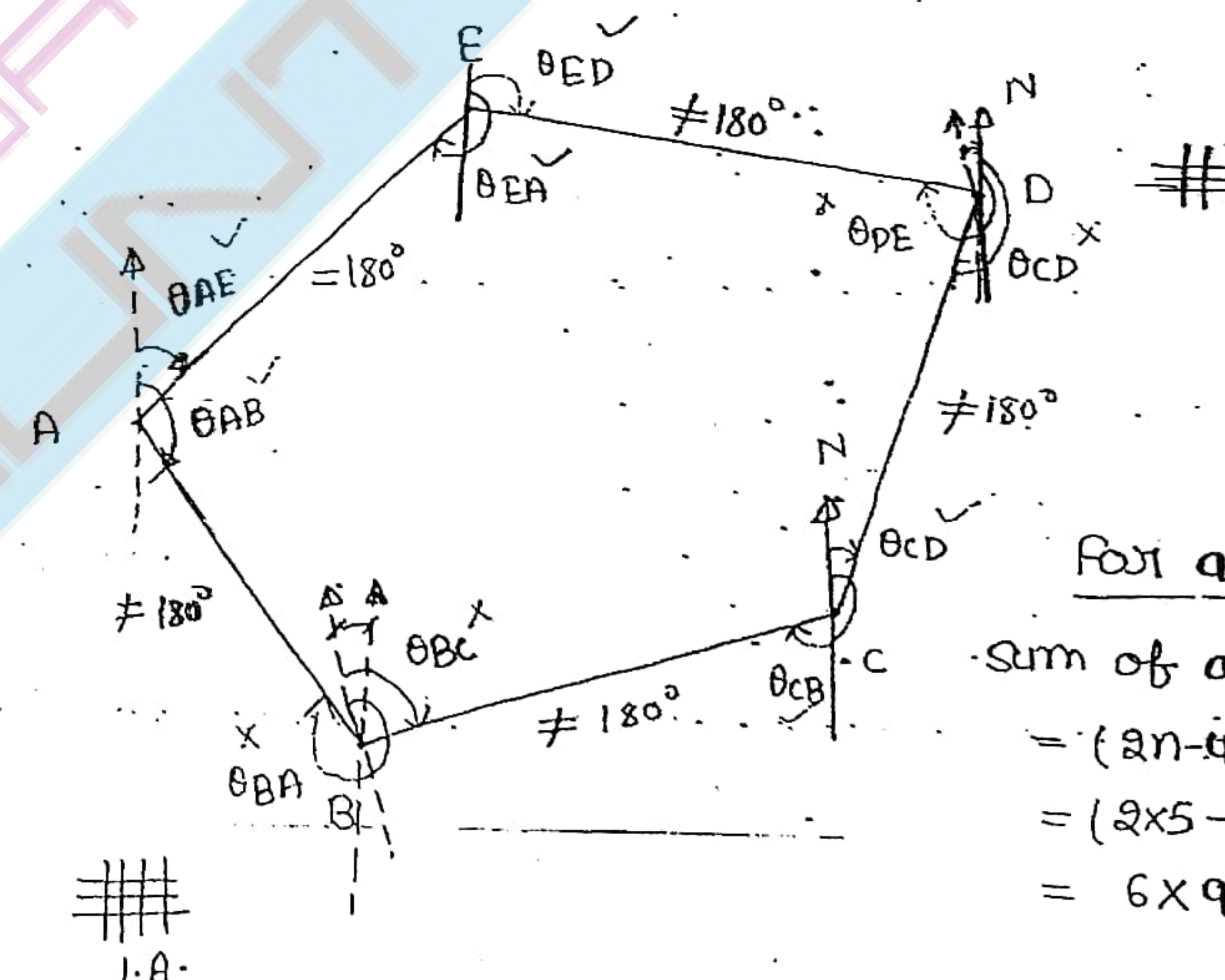
$$B.B = \theta_{BA}$$

For line BA

$$F.B = \theta_{BA}$$

$$B.B = \theta_{AB}$$

Local Attraction :-



For a polygon

Sum of all angles

$$= (2n - 4) \times 90^\circ$$

$$= (2 \times 5 - 4) \times 90^\circ$$

$$= 6 \times 90^\circ = 540^\circ$$

Important points :-

- 1) If difference of F.B & B.B is not equal to  $180^\circ$ , Any one station or both may be affected from local attraction.



(2) If difference of a line

$$F.B - B.B = 180^\circ$$

Both station are free from local attraction.

If AE correct Angle  $\Rightarrow AE - EA = 180^\circ$

Correct Angles :- All angles measured at A & E.

AE - ✓ AB - ✓  
EA - ✓ ED - ✓

(3) If any station is affected from the local attraction, error and correction on all reading taken from that station will be same.

D is affect

Correction in DE = Correction in DE

If B is affected - Correction in BA - Correction in BC

(4) For a closed traverse

$$\text{Sum of all internal angles} = (2n-4) \times 90^\circ$$

Ques: (1) following bearing were taken for a closed traverse

Line	F.B	B.B	difference
AB	75° 5'	254° 20'	x
BC	115° 20'	296° 35'	x
CD	165° 35'	345° 35'	180°
DE	224° 50'	44° 5'	x
EA	304° 50'	125° 5'	x

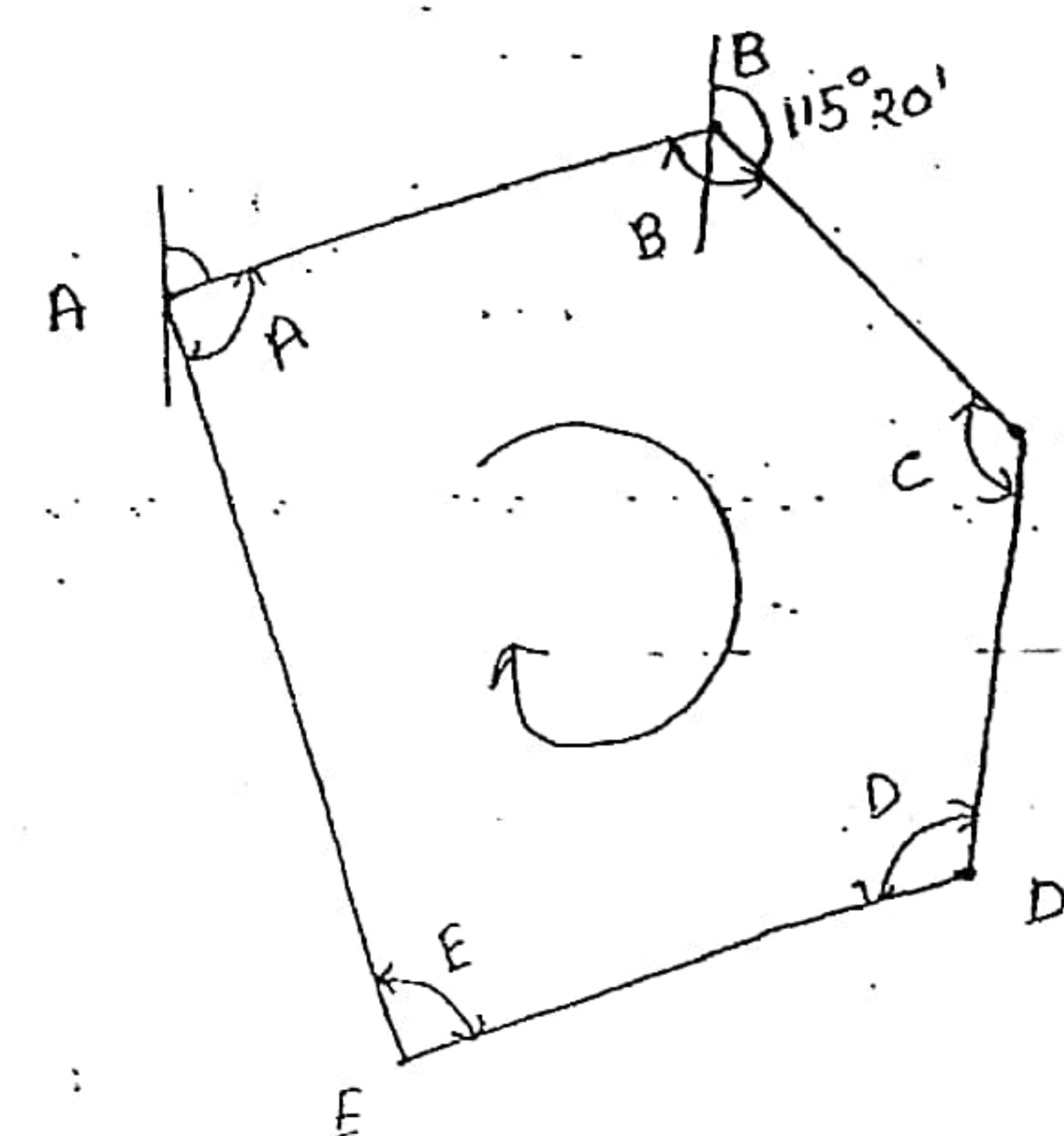
Find out the correct bearing of lines which station are free from local attraction

(#) C & D are free from local Attraction.

Line	Bearing	Correction	Correct Bearing	Calculation
AB	75° 5'	+ 0° 30'	75° 35'	
BA	254° 20'	+ 1° 15'	255° 35'	75° 35' + 180°
BC	115° 20'	1° 15'	116° 35'	
CB	296° 35'	0°	296° 35'	116° 35' + 180°
CD	165° 35'	0°	165° 35'	
DC	345° 35'	0°	345° 35'	
DE	224° 50'	0°	224° 50'	
ED	44° 5'	+ 0° 45'	44° 50'	224° 50' - 180°
EA	304° 50'	+ 0° 45'	305° 35'	
AE	125° 5'	+ 0° 30'	125° 35'	305° 35' - 180°

\*\* Method for Calculation of Internal angle for a closed traverse:

Step (1) Draw the traverse.





Step: 2 Draw a clockwise circle inside the traverse.

Step: 3

Internal Angle.

$$\begin{aligned}\angle A &= AE - AB \\ \angle B &= BA - BC \\ \angle C &= CB - CD \\ \angle D &= DC - DE \\ \angle E &= ED - EA\end{aligned}$$

Step: 4 If any angle is (-)ve then add  $360^\circ$ .

Step: 5 If any angle becomes more than  $360^\circ$ , then deduct  $360^\circ$ .

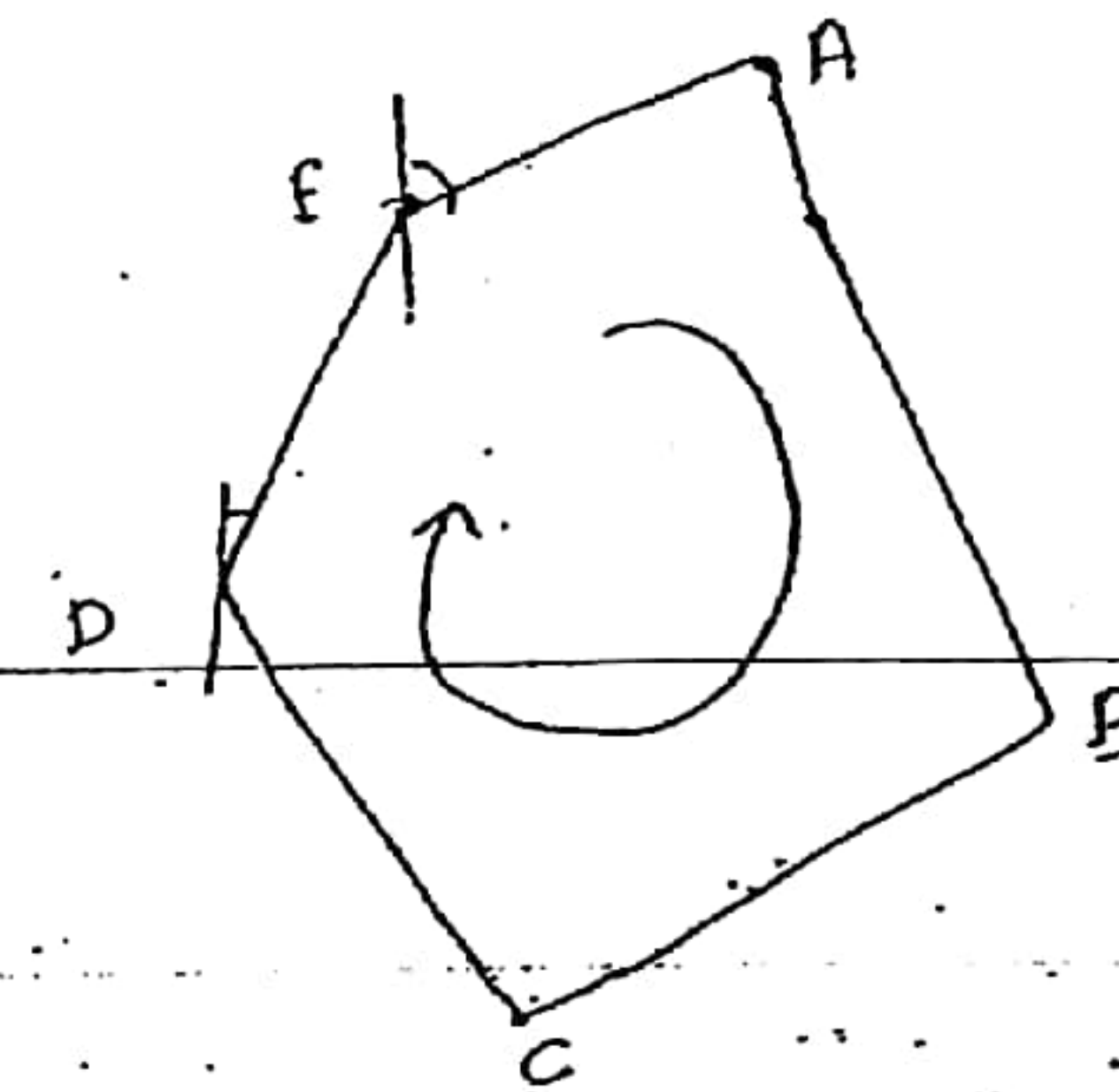
Ques: Following are the bearing of a closed traverse -

Line	F-B	B-C	diff.
AB	$142^\circ 30'$	$322^\circ 30'$	$180^\circ$
BC	$223^\circ 15'$	$44^\circ 15'$	$\times$
CD	$287^\circ$	$107^\circ 45'$	$\times$
DE	$12^\circ 45'$	$193^\circ 15'$	$\times$
EA	$60^\circ$	$239^\circ$	$\times$

Solution:

	Bearing	Correction	Corrected Bearing	Calculation
AB	$142^\circ 30'$	0	$142^\circ 30'$	
BA	$322^\circ 30'$	0	$322^\circ 30'$	
BC	$223^\circ 15'$	0	$223^\circ 15'$	
CB	$44^\circ 15'$	$-1^\circ 0'$	$43^\circ 15'$	$223^\circ 15' - 180$
CD	$287^\circ$	$-1^\circ$	$286^\circ$	
DC	$107^\circ 45'$	$-1^\circ 45'$	$106^\circ$	$286^\circ - 180$
DE	$12^\circ 45'$	$-1^\circ 45'$	$11^\circ 00'$	
ED	$193^\circ 15'$	$-2^\circ 15'$	$191^\circ$	$11^\circ + 180$
EA	$60^\circ$	$-2^\circ 15'$	$57^\circ 45'$	
AE	$239^\circ$	0	$237^\circ 45'$	$57^\circ 45' + 180^\circ$

We have to apply internal angle Method



$$\begin{aligned}\angle A &= AE - AB = 239^\circ - 142^\circ 30' = 96^\circ 30' \\ \angle B &= BA - BC = 322^\circ 30' - 223^\circ 15' = 99^\circ 15' \\ \angle C &= CB - CD = 44^\circ 15' - 287^\circ + 360^\circ = 117^\circ 15' \\ \angle D &= DC - DE = 107^\circ 45' - 12^\circ 45' = 95^\circ \\ \angle E &= ED - EA = 193^\circ 15' - 60^\circ = 133^\circ 15'\end{aligned}$$

$541^\circ 15'$

Sum of all internal angle =  $(2n-4) \times 90 = 540^\circ$

Total error =  $(+1^\circ 15')$  ( $541^\circ 15' - 540^\circ$ )

Total correction =  $(-1^\circ 15')$

Correction per angle =  $\frac{(-1^\circ 15')}{5} = (-) 0^\circ 15'$

Corrected Internal Angles -

$$\begin{aligned}\angle A &= 96^\circ 15' \quad (96^\circ 30' - 0^\circ 15') \\ \angle B &= 99^\circ \quad (99^\circ 15' - 0^\circ 15') \\ \angle C &= 117^\circ \\ \angle D &= 94^\circ 45' \\ \angle E &= 133^\circ\end{aligned}$$



$$\begin{array}{r}
 AB = 142^\circ 30' \\
 + \angle A = 96^\circ 15' \\
 \hline
 AE = 238^\circ 45' \\
 - 180^\circ \\
 \hline
 EA = 58^\circ 45' \\
 \angle E = +133^\circ \\
 \hline
 ED = 191^\circ 45' \\
 - 180^\circ \\
 \hline
 DE = 11^\circ 45'
 \end{array}
 \quad \left| \quad \begin{array}{l}
 AE = \angle A + AB \\
 ED = \angle E + EA
 \end{array}
 \right.$$

$$\begin{array}{r}
 DE = 11^\circ 45' \\
 \angle D = +94^\circ 45' \\
 \hline
 DC = 106^\circ 30' \\
 + 180^\circ \\
 \hline
 CD = 286^\circ 30' \\
 \angle C = +117^\circ \\
 \hline
 CB = 403^\circ 30' \\
 - 360^\circ \\
 \hline
 CB = 43^\circ 30' \\
 + 180^\circ \\
 \hline
 BC = 223^\circ 30' \\
 \angle B = +99^\circ \\
 \hline
 BA = 322^\circ 30' \\
 - 180^\circ \\
 \hline
 AB = 142^\circ 30'
 \end{array}
 \quad \left| \quad \begin{array}{l}
 DC = \angle D + DE \\
 CB = \angle C + CD \\
 BA = \angle B + CB
 \end{array}
 \right.$$

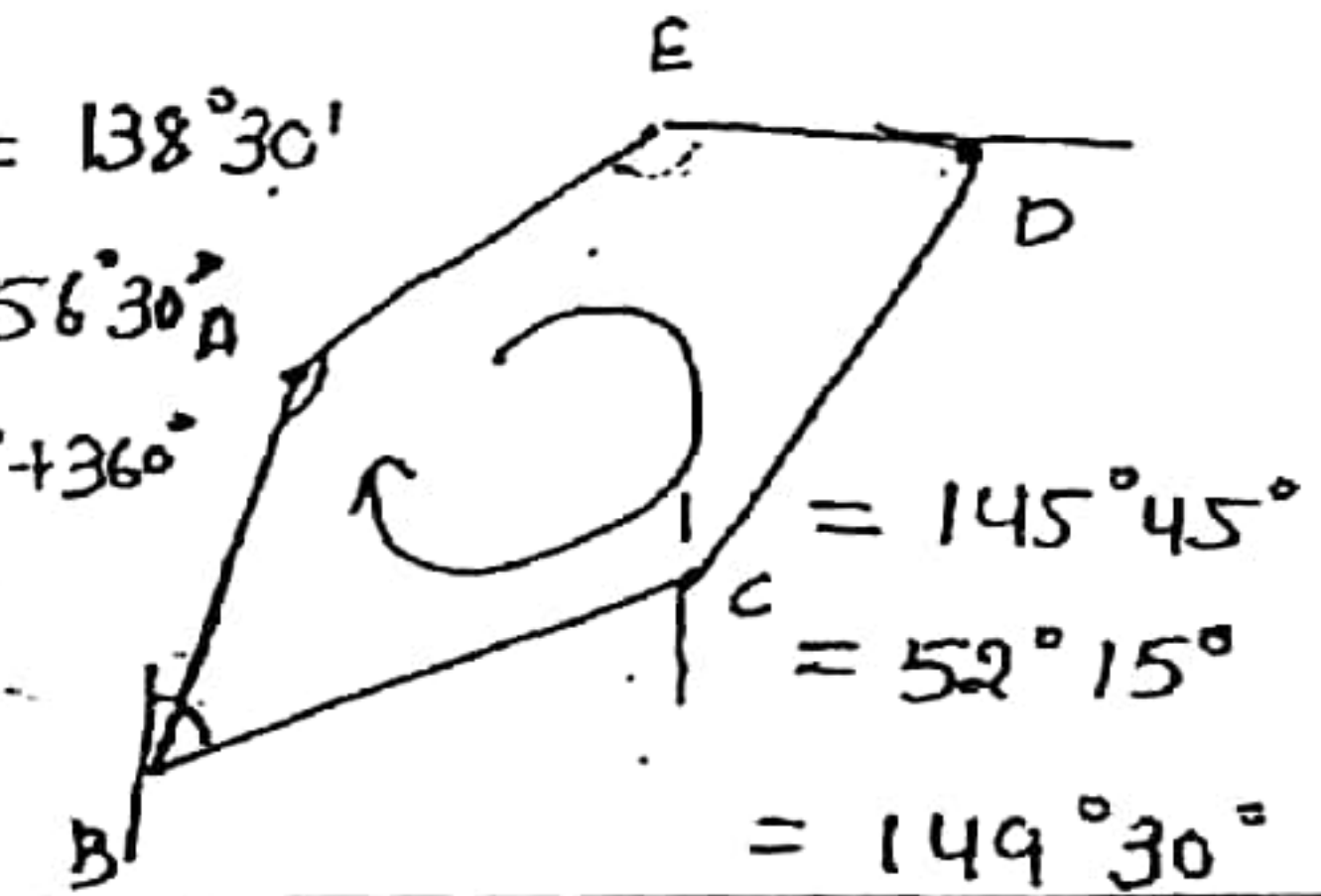
$$\begin{array}{r}
 BC = 223^\circ 30' \\
 \angle B = +99^\circ \\
 \hline
 BA = 322^\circ 30' \\
 - 180^\circ \\
 \hline
 AB = 142^\circ 30'
 \end{array}$$

Ques: (3) Work out the correct bearing of a closed traverse.

	FB	B.B.	
AB	191°30'	13°	178°30' (1°30') (less error)
BC	69°30'	246°30'	-177° (3°)
CD	32°15'	210°30'	178°15' (1°45')
DE	262°45'	80°45'	182° (2°)
EA	230°15'	53°	177°15' 2°45'

Sol<sup>n</sup>: Internal Angle Method:

$$\begin{array}{l}
 \angle A = AB - AE = 191^\circ 30' - 53^\circ = 138^\circ 30' \\
 \angle B = BC - BA = 69^\circ 30' - 13^\circ = 56^\circ 30' \\
 \angle C = CD - CB = 32^\circ 15' - 246^\circ 30' + 360^\circ = 145^\circ 45' \\
 \angle D = DE - DC = 262^\circ 45' - 210^\circ 30' = 52^\circ 15' \\
 \angle E = EA - ED = 230^\circ 15' - 80^\circ 45' = 149^\circ 30'
 \end{array}$$



$$= 542^\circ 30'$$

$$\text{Total error} = +2^\circ 30'$$

$$\text{Correction} = (-) 2^\circ 30'$$

$$\text{Correction per angle} = (-) 0^\circ 30'$$

Corrected Angle -

$$\begin{array}{r}
 \angle A = 138^\circ \\
 \angle B = 56^\circ \\
 \angle C = 145^\circ 15' \\
 \angle D = 51^\circ 45' \\
 \angle E = 149^\circ \\
 \hline
 540^\circ
 \end{array}$$

$$\frac{1^\circ 30'}{2} = 0^\circ 45'$$

AB has least error — (when any line is shorter than the others, the error in that line is less)

$$\begin{array}{l}
 AB = 191^\circ 30' + 0^\circ 45' = 192^\circ 15' \\
 BA = 13^\circ - 0^\circ 45' = 12^\circ 15'
 \end{array}
 \left. \begin{array}{l} \text{diff} \\ \end{array} \right\} = 180^\circ$$

$$\begin{array}{r}
 AB = 192^\circ 15' \\
 - \angle A = 138^\circ \\
 \hline
 AE = 53^\circ 45' \\
 + 180^\circ \\
 \hline
 EA = 234^\circ 15'
 \end{array}
 \quad \left| \quad \begin{array}{l}
 AE = AB - \angle A
 \end{array}
 \right.$$



$$\begin{array}{r} EA = 234^{\circ}15' \\ - \angle E = -149^{\circ} \\ \hline \end{array}$$

$$ED = EA - \angle E$$

$$\begin{array}{r} ED = 85^{\circ}15' \\ + 180^{\circ} \\ \hline \end{array}$$

$$\begin{array}{r} DE = 265^{\circ}15' \\ \angle D = -51^{\circ}45' \\ \hline \end{array}$$

$$DC = DE - \angle D$$

$$\begin{array}{r} DC = 213^{\circ}30' \\ = -180^{\circ} \\ \hline \end{array}$$

$$\begin{array}{r} CD = 33^{\circ}30' \\ \angle C = (-)145^{\circ}15' \\ \hline \end{array}$$

$$CB = CD - \angle C$$

$$\begin{array}{r} = -111^{\circ}45' \\ + 360^{\circ} \\ \hline \end{array}$$

$$\begin{array}{r} CB = 248^{\circ}15' \\ - 180^{\circ} \\ \hline \end{array}$$

$$BC = 68^{\circ}15'$$

$$BA = BC - \angle B$$

$$\angle B = 56^{\circ}$$

$$\begin{array}{r} BA = 12^{\circ}15' \\ + 180^{\circ} \\ \hline \end{array}$$

$$AB = 192^{\circ}15'$$

Some value of  
AB is not  
Correct.

Ques: (4)

ES-2002  
2(c) 10 marks

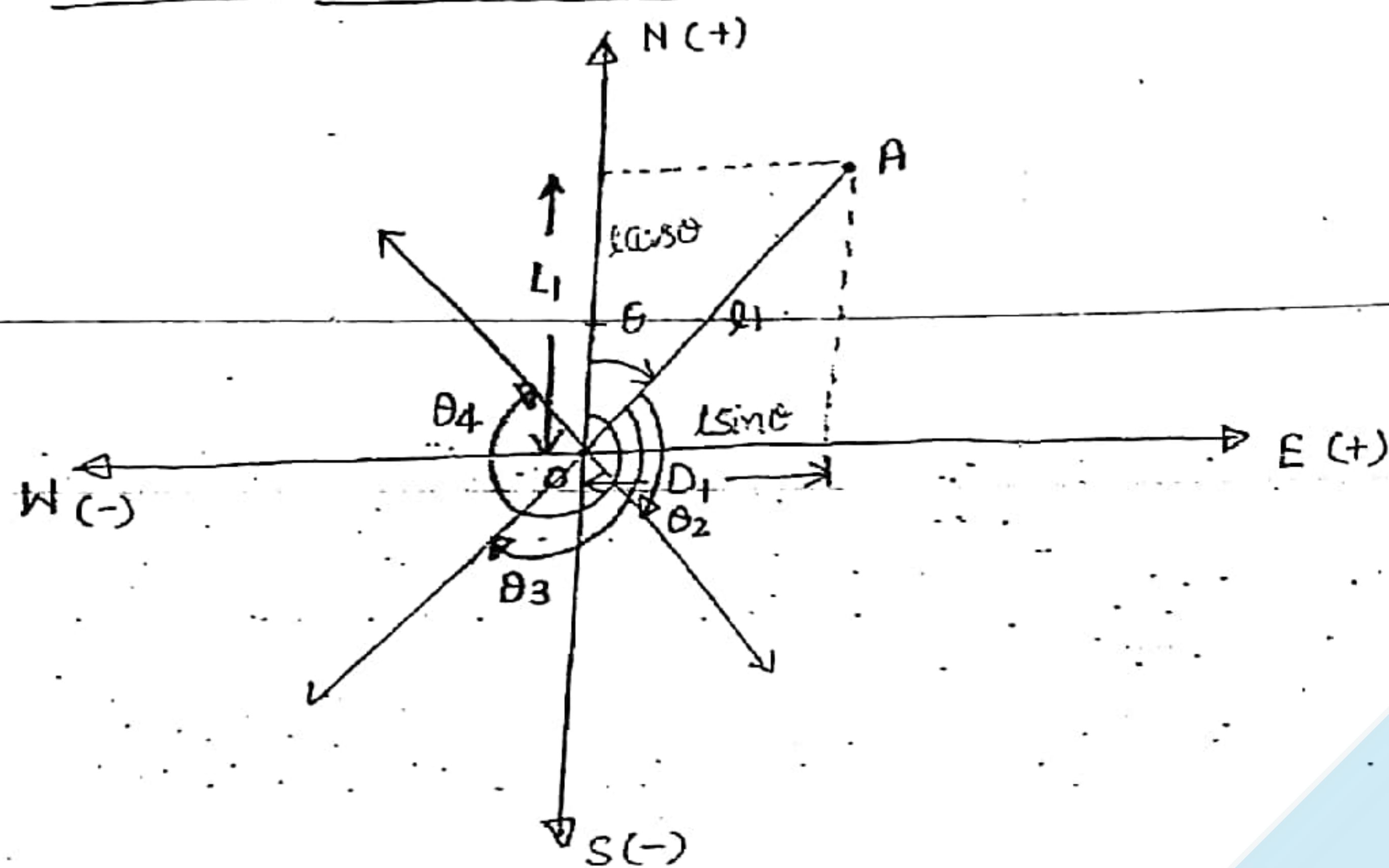
Ques: (5)

ES-2005  
8(c)



## TRAVERSE SURVEY

### ⊕ Latitude and Departure :-



Latitude of a line is the projection on N-S line.

$$L = l \cos \theta$$

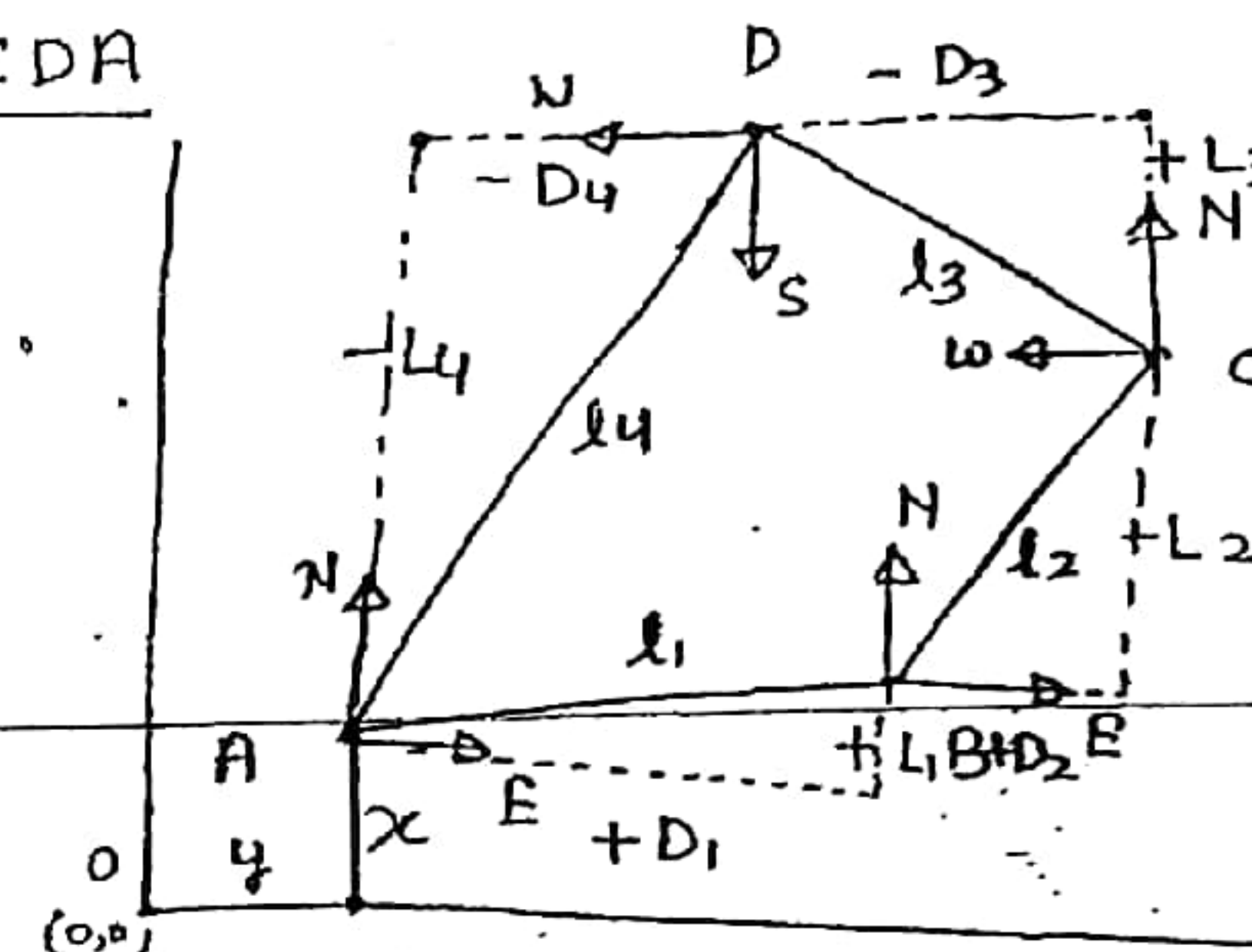
Departure : projection on E-W line.

$$D = l \sin \theta$$

WCB	QSB	Latitude	Departure
0 to 90°	N $\theta_1$ E	(+) $l \cos \theta$	(+) $l \sin \theta$
90 to 180°	S $\theta_2$ E	(-) $l \cos \theta$	(+) $l \sin \theta$
180° to 270°	S $\theta_3$ W	(-) $l \cos \theta$	(-) $l \sin \theta$
270° to 360°	N $\theta_4$ W	(+) $l \cos \theta$	(-) $l \sin \theta$

### ⊕ For a closed traverse :-

For ABCDA



Dependent Latitude / Departure = The value of latt. / Dep. of each line separately.

For DA

$$\text{Lat} = -L_4 = -l_4 \cos \theta_4$$

$$\text{Dep} = -D_4 = -l_4 \sin \theta_4$$

### Independent Co-ordinate :-

→ Co-ordinate of different points w.r.t. one fixed origin.

Point	Lat.	Dep.
A	x	y
B	x + L <sub>1</sub>	y + D <sub>1</sub>
C	x + L <sub>1</sub> + L <sub>2</sub>	y + D <sub>1</sub> + D <sub>2</sub>
D	x + L <sub>1</sub> + L <sub>2</sub> + L <sub>3</sub>	y + D <sub>1</sub> + D <sub>2</sub> + D <sub>3</sub>

### For a closed Traverse :-

$$\sum \text{Lat} = 0$$

$$\sum \text{Dep} = 0$$

$$l_1 \cos \theta_1 + l_2 \cos \theta_2 + \dots = 0 \quad \text{--- (I)}$$

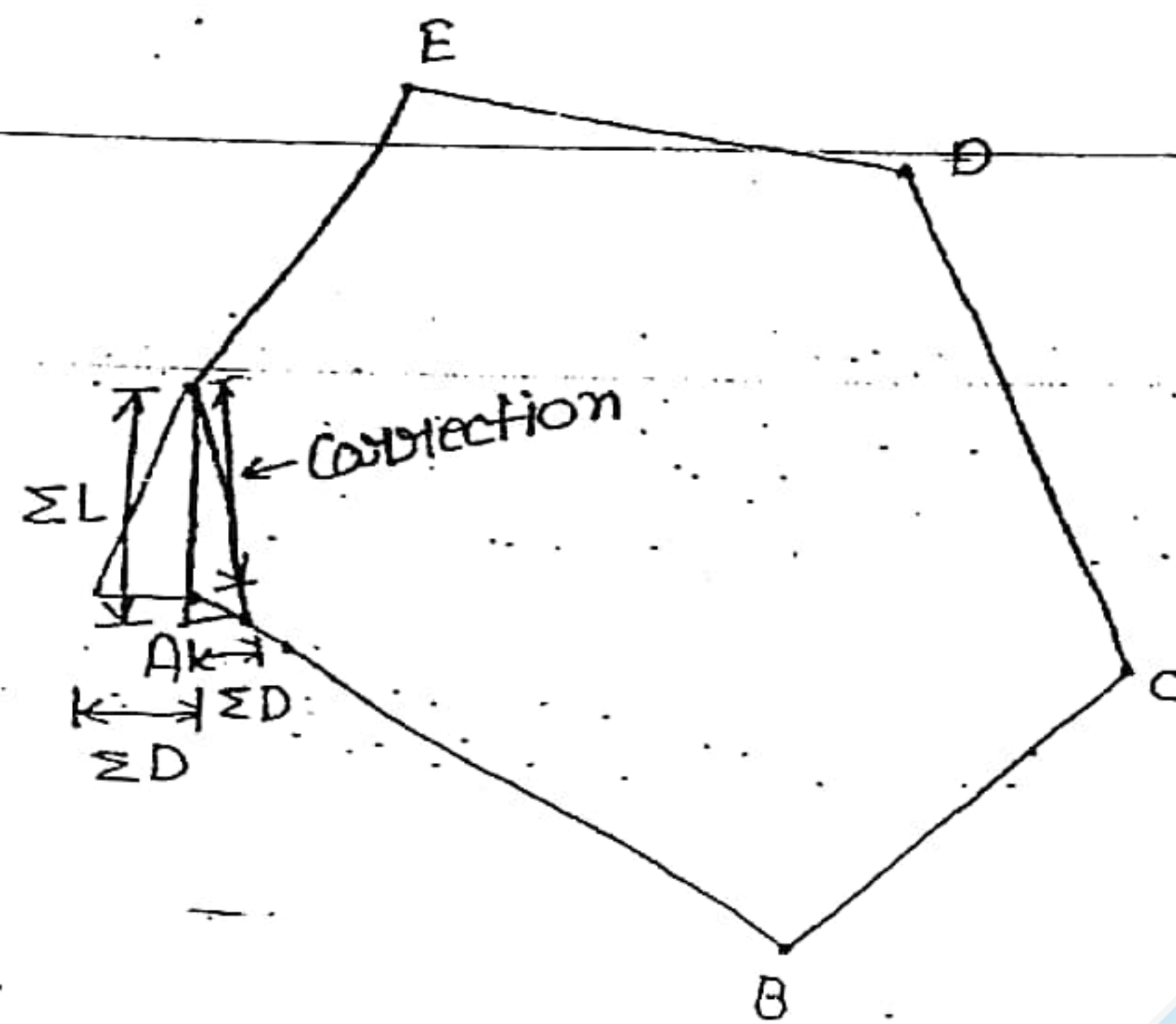
$$l_1 \sin \theta_1 + l_2 \sin \theta_2 + \dots = 0 \quad \text{--- (II)}$$



### ⊕ Closing Error :-

$$\text{If } \sum \text{Lat} \neq 0 \\ \sum \text{Dep} \neq 0$$

Then there are some errors in measurements.



⇒ The traverse drawn on the paper will not close at end.  
Methods for correcting a closing error :-

- (1) Bowditch Method.
- (2) Transit Method.
- (3) Graphical method.
- (4) Axis method.

(i) Bowditch Method :- This method is used when linear and angular both measurements have been taken with equal degree of precision.

→ The corrections are based on length of different lines.

$\sum L$  = Total error in latitude.  
(Adding all lat. with +/-)

$\sum D$  = Total error in departure.  
(using +/- sign)

### ⊕ Correction in latitude of a line

$$C_L = \frac{l_1}{\sum l} \times \sum L$$

$$(\sum l = l_1 + l_2 + l_3 + \dots + l_n)$$

### ⊕ Correction in departure -

$$C_D = \frac{l_1}{\sum l} \times \sum D$$

29/12/2015

### 2) Transit Method :-

→ This method is used when angular measurement are more accurate than linear measurement.

eg,

$\sum L$  = Total error in latitude  
(Adding with +/- sign)

$\sum D$  = Total error in departure  
(Adding with +/- sign)

$L_T$  = Total sum of latitude (without sign, consider all +ve)

$D_T$  = Total sum of departure (without sign, consider all +ve)



Correction in Latitude of a line s

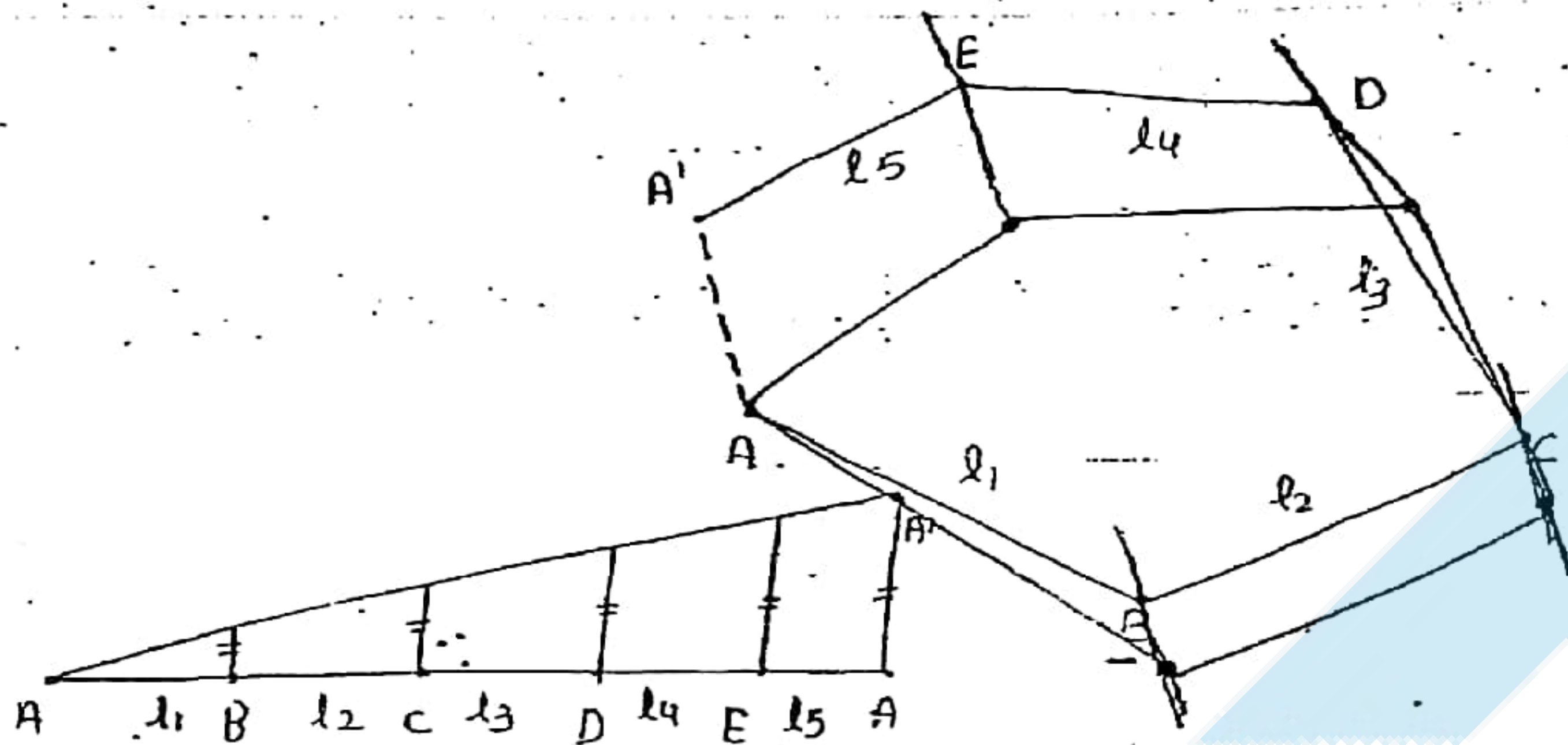
$$C_L = \frac{L_i}{L_T} \times \Sigma L$$

Correction in Departure of line s

$$C_D = \frac{D_i}{D_T} \times \Sigma D$$

### (3) Graphical Method s-

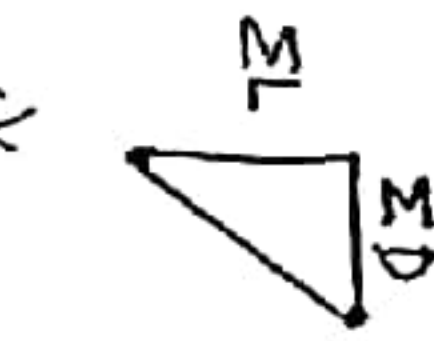
→ It is based on bowditch method.



Ques: ① A closed traverse has following length and bearing. Find out the closing error and correct the traverse for closing error by - (a) Bowditch Method (b) Transit method.

Line	length	Bearing
AB	160	46°
BC	190	130°
CD	200	220°
DA	180	320°

Line	length	Bearing	Lat.	Dep.	Correction in Lat.	corrected Lat.	corrected Lat	corrected Dep.
AB	160	46°	111.15	115.09	(+) 5.76	(-) 3.59	116.91 (111.15+5.76)	111.50
BC	190	130°	-122.13	145.54	(+) 6.85	(-) 4.26	-115.28	141.28
CD	200	220°	-153.21	-128.56	(+) 7.29	(-) 4.49	-146.0	-133.05
DA	180	320°	137.89	-115.70	(+) 6.48	(-) 4.02	144.37	-119.74
$\Sigma L = 730$			$\Sigma L = -26.30$	$\Sigma D = 16.38$	$\frac{L_i}{L_T} \times \Sigma L$		0	0



$$\text{closing error} = \sqrt{(26.30)^2 + (16.38)^2} = 30.98m$$

For Transit Error →

$$L_T = 524.38' (\text{wologism})$$

$$L_D = 504.89 (\text{wologism})$$

Line	Lat.	Dep.	corrected Lat	corrected Dep.
AB	111.15	115.09	116.91	111.50
BC	-122.13	145.54	-115.28	141.28
CD	-153.21	-128.56	-146.0	-133.05
DA	137.89	-115.70	144.37	-119.74



Ques (2) A closed traverse has following length and bearing calculate missing values —

Line	Length	Bearing	Lat	Dep.
PA	$x$	$35^\circ$	$0.819x$	$0.57x$
QR	280	$80^\circ$	48.62	275.75
RS	350	$165^\circ$	-338.07	90.59
ST	$y$	$238^\circ$	$-0.53y$	$-0.85y$
TPP	275	$310^\circ$	176.76	-70.66

$$\sum Lat = 0$$

$$0.819x - 0.53y - 112.69 = 0 \quad \text{--- (I)}$$

$$\sum Dep = 0$$

$$0.57x - 0.85y + 155.68 = 0 \quad \text{--- (II)}$$

$$x = -452.47 \text{ m}$$

$$y = -486.57 \text{ m}$$

Ques : (3) A closed traverse has following length & bearing —

Line	length	Bearing	Lat.	Dep.
AB	240 m	$86^\circ$	16.74	239.42
BC	$x$	$43^\circ$	$0.73x$	$0.68x$
CD	140 m	$310^\circ$	89.99	-107.25
DE	225 m	$300^\circ$	112.5	-194.86
EF	160 m	$\theta$	$160 \cos \theta$	$160 \sin \theta$
FA	200	$165^\circ$	-193.185	51.76

$$26.045 + 0.73x - 10.93 + 160 \sin \theta + 0.68x + 160 \cos \theta = 0$$

$$\sum Lat = 0$$

$$0.73x + 160 \cos \theta + 26.04 = 0 \quad \text{--- (I)}$$

$$\sum Dep = 0$$

$$0.68x + 160 \sin \theta - 10.93 = 0 \quad \text{--- (II)}$$

$$160 \cos \theta = -(26.04 + 0.73x)$$

$$160 \sin \theta = (10.93 - 0.68x)$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$(160 \cos \theta)^2 + (160 \sin \theta)^2 = (0.73x + 26.04)^2 + (0.68x - 10.93)^2$$

$$160^2 = 0.99x^2 + 797.546 + 23.153x$$

$$x = 79.77, -310.92$$

$$x = 146.31$$

$$\cos \theta = \frac{-0.73x - 26.04}{160}$$

$$= -0.830289$$

$$\sin \theta = \frac{-0.68x + 10.93}{160}$$

$$= (-) 0.55369$$

$$\tan \theta = \frac{0.55369}{0.830289} = 0.66687$$

$$\theta = 33^\circ 41' 52''$$

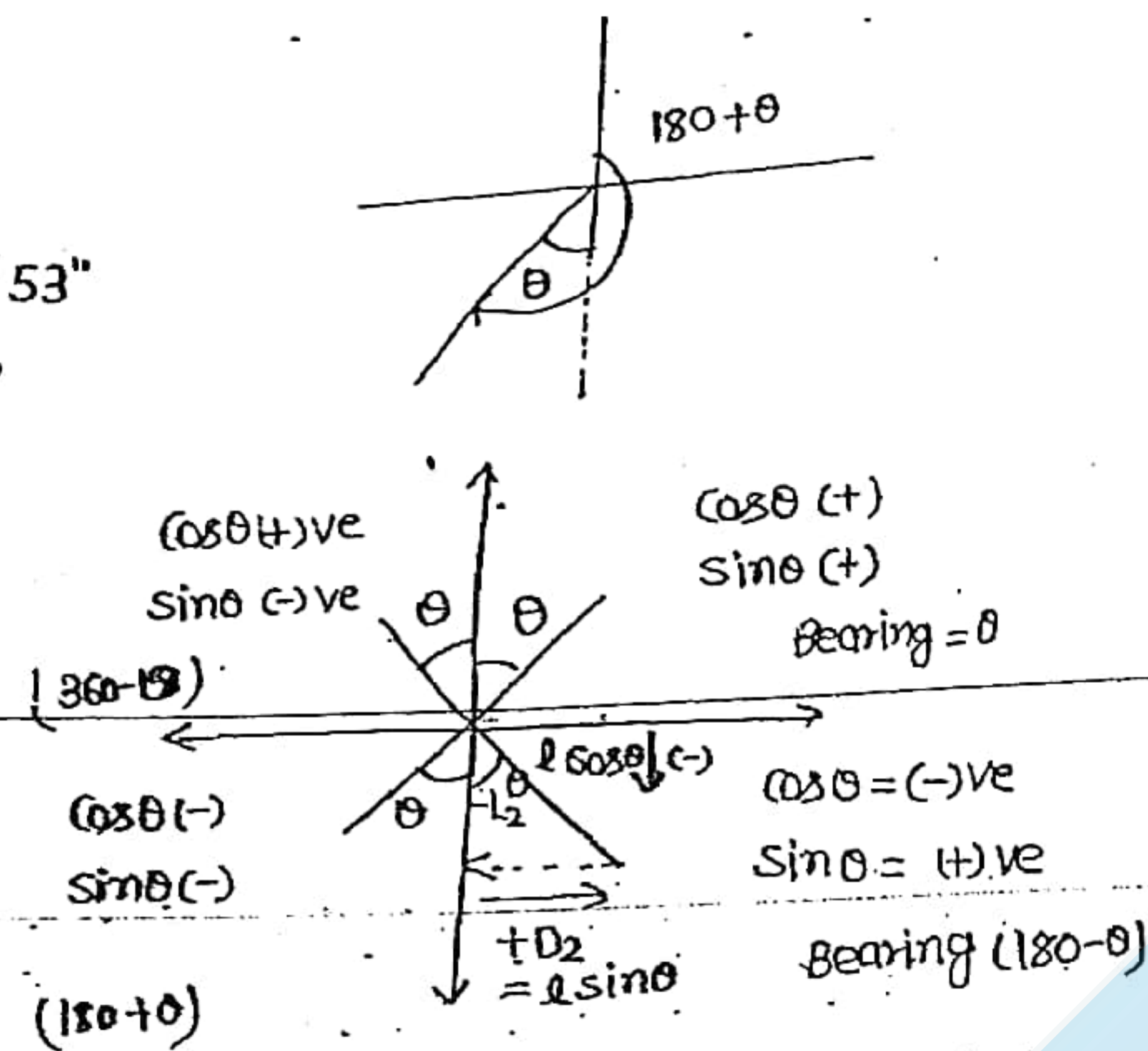


Bearing of line

$$= 180 + \theta$$

$$= 180 + 33^\circ 41' 53''$$

$$= 213^\circ 41' 53''$$



Ques 1(c) A closed traverse as following length & bearings -  
ES-2000

Line	Length	Bearing	Lat.	Dep.
AB	200	$\theta$	$200 \cos \theta$	$200 \sin \theta$
BC	98	$178^\circ$	-97.94	3.42
CD	$x$	$270^\circ$	0	$-x$
DA	86.4	$1^\circ$	86.38	1.51

$$\sum \text{Lat} = 0$$

$$\sum \text{Dep} = 0$$

$$200 \cos \theta - 11.56 = 0$$

$$200 \sin \theta - x + 4.93 = 0$$

$$\cos \theta = \frac{11.56}{200}$$

$$200 \sin \theta = x - 4.93$$

$$\cos \theta = 0.0578$$

$$(200 \sin \theta)^2 + (200 \cos \theta)^2 = (x - 4.93)^2 + (11.56)^2$$

$$(200)^2 = x^2 + 24.301 - 9.86x + 133.434$$

$$x^2 - 9.86x - 39842.065 = 0$$

## LEVELLING

$$x_1 = 204.6 \text{ m}$$

$$200 \sin \theta = x - 4.93$$

$$\sin \theta = \frac{199.67}{200}$$

$$\sin \theta = 0.9986$$

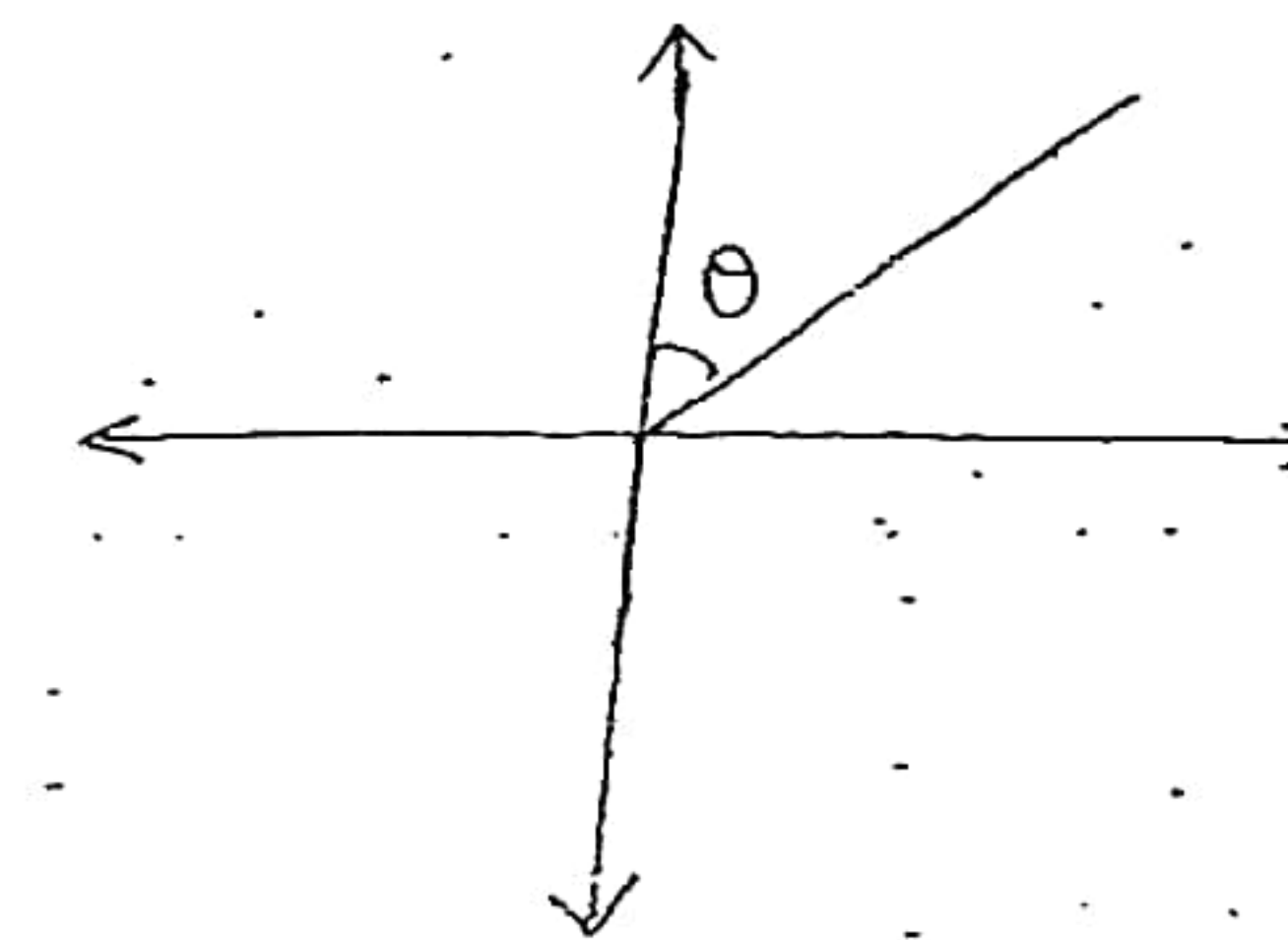
$$\cos \theta = 0.0578$$

$$\tan \theta = \frac{0.9986}{0.0578} = 17.276$$

$$\theta = 86.68^\circ$$

$$\theta = 86^\circ 41' 13'' \text{ both angle (+ve)}$$

$$\text{Bearing} = \theta = 86^\circ 41' 13''$$





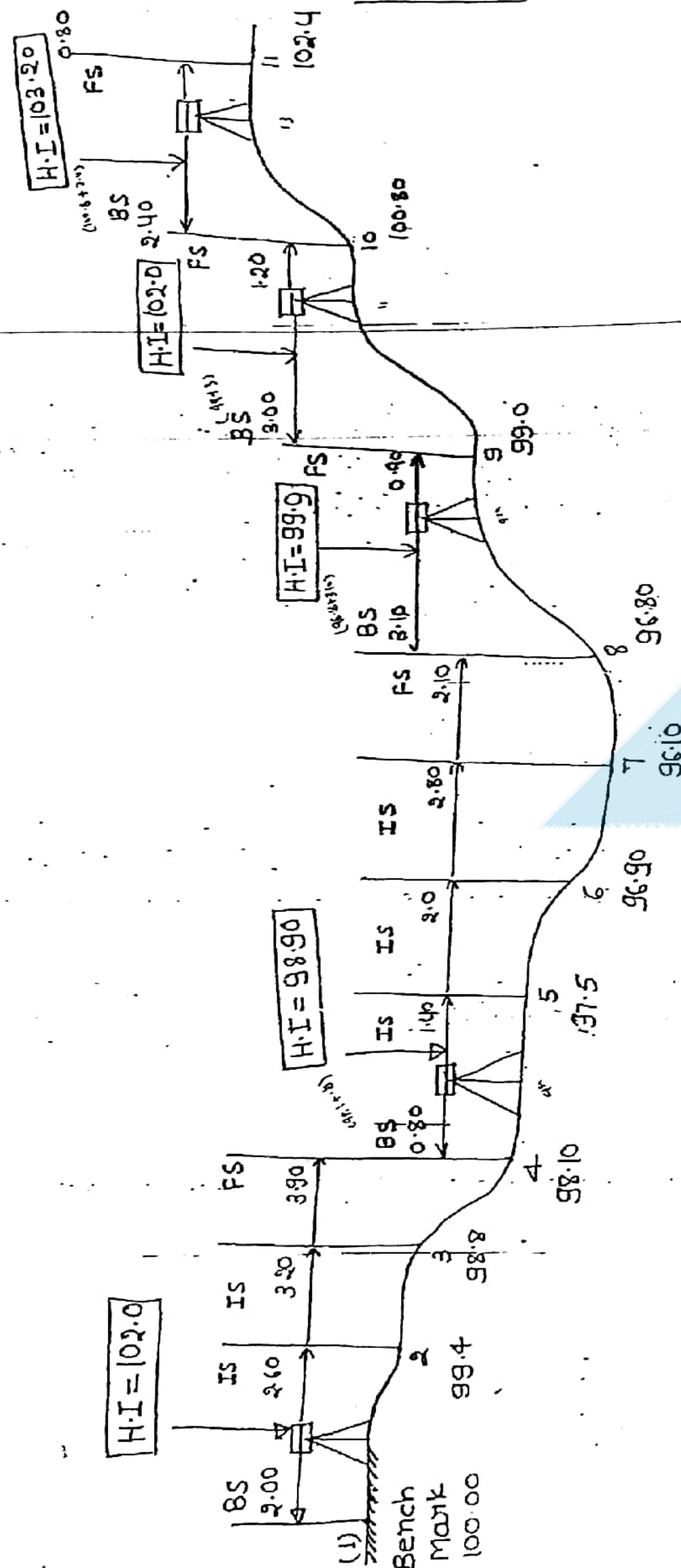
Bench  
Mark  
100.00

LEVELLING

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LEVELLINGImportant definition :-

- (1) Bench Mark :- A fixed location on ground of known R.L., from where levelling work can be started.
- (2) Reduced Level (RL) :- Height of any point w.r.t. either mean sea level or w.r.t. any bench mark of known R.L. is called Reduced level.
- (3) Back Sight :- First reading taken after setting up the instrument at any station.
- (4) Fore Sight :- Last reading taken from any instrument location.
- (5) Intermediate Sight :- All other reading than B.S./F.S. are called I.S.

(6) Height of Instrument Method :-

$$H.I. = \left( \begin{array}{l} \text{R.L. of Bench Mark} \\ \text{or previous station} \end{array} + B.S. \right)$$

$$\text{R.L. of Next point} = H.I. - I.S./F.S.$$

(7) Rise & Fall Method :-

$$\text{Rise / Fall of this point} = \left( \begin{array}{l} \text{Previous} \\ \text{reading} \end{array} \right) - \left( \begin{array}{l} \text{Next} \\ \text{Reading} \end{array} \right)$$

(+)ve  $\rightarrow$  Rise.

(-)ve  $\rightarrow$  Fall.

$$= \left( \begin{array}{l} \text{Staff reading} \\ \text{of previous point} \end{array} \right) - \left( \begin{array}{l} \text{Staff reading} \\ \text{at this point} \end{array} \right)$$



R.L. of this point = R.L. of previous point + Rise (or) - Fall.

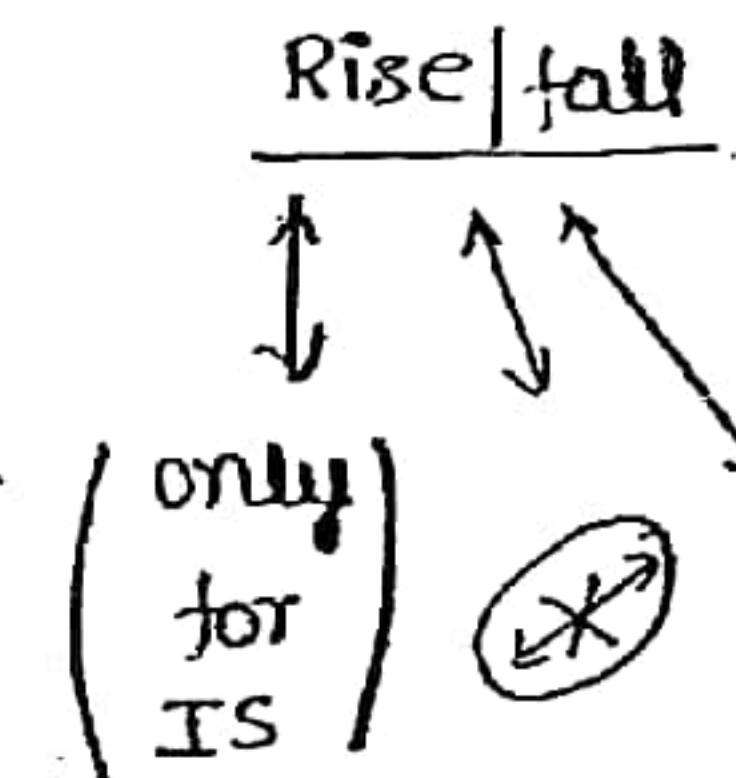
Ques 8 (J) Following readings were taken in a fly levelling using a staff of 4.0 m length.

BS 2.00, IS 2.60, IS 3.20, FS 3.90, BS 0.80, IS 1.40, IS 2.0, IS 2.80, FS 2.10, BS 3.10, FS 0.90, BS 3.00, FS 1.20, BS 2.40, FS 0.80

Instrument was shifted after 4<sup>th</sup>, 9<sup>th</sup>, 11<sup>th</sup> & 13<sup>th</sup> reading.

1<sup>st</sup> reading was taken on a B.M. of R.L. = 100.00 m. Fill the level book and calculate R.L. of different points.

Point	BS	IS	FS	H.I	R.L	Rise	Fall	R.L
1	2.00	2.60		102.00	100.00	X	X	100.00
2		2.60			99.4		0.6	99.4
3		3.20			98.8		0.6	98.8
4	0.80		3.90	98.9	98.1		0.70	98.1
5		1.40			97.5		0.60	97.5
6		2.0			96.9		0.6	96.9
7		2.80			96.1		0.8	96.1
8	3.10		2.10	95.9	96.80	0.7		96.8
9	3.00		0.90	102.0	99.0	2.20		99.0
10	2.40		1.20	103.2	100.8	1.80		100.8
11			0.80		102.40	1.60		102.40
12	11.3		8.9					



#### ⊕ Arithmetic check

$$= \sum BS - \sum FS = 11.30 - 8.90 = 2.40$$

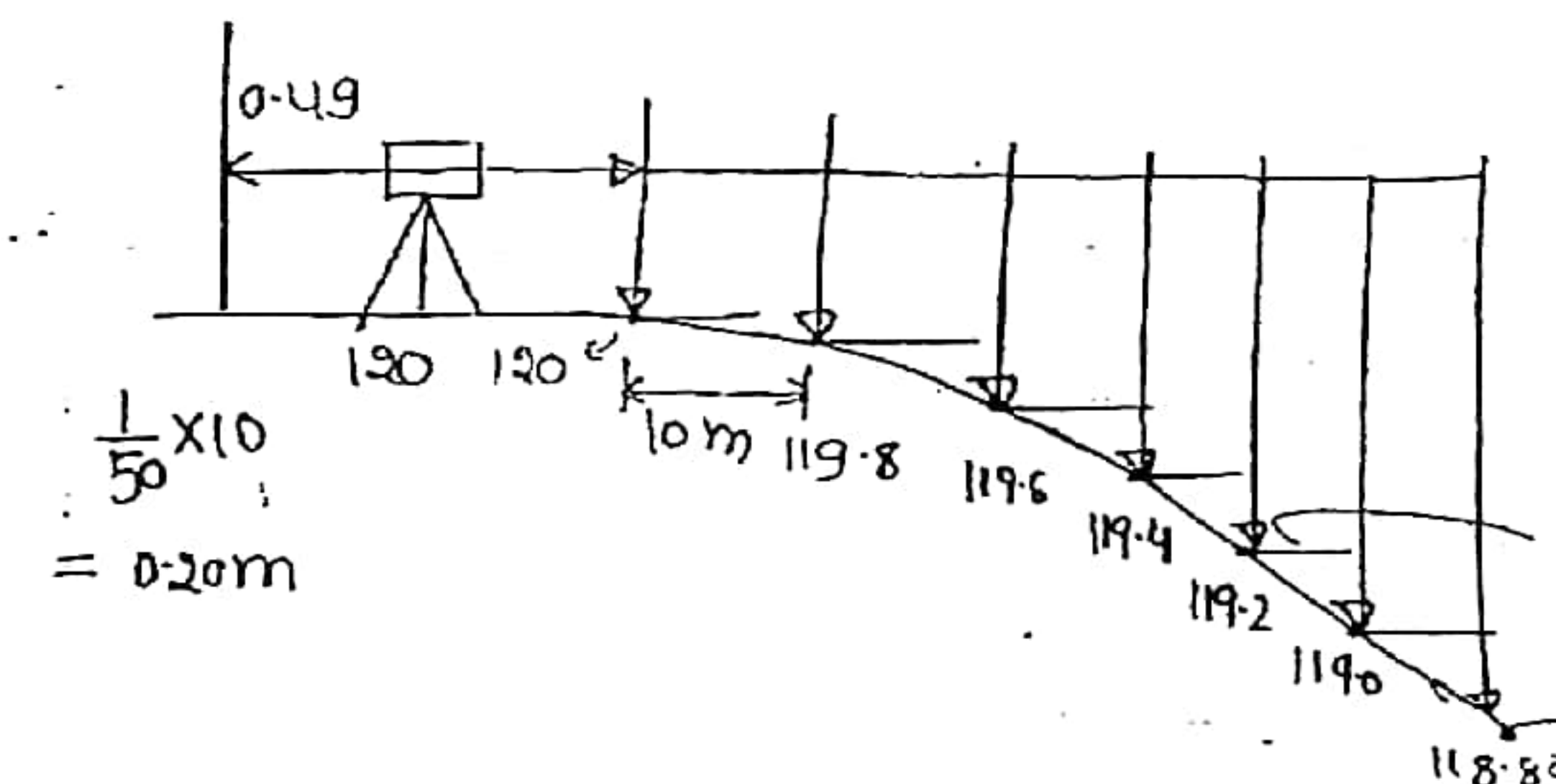
$$= \left( \text{Last R.L} \right) - \left( \text{1st R.L} \right) = 102.4 - 100.00 = 2.4$$

$$= \sum \text{Rise} - \sum \text{Fall} = 6.3 - 3.9 = 2.4$$

Ques: 7(b) In running fly level from a bench mark, of R.L. 120.75 m. Following readings were taken —

B.S 0.85 1.285 1.182 0.965 0.49  
F.S 0.555 1.150 1.945 1.755 —

From the 1<sup>st</sup> position of instrument seven pegs at 10 m interval are to be set out on a uniform falling gradient of 1 in 50. R.L. of 1<sup>st</sup> peg is 120.0 m. work out the staff reading for top of the pegs. fill level book and apply arithmetic check.





Point	BS	IS	FS	H.I	R.L	Rise	Fall	R.L
1	0.85				120.75	x	x	
2	1.285		0.555		121.015	0.295		
3	1.185		1.150		121.180	0.135		
4	0.965		1.945		120.417		0.763	
5	0.49		1.755		119.627		0.790	
6		0.117			120.00	0.373		
7		0.317			119.8		0.20	
8		0.517			119.6		0.20	
9		0.717			119.4		0.20	
10		0.917			119.2		0.20	
11		1.117			119.0		0.20	
12			1.317		118.8		0.20	
$\Sigma BS = 4.772$		$\Sigma FS = 6.722$		$0.803$		$2.753$		

Rise/fall for point No-6

$$= 120.00 - 119.627$$

$$= 0.373$$

$$0.49 - x = 0.373$$

$$x = 0.49 - 0.373$$

$$x = 0.117$$

check:-

$$\Sigma BS - \Sigma FS$$

$$= 4.772 - 6.722$$

$$= -1.95$$

$$\text{last RL} - 1^{\text{st}} \text{RL}$$

$$= 118.8 - 120.75$$

$$= -1.95$$

$$\Sigma \text{Rise} - \Sigma \text{fall} = 0.803 - 2.753$$

$$= -1.95$$

(OK)

Ques: (2) Find out missing values in a level book as shown below -

Points	BS	IS	FS	R.L	Rise	Fall
① B.M	2.40			120.00		
2)	5.65		2.20	121.2	1.20	
3)		2.85		124.00	2.80	
4)		1.65		125.00	3.00	
5)	3.55		2.80	125.05		0.15
6)			2.60	126.00	0.95	

Missing Value -

$$⑤ \quad 2.40 - 1.20 = 1.20 \text{ (Rise)}$$

$$⑥ \quad 12.4 - 121.2 = 2.80 \text{ (Rise)}$$

$$⑦ \quad 127 - 124 = 3 \text{ (Rise)}$$

$$⑧ \quad 126.8 - 127.0 = -0.2 \text{ (fall)}$$

$$⑨ \quad 125.0 - 126.8 = -1.8 \text{ (fall)}$$

$$(1) \quad = 2.80 + 2.85 = 5.65 \checkmark$$

$$(2) \quad 120.0 + 1.20 = 121.20 \text{ m } \checkmark$$

$$⑤ \quad 1.80 = 2.80 - 1.00 \quad (3) \quad 1.80 - 0.20 = 1.6$$

$$(3) \quad 2.85 - 1.2 = 1.65$$

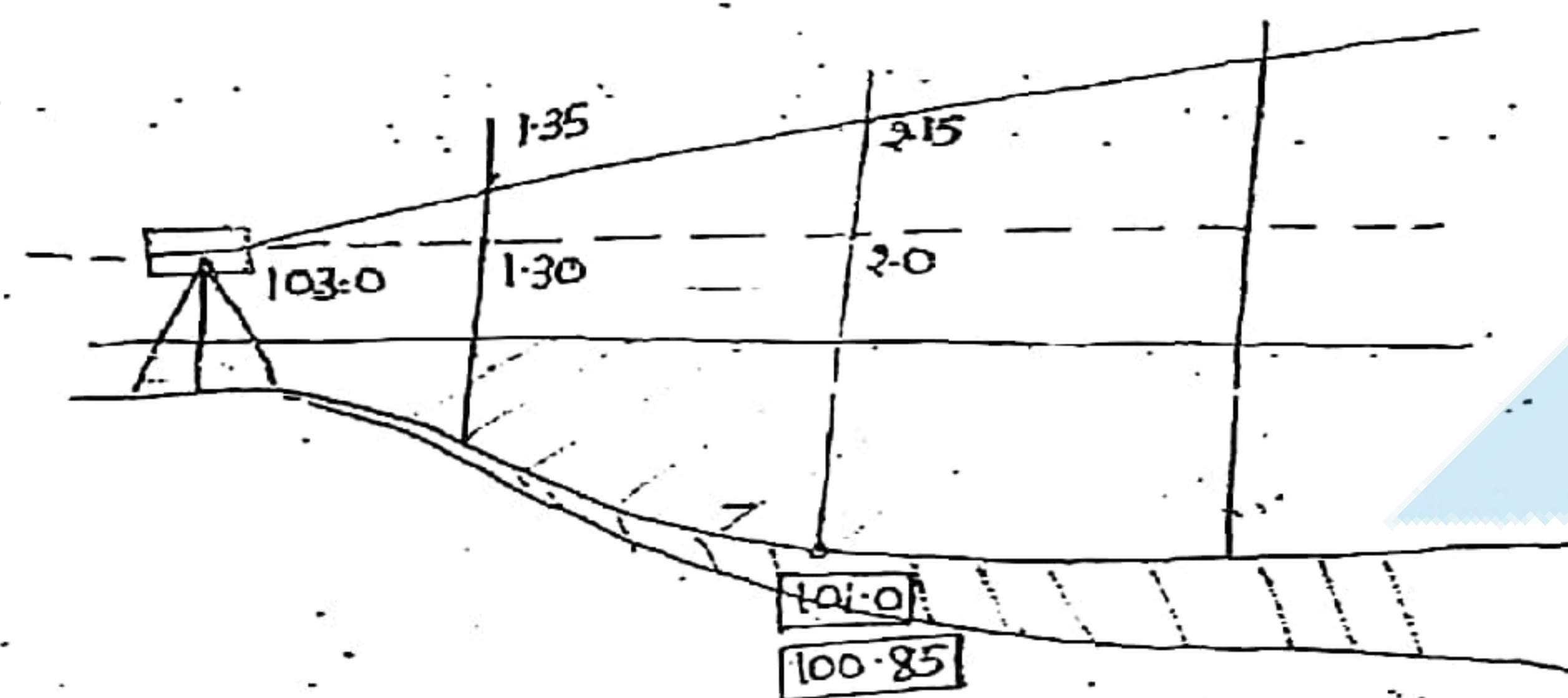
$$③ \quad \begin{array}{r} 2.85 \\ - 1.2 \\ \hline 1.65 \end{array}$$



### Reciprocal Levelling :-

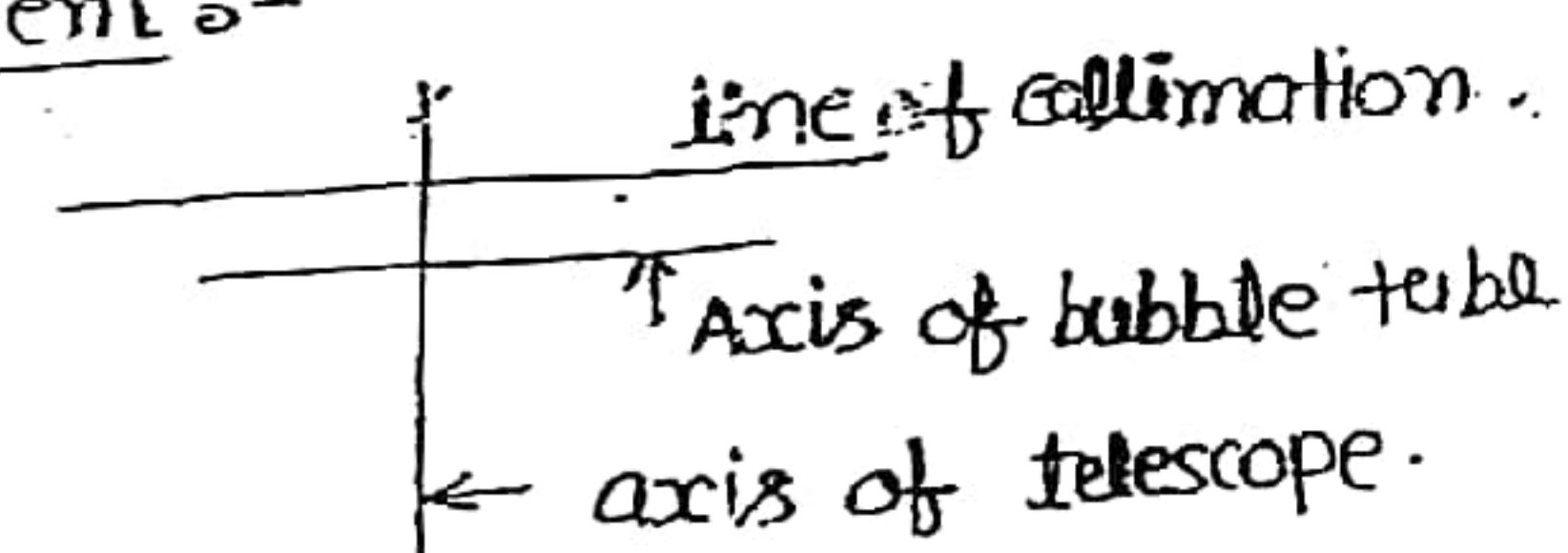
→ Reciprocal levelling is used to find out any error in the levelling instrument and to eliminate the effect of such error and other error like due to earth curvature & refraction.

→ If line of sight is not horizontal when bubble is showing in centre. The instrument is faulty. In this case all the readings taken will be wrong.


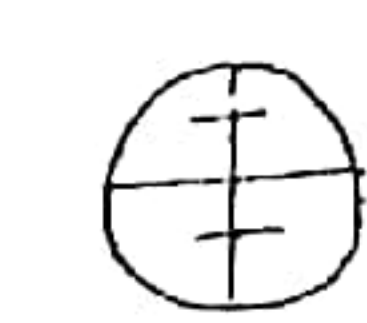


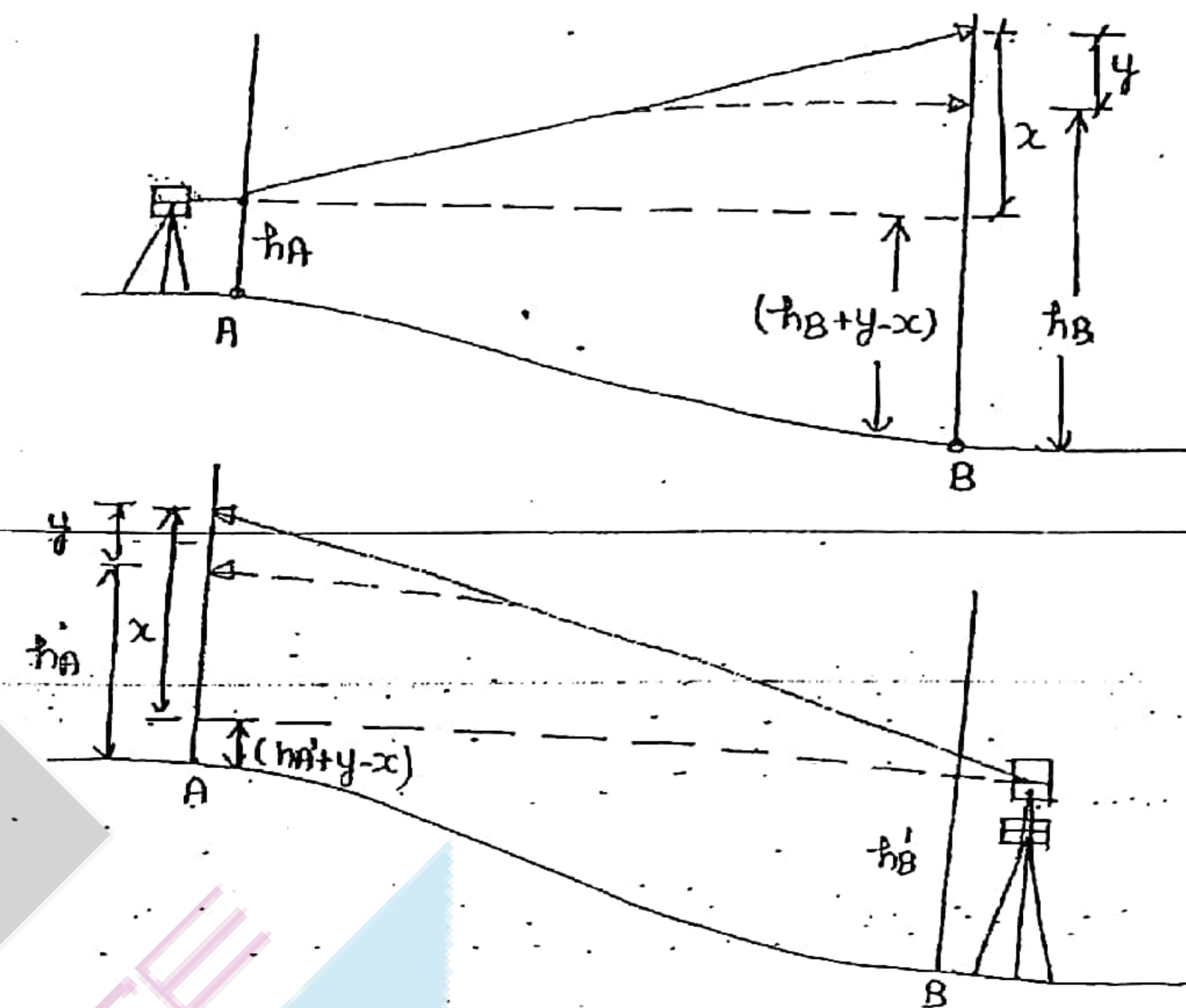
### Adjustments :-

→ (i) Permanent Adjustment :-



→ (a) Temporary Adjustment :-

- centring
- levelling → 
- Removal of parallax - crosshair (disturbance) (focusing of crosshair) 



Two point 'A' & 'B' are selected at some distance, and staff readings are noted keeping the instrument 1<sup>st</sup> near A & then near B.

	Staff reading at 'A'	Staff reading at 'B'
Instru. A	$h_A$ ✓	$h_B$
Instru. B	$h_A'$	$h_B'$ ✓

⊕ When the Instrument is at A

correct reading at A =  $h_A$

" " " B should be =  $(h_B + y - x)$

exact difference of level b/w A & B

$$H = (h_B + y - x) - h_A' \quad \text{--- ①}$$



⊕ When instrument is at B -

Correct reading at A should be  $= (h_A' + y - x)$

" " " B  $= h_B'$  " =

exact difference of level

$$H = h_B' - (h_A' + y - x)$$

$$H = (h_B' - h_A' - y + x) \quad \text{--- (2)}$$

Add (1) & (2)

$$2H = h_B + y - x - h_A + h_B' - h_A' - y + x$$

$$H = \frac{(h_B - h_A) + (h_B' - h_A')}{2}$$

Ques: (1) For a reciprocal levelling, following readings were taken -

	Staff reading at (A)	Staff reading at (B)	difference
Instrument (A)	1.82 $h_A$	2.72 $(2.785)$ $(\downarrow) h_B' \text{ Error } = 0.065$	0.90
Instrument (B)	0.92 $h_A$ $0.985 \text{ error } = 0.065 (\downarrow)$	1.95 $h_B'$	1.03

0.965 (Avg) exact difference  
R.L B = 199.035

If R.L of point A is 200.00 m and distance b/w (A) & (B) is 260 m. Calculate the exact R.L of point B & angular error in the instrument. Neglect the effect of curvature and refraction.

Sol<sup>n</sup> Exact difference of level b/w A & B.

$$H = \frac{(h_B - h_A) + (h_B' - h_A')}{2} = \frac{(2.72 - 1.82) + (1.95 - 0.92)}{2}$$

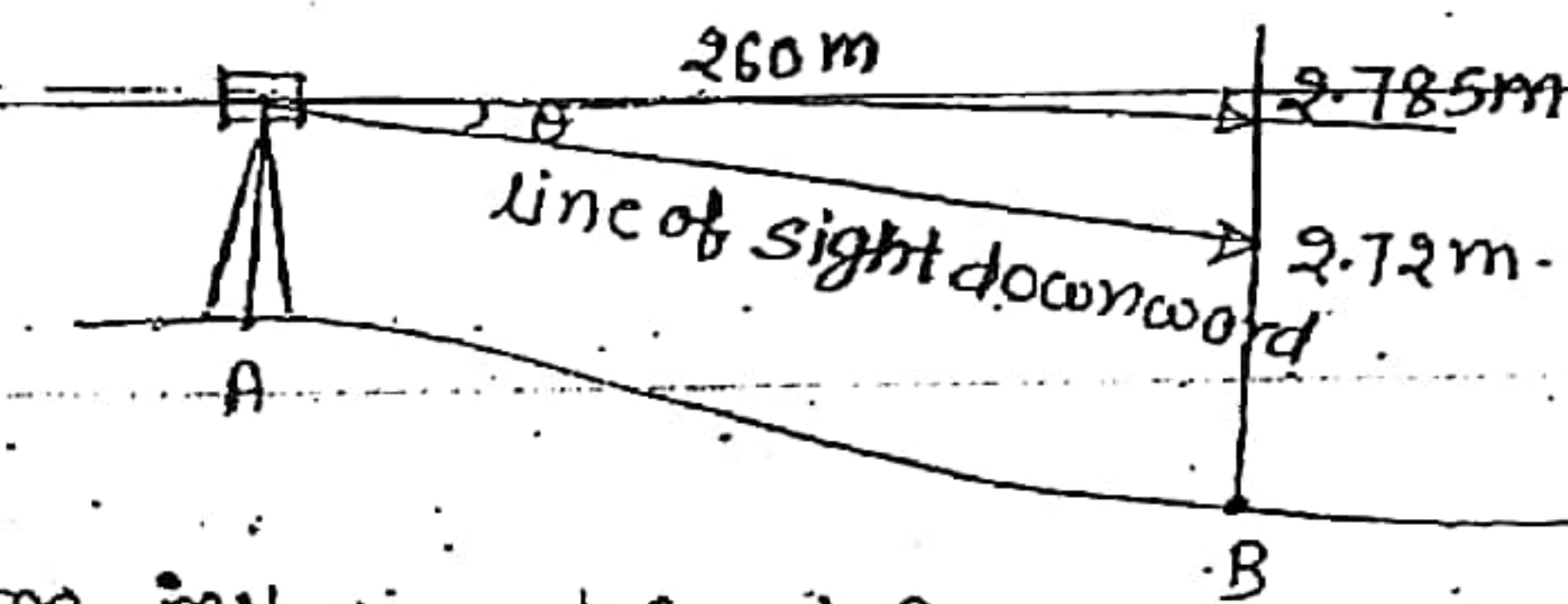
$$H = 0.965$$

R.L of A = 200.00 m.

∴ R.L of B = 200.00 (R.L of A) - difference of level

$$\begin{aligned} \text{(B is below A)} &= 200.00 - 0.965 \\ &= 199.035 \text{ m.} \end{aligned}$$

Error in Instrument :-



when instrument is at A -

Correct Reading at A = 1.82

exact Reading diff of level = 0.965

$$\begin{aligned} \text{exact Reading at B should be} &= 1.82 + 0.965 \\ &= 2.785 \text{ m} \end{aligned}$$

Reading taken at B = 2.72 < 2.785 m.  
line of sight

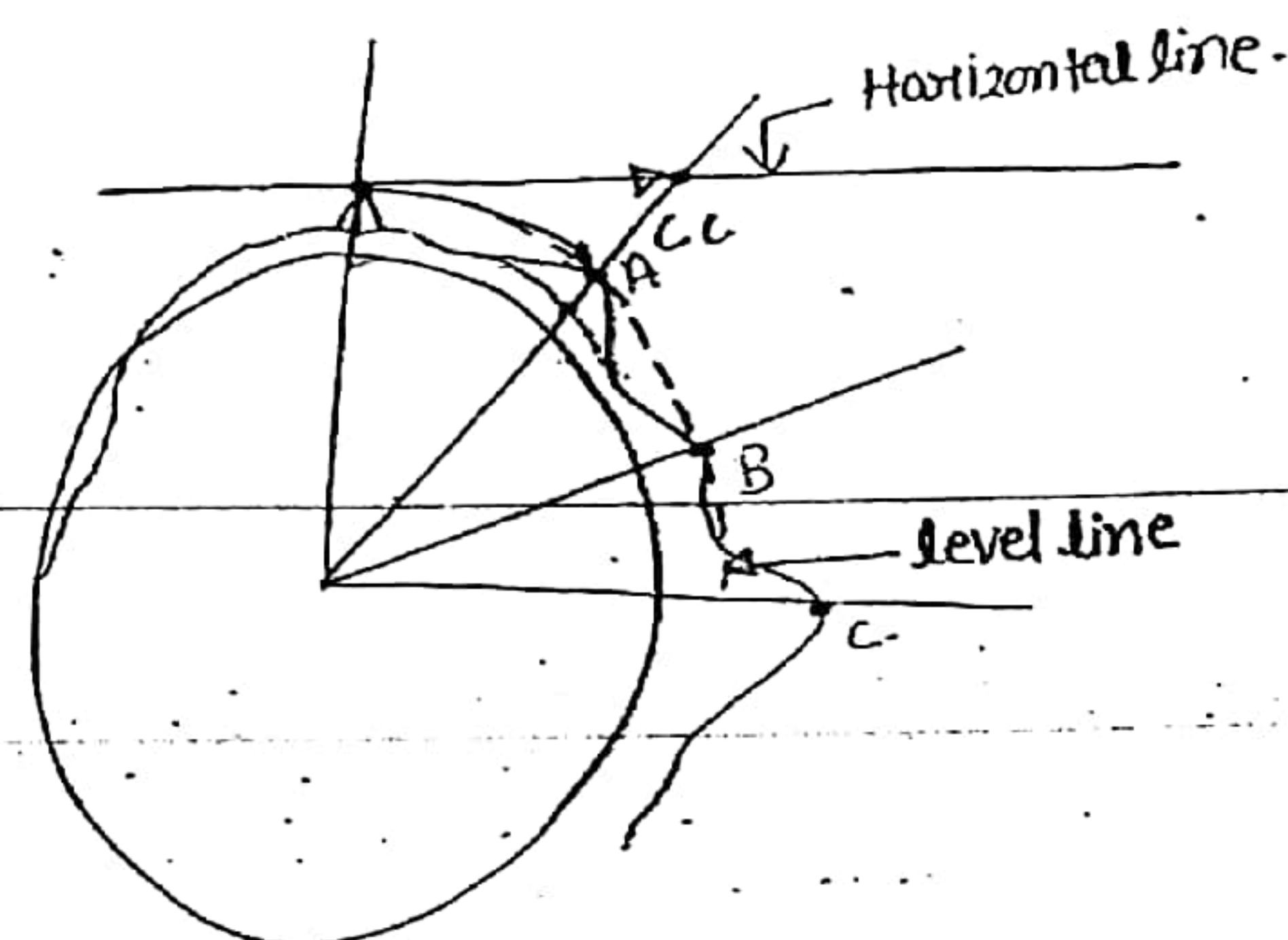
error in inclination :-

$$\begin{aligned} &= \frac{\text{difference of Reading}}{\text{distance}} = \frac{2.785 - 2.72}{260} \\ &= \frac{1}{4000} \text{ Ans} \end{aligned}$$



⊕ Correction due to earth curvature and refraction :-

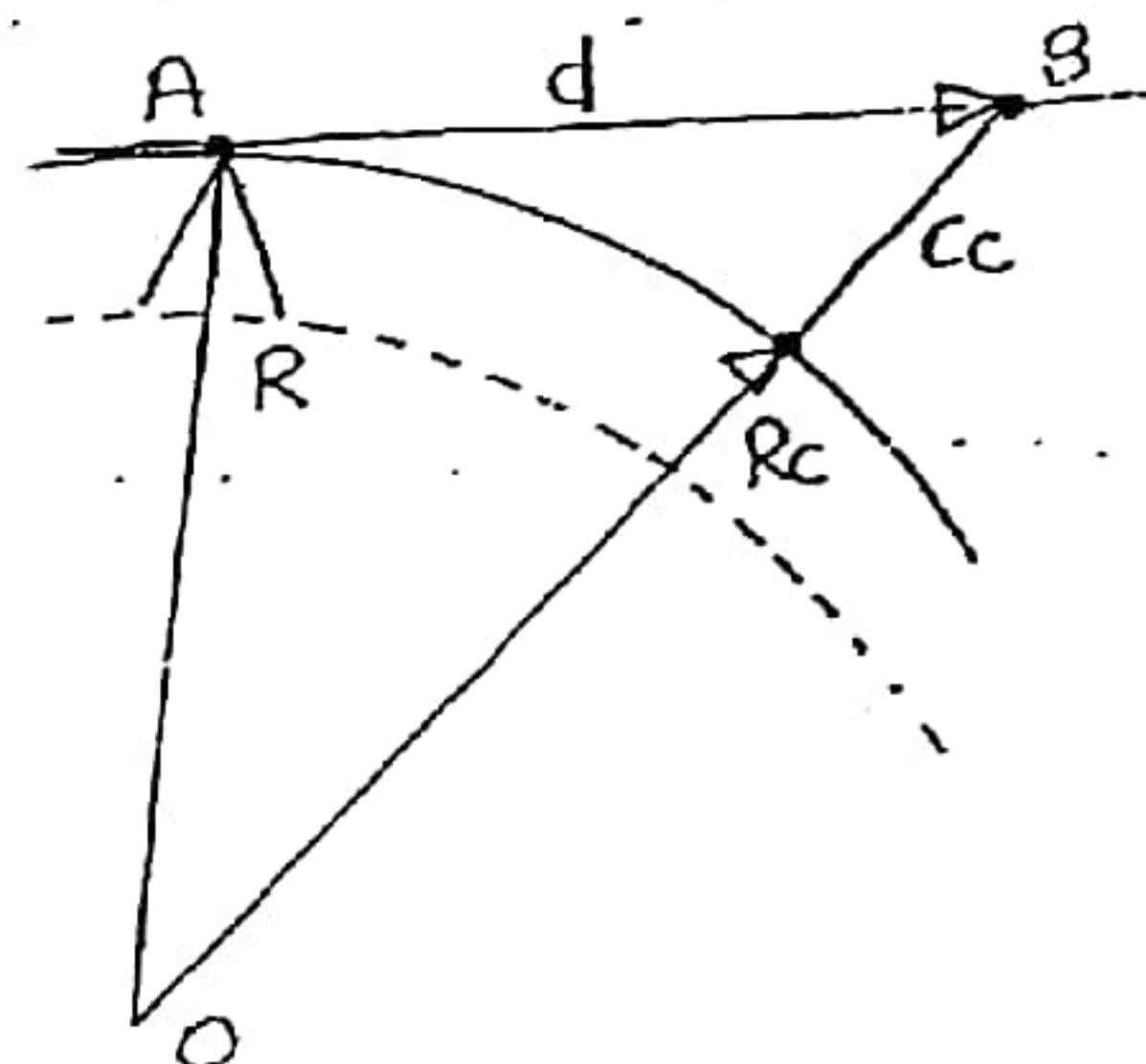
(1) Due to earth curvature :-



(#) Horizontal line :- A line tangent to any point on earth surface. line of sight setup by an instrument will show a horizontal line.

(#) level line :- A line lld to earth surface. The R.L of different points should be measured w.r.t level line.

(#) Correction due to curvature :- Is the difference b/w horizontal line and level line for any point.



$$R \neq R_c$$

$$(R + C_c)^2 = R^2 + d^2$$

$$R^2 + C_c^2 + 2 \cdot R \cdot C_c = R^2 + d^2$$

$$C_c(2R + C_c) = d^2$$

$$C_c = \frac{d^2}{2R}$$

Correction due to curvature =  $\frac{d^2}{2R}$

$$C_c = \frac{d^2}{2 \times 6370} \text{ km} = \frac{d^2}{2 \times 6370} \times 1000 \text{ meter}$$

$$C_c = (-) 0.0785 d^2$$

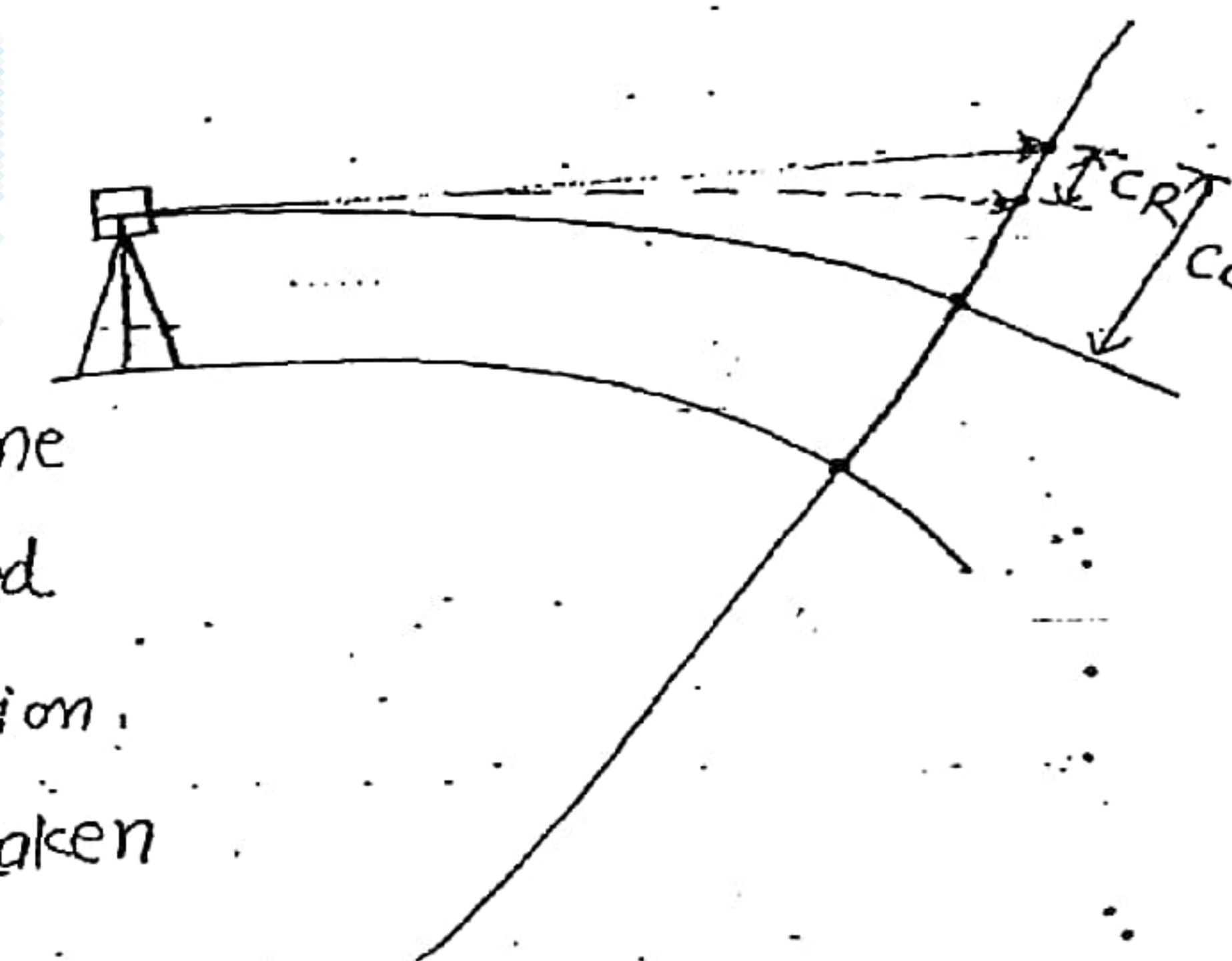
This correction is always (-)ve for staff reading.

$$R.L = H.I - (h_1 - C_c)$$

$$R.L = H.I - h_1 + C_c$$

This correct<sup>n</sup> will become (+)ve for R.L.

(2) Correction due to Refraction :-



Due to refraction, line of sight get deflected in downward direction, thus the reading taken is less.

This correction shall be (+)ve for staff reading.

$$C_R = (+) \frac{1}{7} \times \frac{d^2}{2R} = \frac{1}{7} \times C_c$$

$$C_R = \frac{1}{7} C_c$$

$$C_R = (+) \frac{1}{7} \times 0.0785 d^2$$

$$C_R = (+) 0.0112 d^2 \quad \text{--- (2)}$$



(3) Combined Correction due to Curvature & Refraction :

$$C = C_c + C_R$$

$$= -\frac{d^2}{2R} + \frac{1}{7} \frac{d^2}{2R}$$

$$C = -\frac{6}{7} \cdot \frac{d^2}{2R}$$

$$= -\frac{6}{7} \times 0.0785 d^2$$

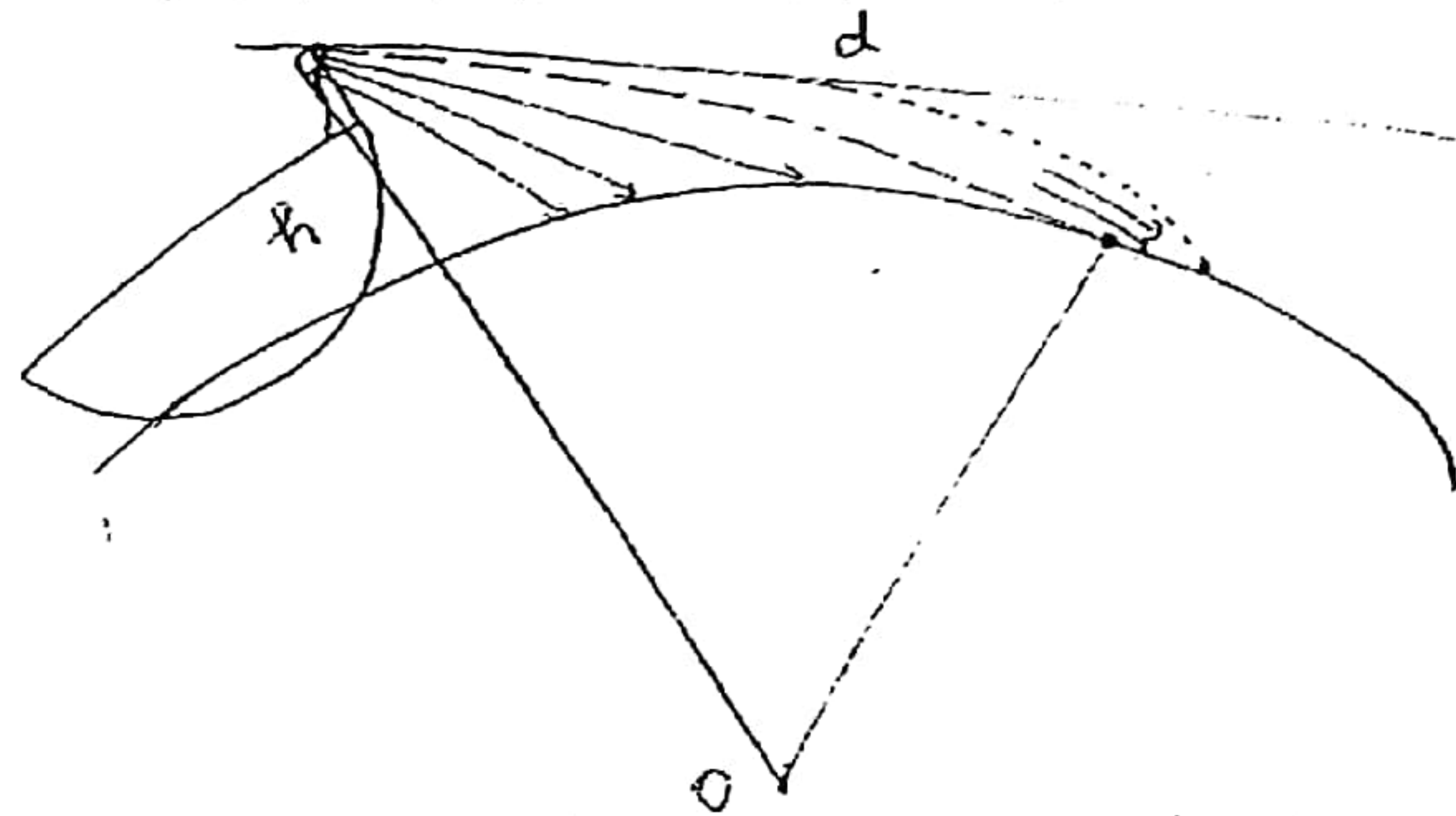
$$C = (-) 0.0673 d^2 \quad \text{--- (3)}$$

Example : Calculate  $C_c$ ,  $C_R$  &  $C$  due to curvature & refraction.

For (i) 120 m (ii) 1500 m

	For 120 m	For 1500 m
$d$ in km	0.12 km	1.5 km
$C_c = 0.0785 d^2$	$= 1.13 \times 10^{-3} \text{ m}$ $= (-) 0.113 \text{ cm}$	$= 0.1766 \text{ m} = (-) 17.66 \text{ cm}$
$C_R = \frac{1}{7} C_c$	$+ 0.016 \text{ cm}$	$(+) 2.52 \text{ cm}$
Combined	$(-) 0.096 \text{ cm}$	$(-) 15.14 \text{ cm}$

(#) Distance of Visible horizon :-



$\Rightarrow$  Distance of visible horizon is max<sup>m</sup> distance upto which A person can see on the surface of earth.

(#) If only earth curvature is considered -

$$h = C_c = \frac{d^2}{2R} = 0.0785 \frac{d^2}{\text{km}}$$

$$d = \sqrt{\frac{h}{0.0785}}$$

$$d = 3.57 \sqrt{h}$$

$$d = 3.57 \sqrt{h} \quad \begin{matrix} d \rightarrow \text{in km} \\ h \rightarrow \text{in m} \end{matrix}$$

(#) If combined effect of earth curvature & refraction is considered.

The person will be able to see some more distance.

$$h = \frac{6}{7} \frac{d^2}{2R} = 0.0673 d^2$$

$$d = \sqrt{\frac{h}{0.0673}} = 3.855 \sqrt{h}$$

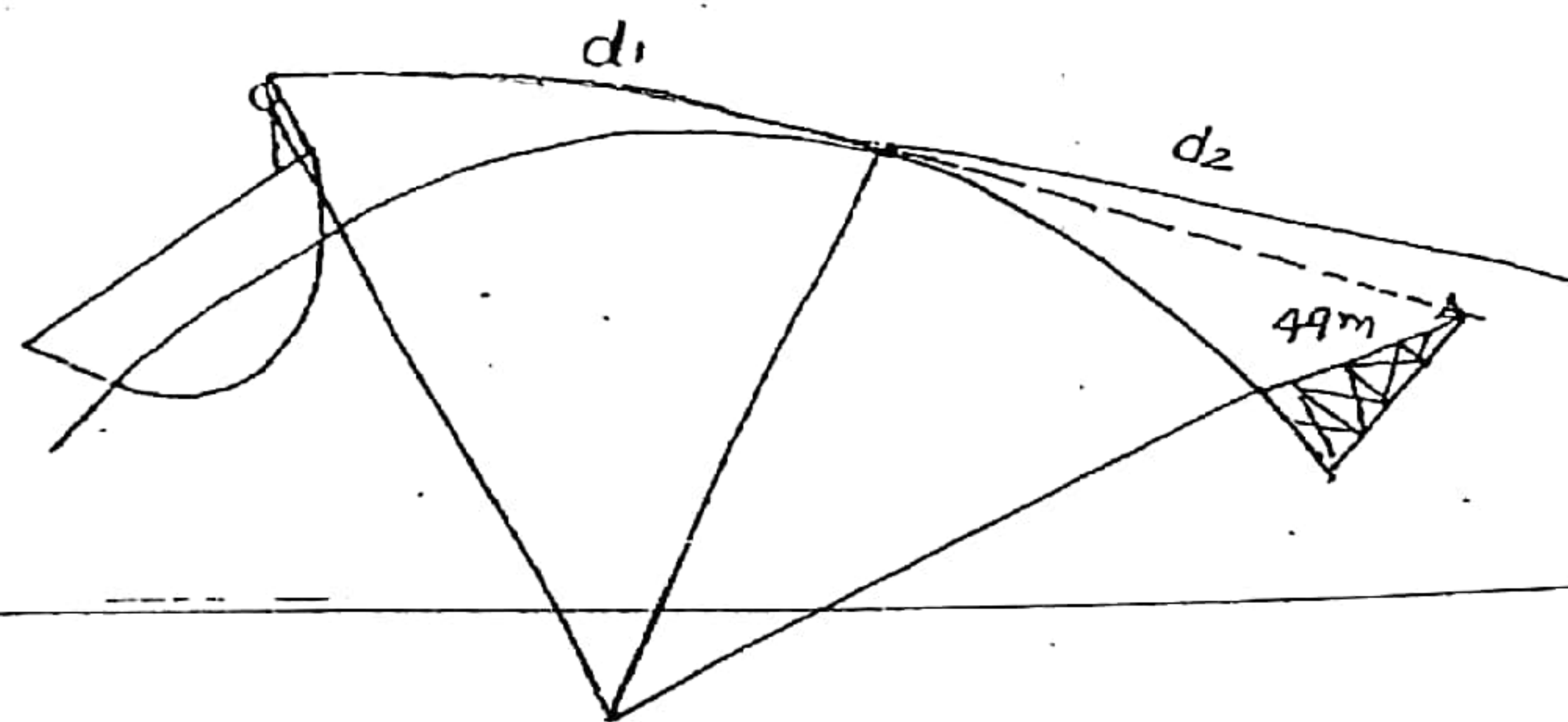
$$d = 3.855 \sqrt{h} \quad \text{--- Distance of visible horizon}$$

$$\begin{matrix} h - \text{in m} \\ d - \text{in km} \end{matrix}$$

ES-98

Ques :- An observer standing on the deck of a ship just sees a light house. The top of light house is at 79 m above sea level and the ht. of observer's eye is 9 m above sea level. Find the distance of observer from light house.





Total distance

$$= d_1 + d_2$$

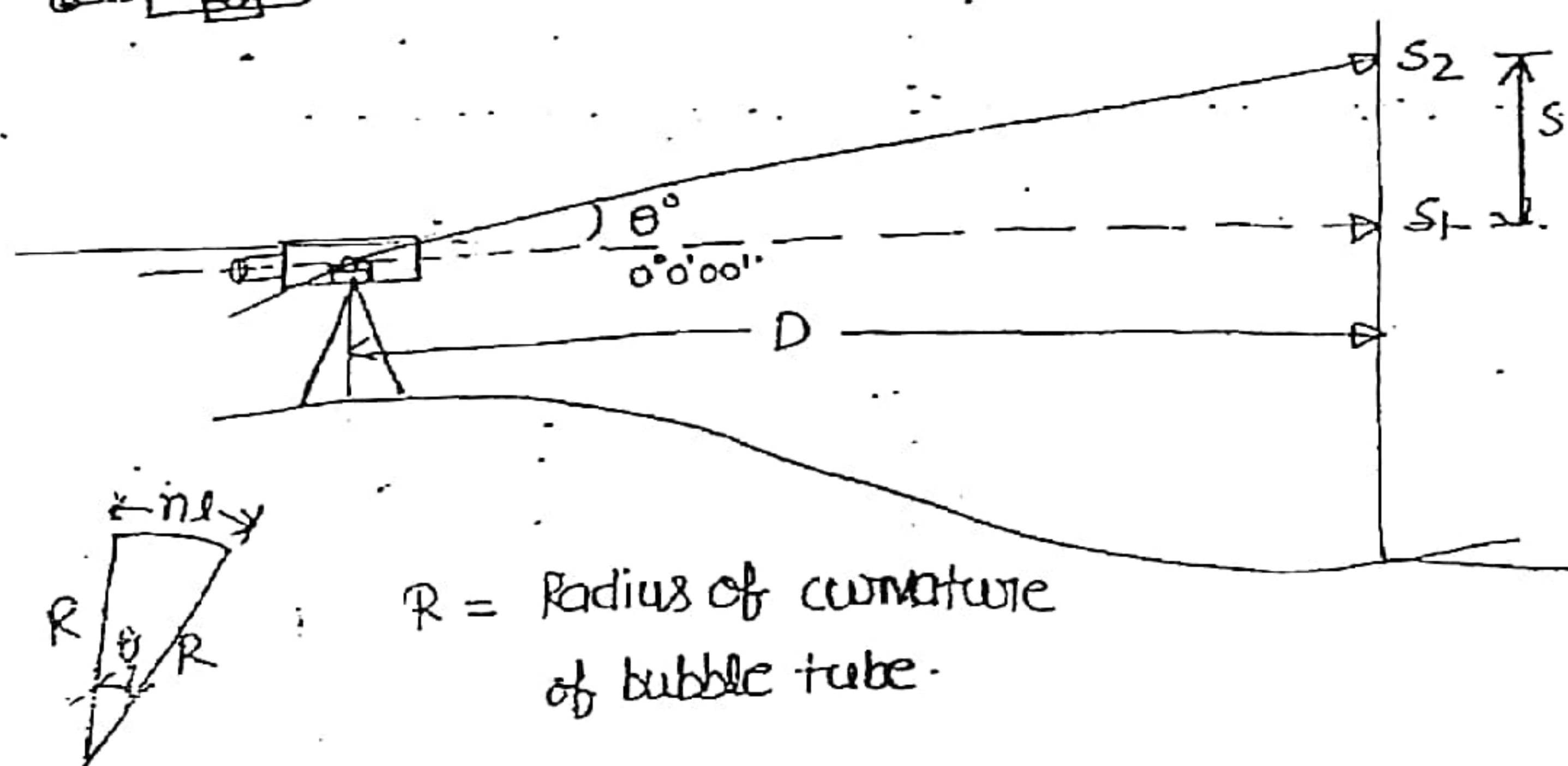
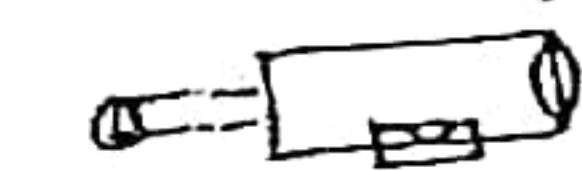
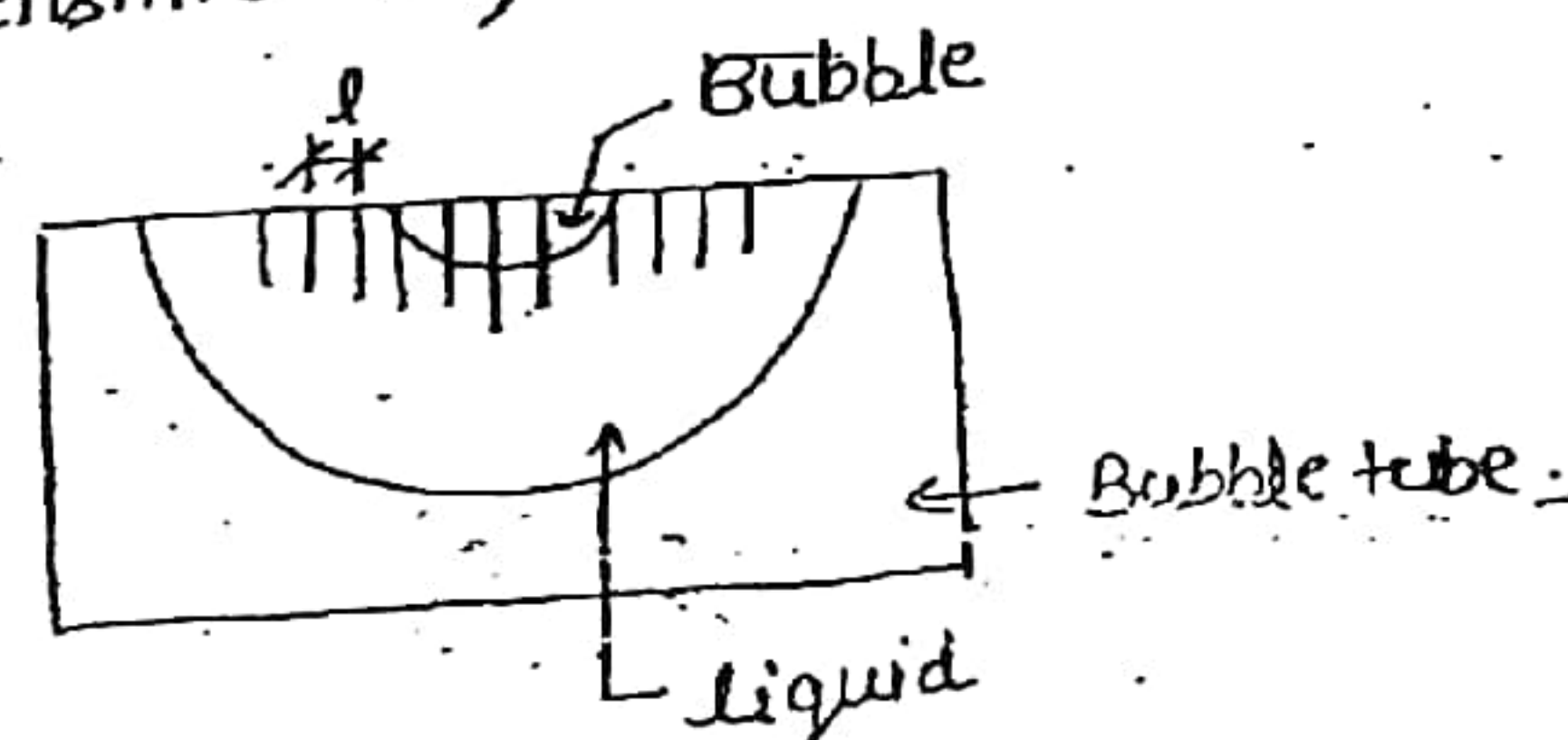
$$= 3.855 \sqrt{h_1} + 3.855 \sqrt{h_2}$$

$$= 3.855 (\sqrt{49} + \sqrt{9}) = 3.855 (7+3)$$

$$= 38.55 \text{ km}$$

3/12/13

⊕ Sensitivity of a Bubble tube  
(Also called Sensitiveness)



R = Radius of curvature of bubble tube.

If a staff reading  $s_1$  is taken from an instrument when bubble is just in centre. Now, if the telescope is rotated by  $\theta$  angle so that the staff reading changes to  $s_2$ . Distance of staff = D from instrument.

In this case the bubble has moved by n divisions.

If l = length of one division —

$$\theta = \frac{s}{D} = \frac{nl}{R} \quad \text{--- (1) Angle of rotation.}$$

sensitivity or sensitiveness of bubble tube is the angle of movement due to one division movement of bubble (say  $\alpha$  angle).

Sensitiveness —

$$\alpha = \frac{\theta}{n} = \frac{s}{nD} = \frac{l}{R} \quad \text{--- (2)}$$

Ques: 1) The staff reading taken from an instrument when bubble was at centre on a staff kept at 250m distance was 1.62m. The bubble was then moved by 6 division & staff reading changed to 1.95m. If length of one division of bubble tube is 2.50mm calculate —

(i) sensitiveness of a bubble tube.

(ii) Radius of curvature of bubble tube.

Ans

l = 2.5 mm (length of one division)

n = 6 (no. of division moved).

D = 250m

s = staff intercept =  $s_2 - s_1$

$$= 1.95 - 1.62 = 0.33 \text{ m}$$



(i) Sensitiveness of bubble table

$$\alpha = \frac{S}{mD} = \frac{0.33}{6 \times 250} = \frac{11}{50000} = 2.2 \times 10^{-4} \text{ rad.}$$

$$= (2.2 \times 10^{-4}) \left( \frac{180}{\pi} \times 60 \times 60 \right) \text{ sec.}$$

$$= 45''.38$$

(ii) Radius of curvature -

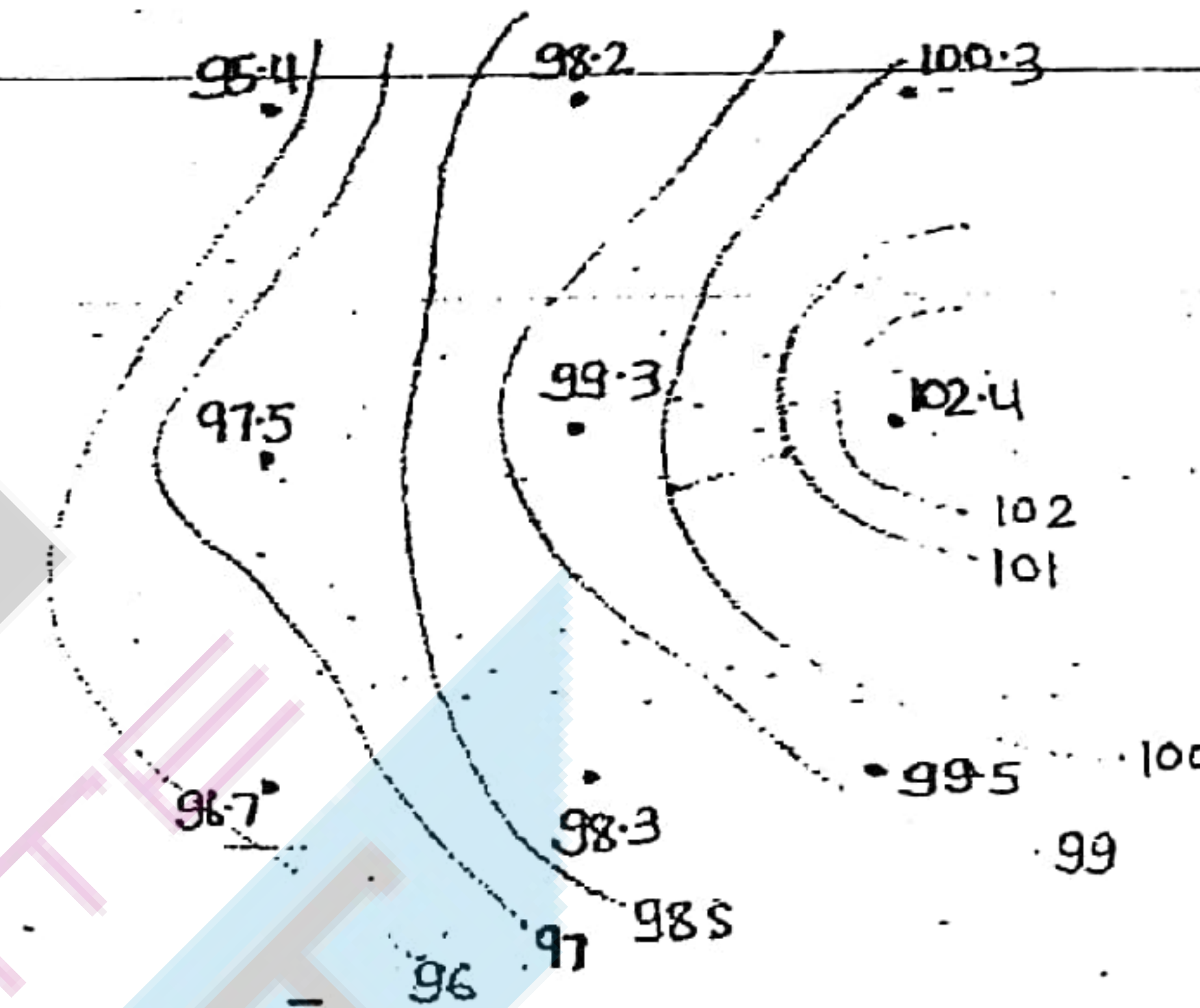
$$R \Rightarrow \alpha = \frac{1}{R} = 2.2 \times 10^{-4}$$

$$R = \frac{1}{\alpha} = \frac{25 \text{ mm}}{2.2 \times 10^{-4}} = 11363.64 \text{ mm}$$

$$= 11.364 \text{ m}$$

## CONTOURS

Contours are the locus of equal elevation points on ground surface.



On a particular contour R.L of all points will be same

### Important Terms :-

1) Contour Interval :- Difference of R.L. b/w two consecutive contour for a drawing is called contour interval. For one drawing contour interval should be kept same at all location.

For ex: In fig - C.I. = 1m

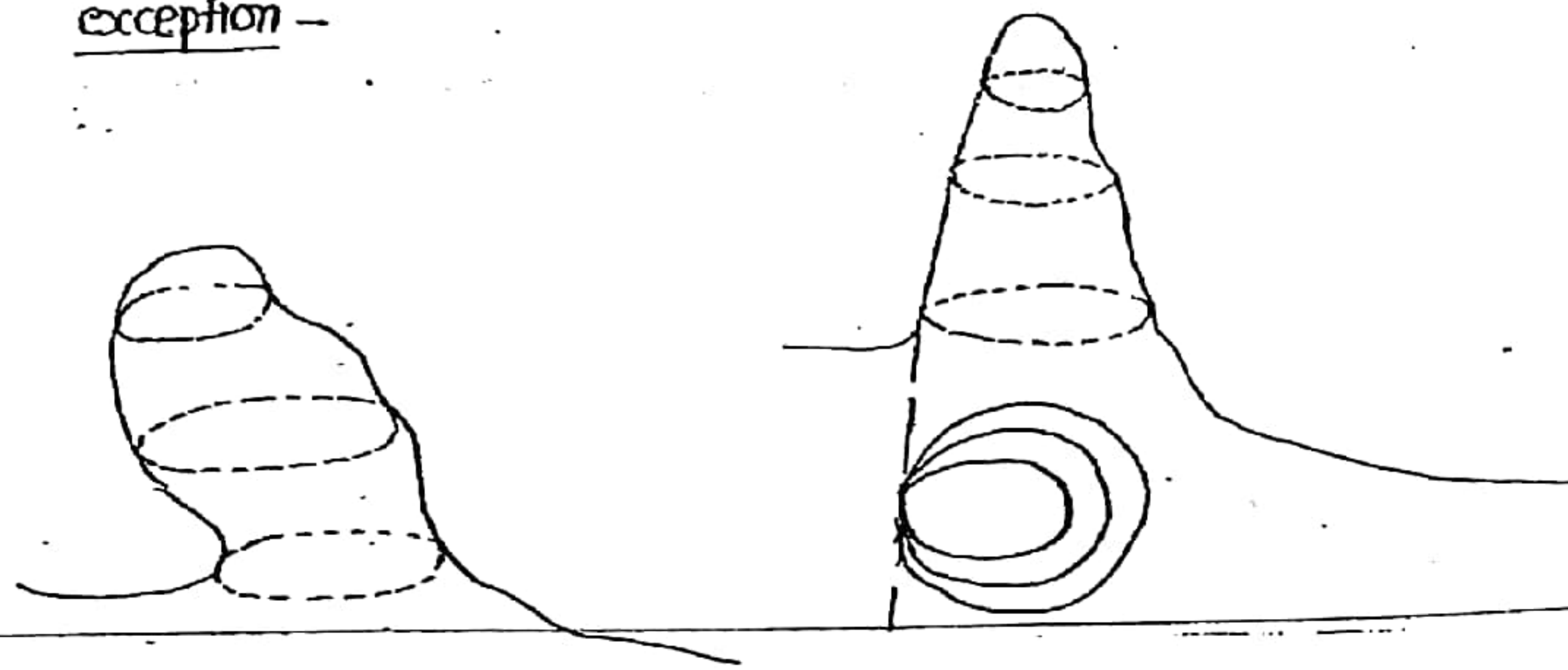
2) Horizontal Equivalent :- It is the horizontal distance b/w any 2 point on two consecutive contour.

3) Properties :-

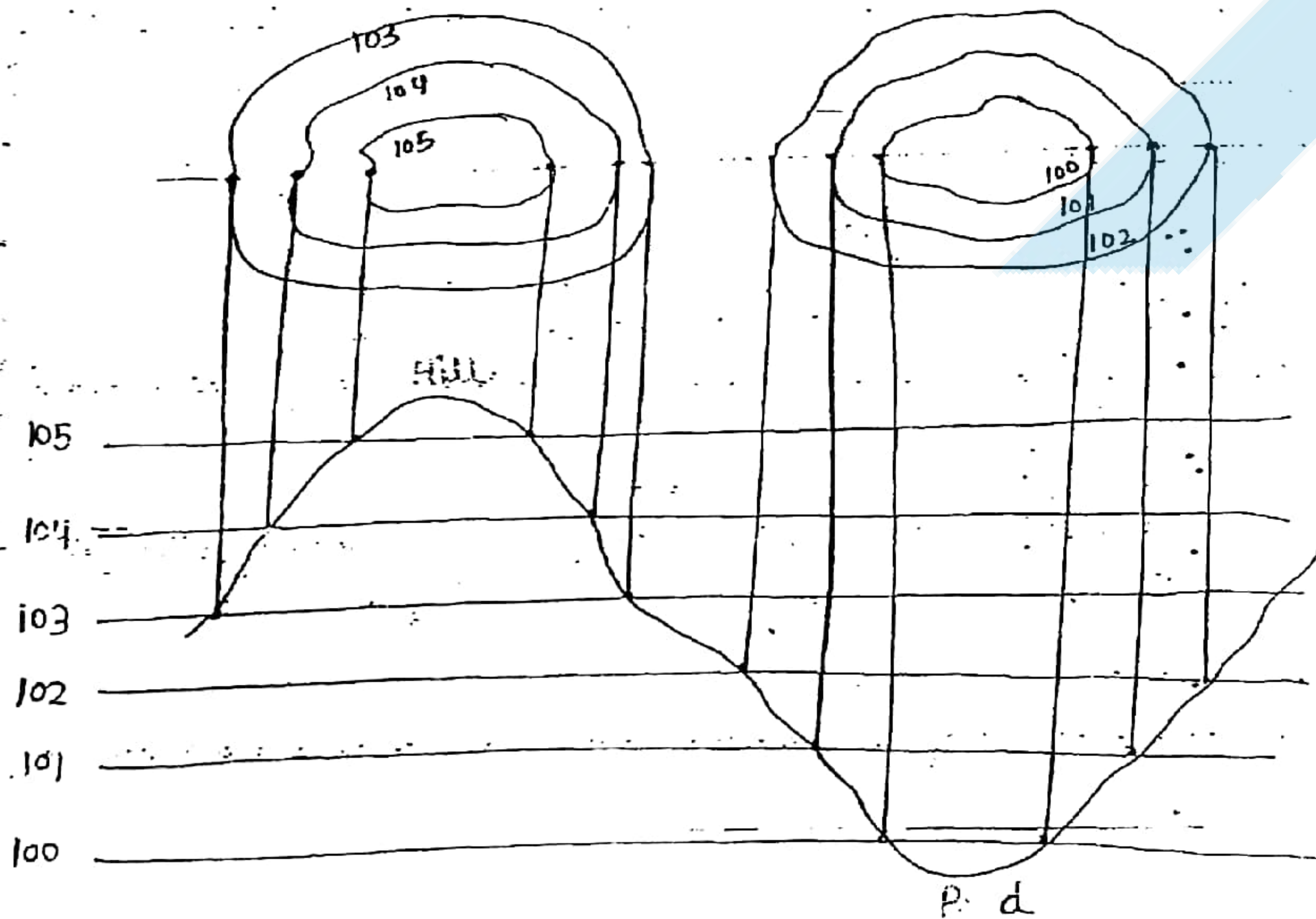
(a) Two contour does not cross each other / does not meet at any point.



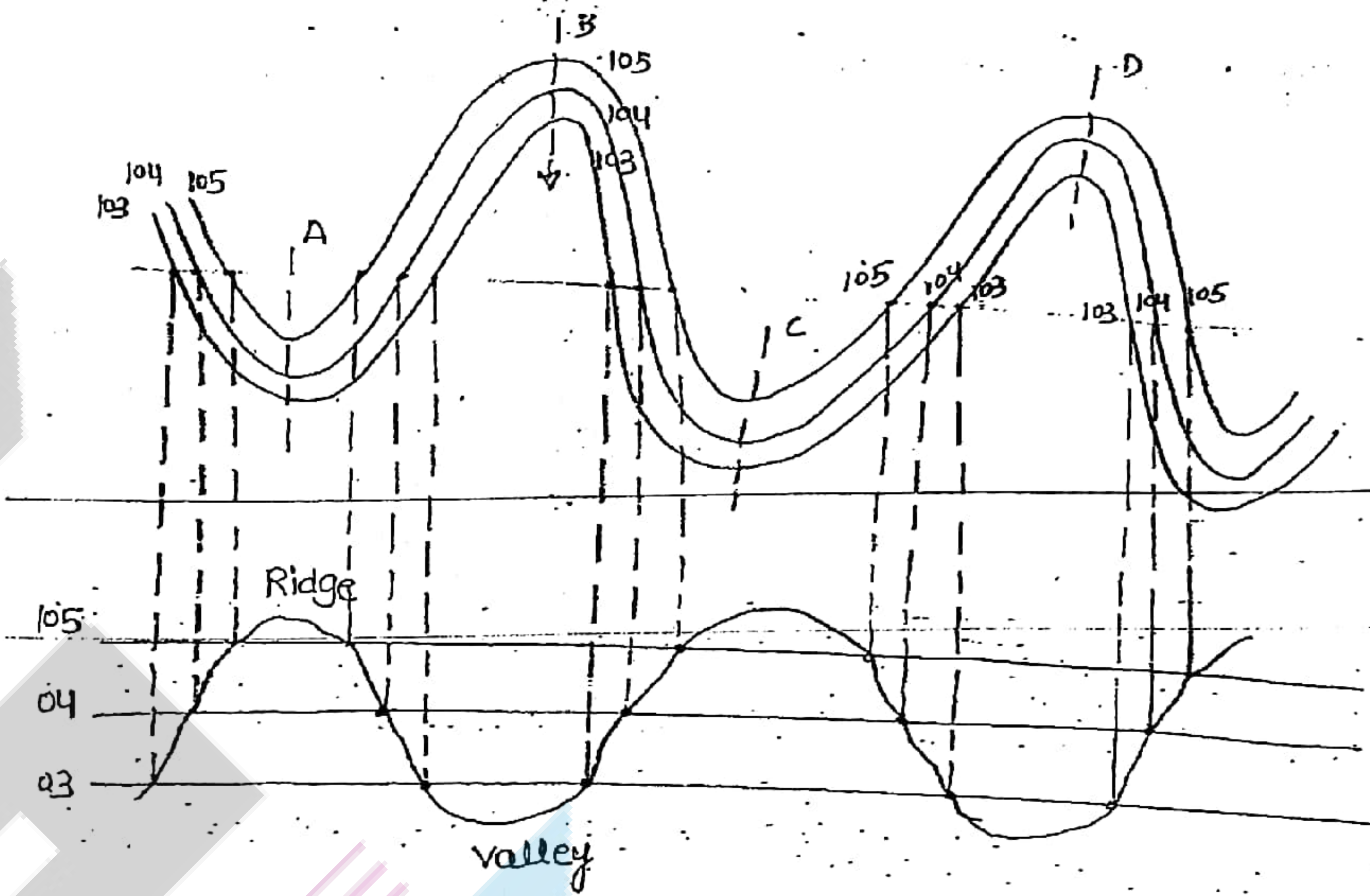
exception -



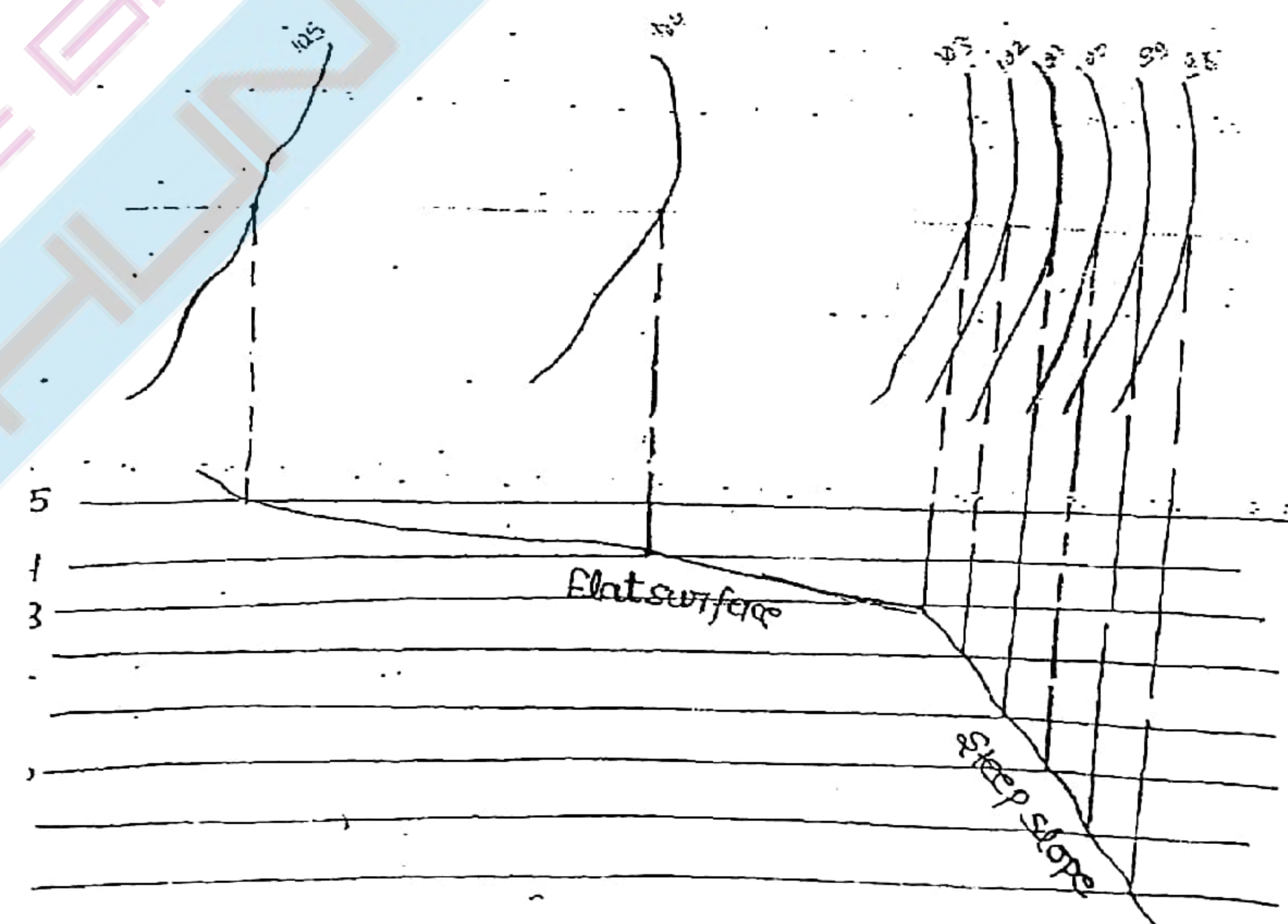
(b) closed contours show either a hill or a pond.



(c) Ridge line / valley lines

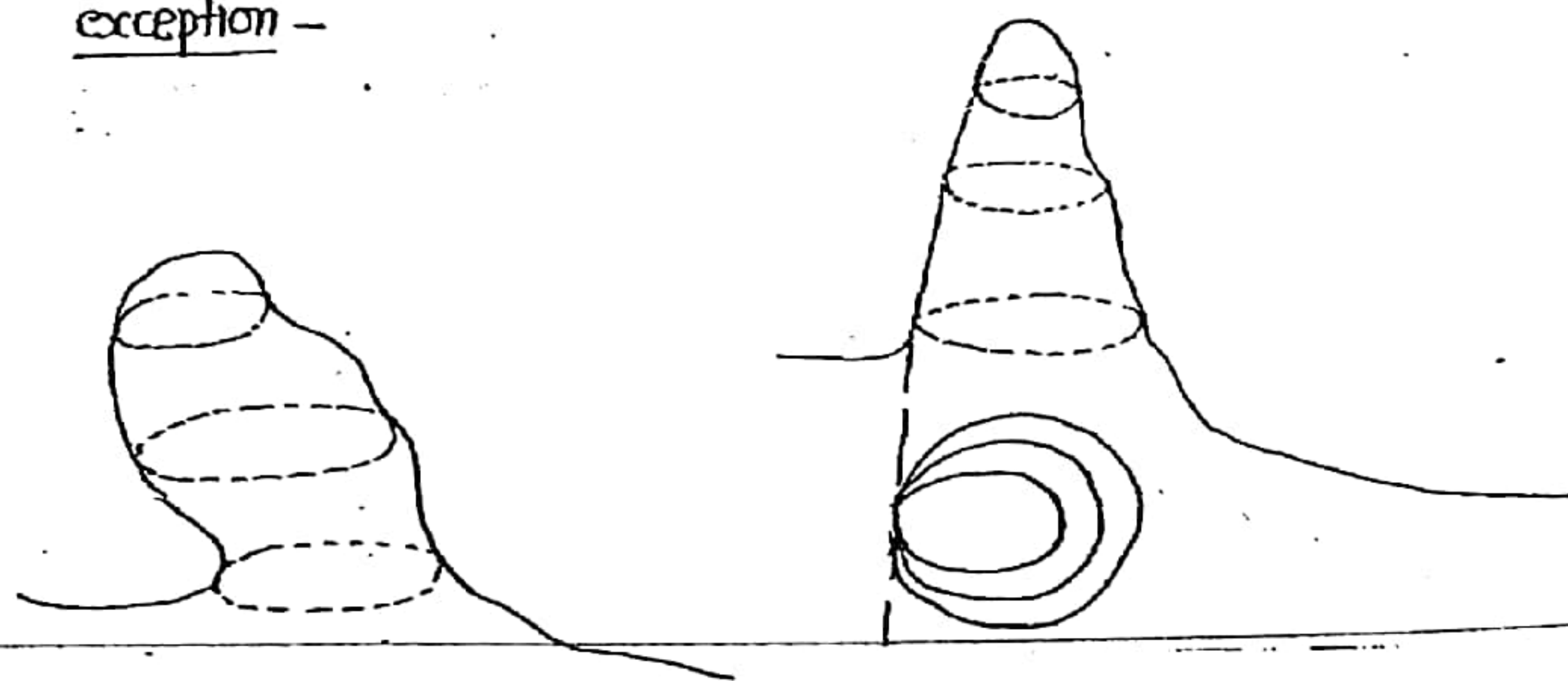


(d) Distant contours — Flat surface  
close contours — Steep slope

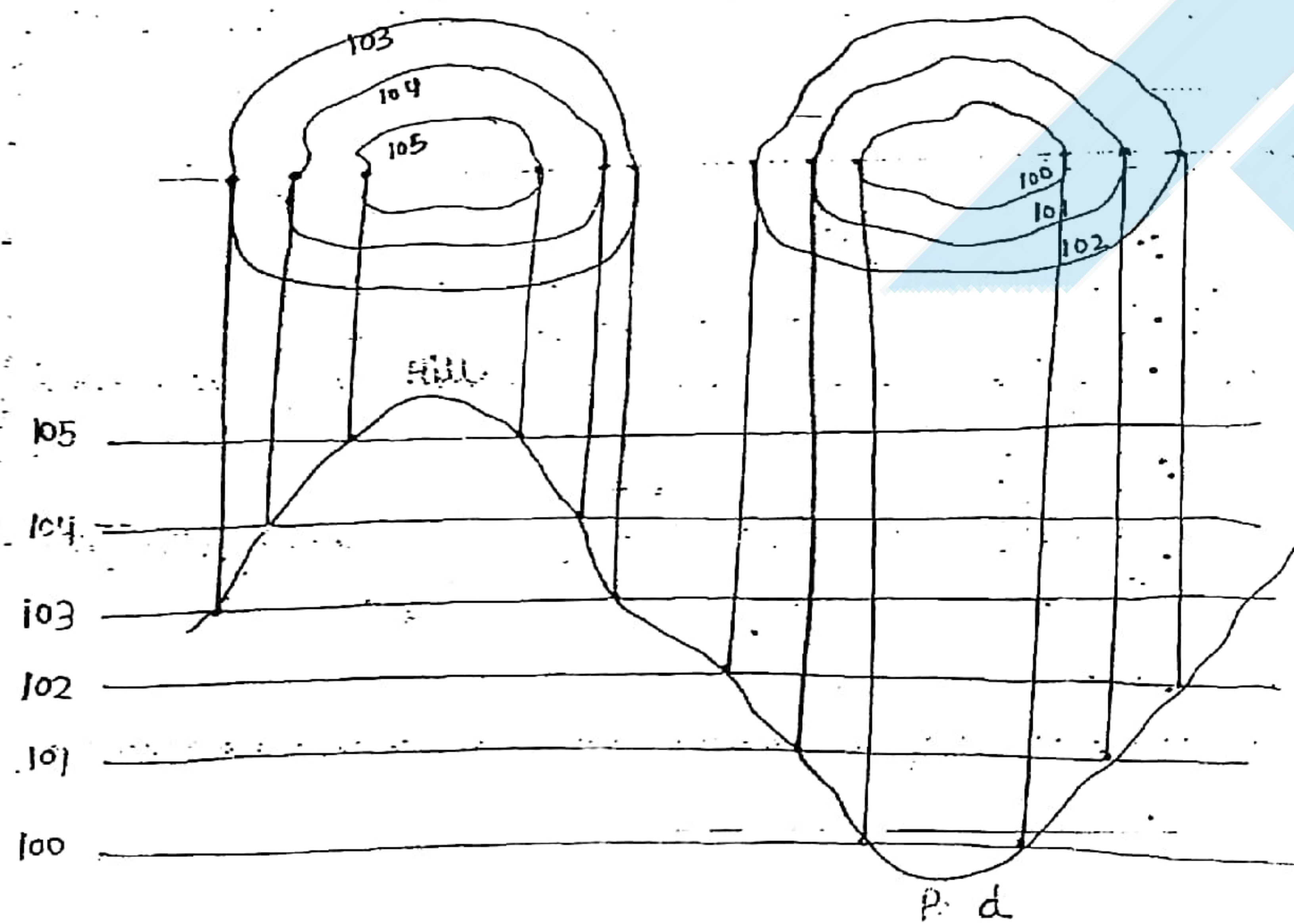




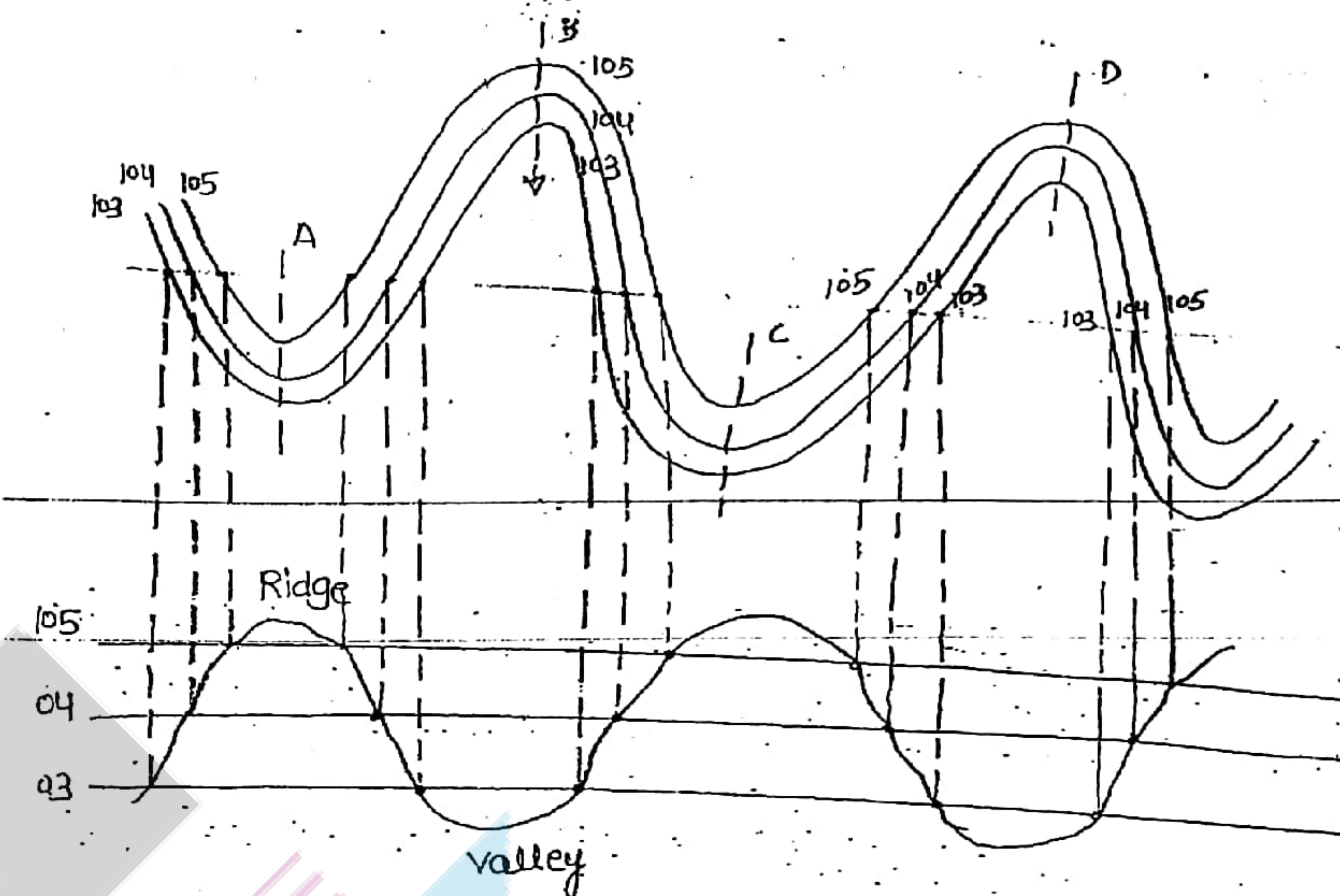
exception -



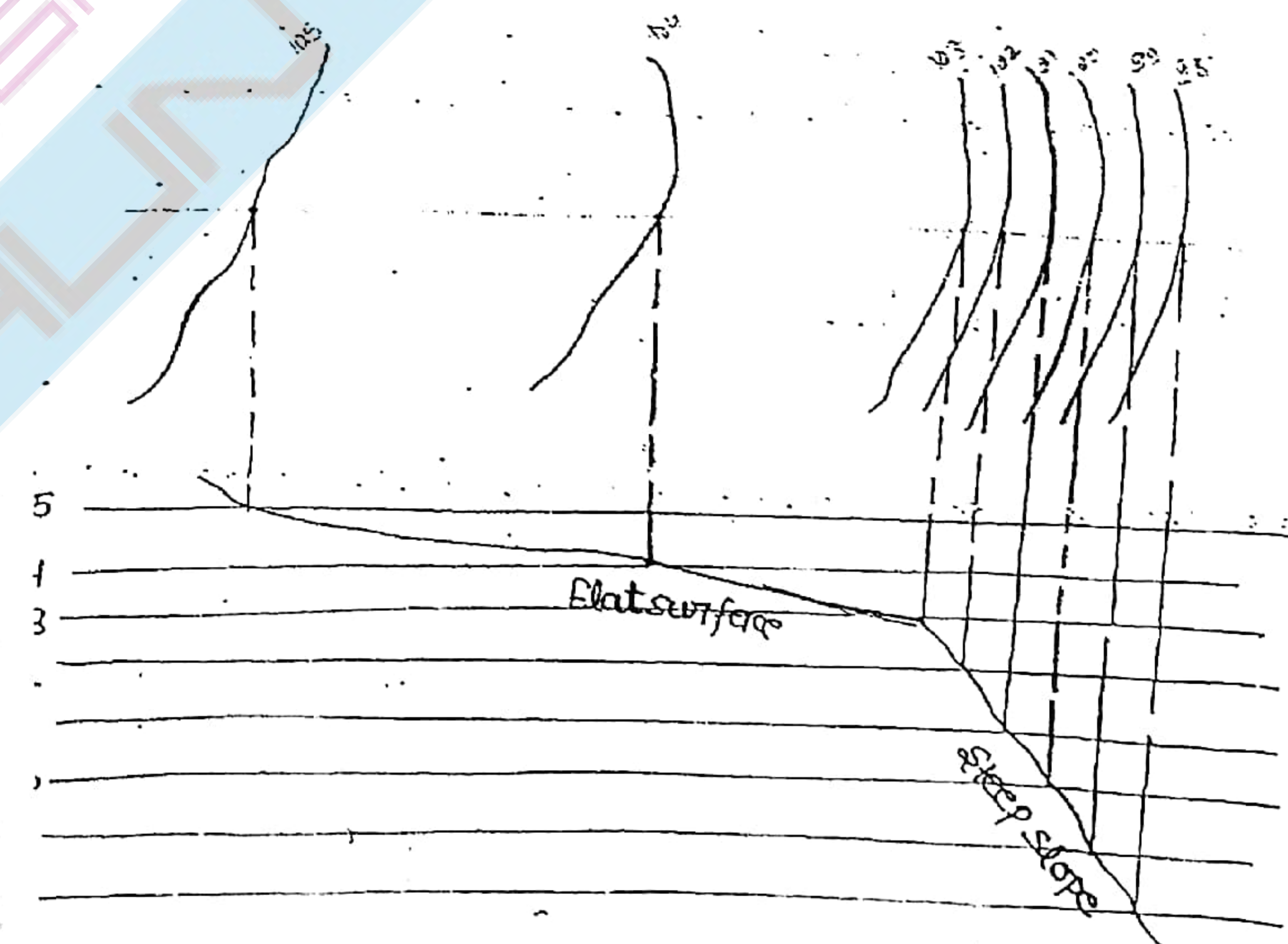
(b) closed contours show either a hill or a pond.



(c) Ridge line / valley lines

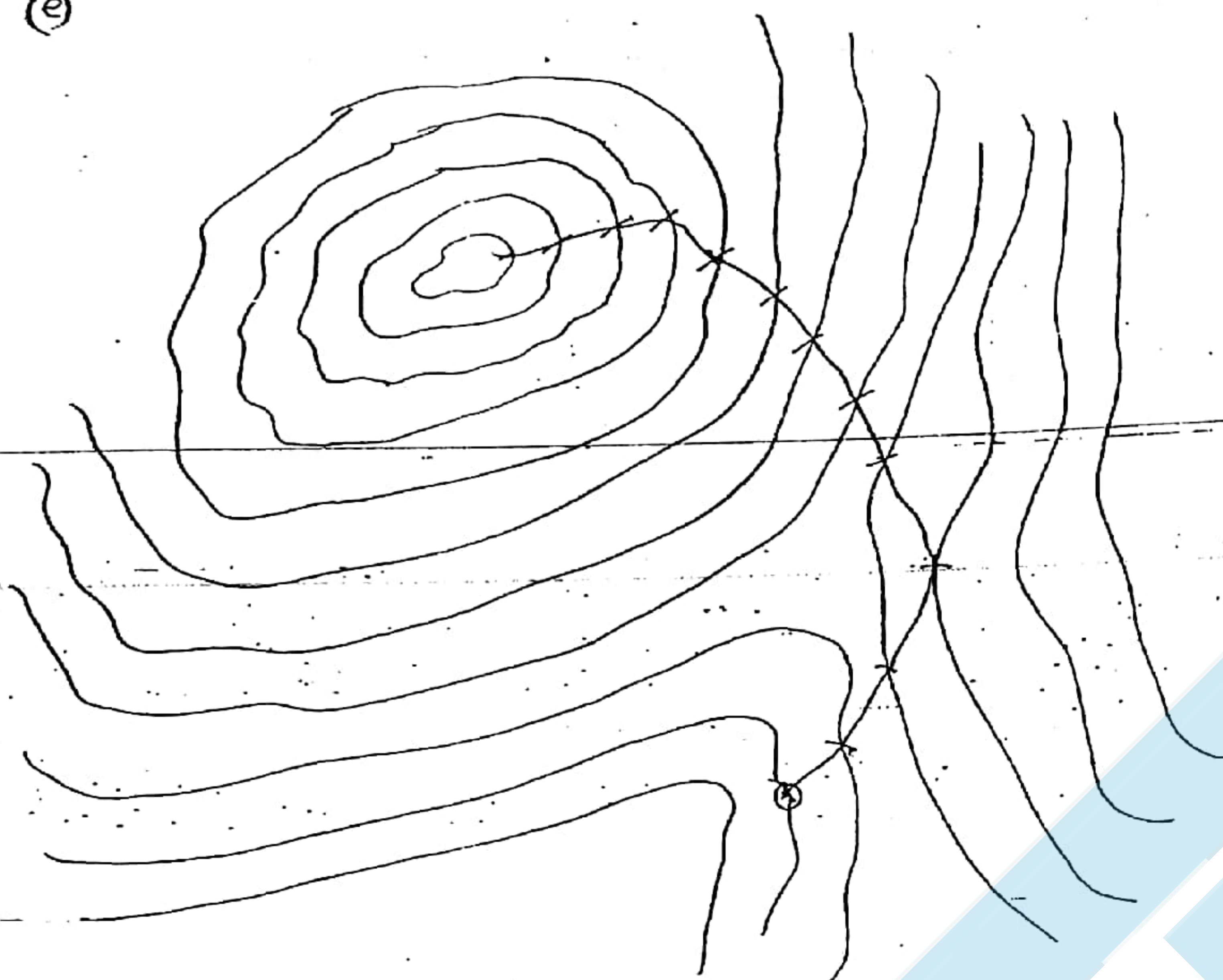


(d) Distant contours — Flat surface  
close contours — steep slope





(e)



How to set alignment of a road in hilly area using a contour plan.

on a uniform gradient

Ex. gradient = 1 in 30.

## AREA / VOLUME

(1) Area —

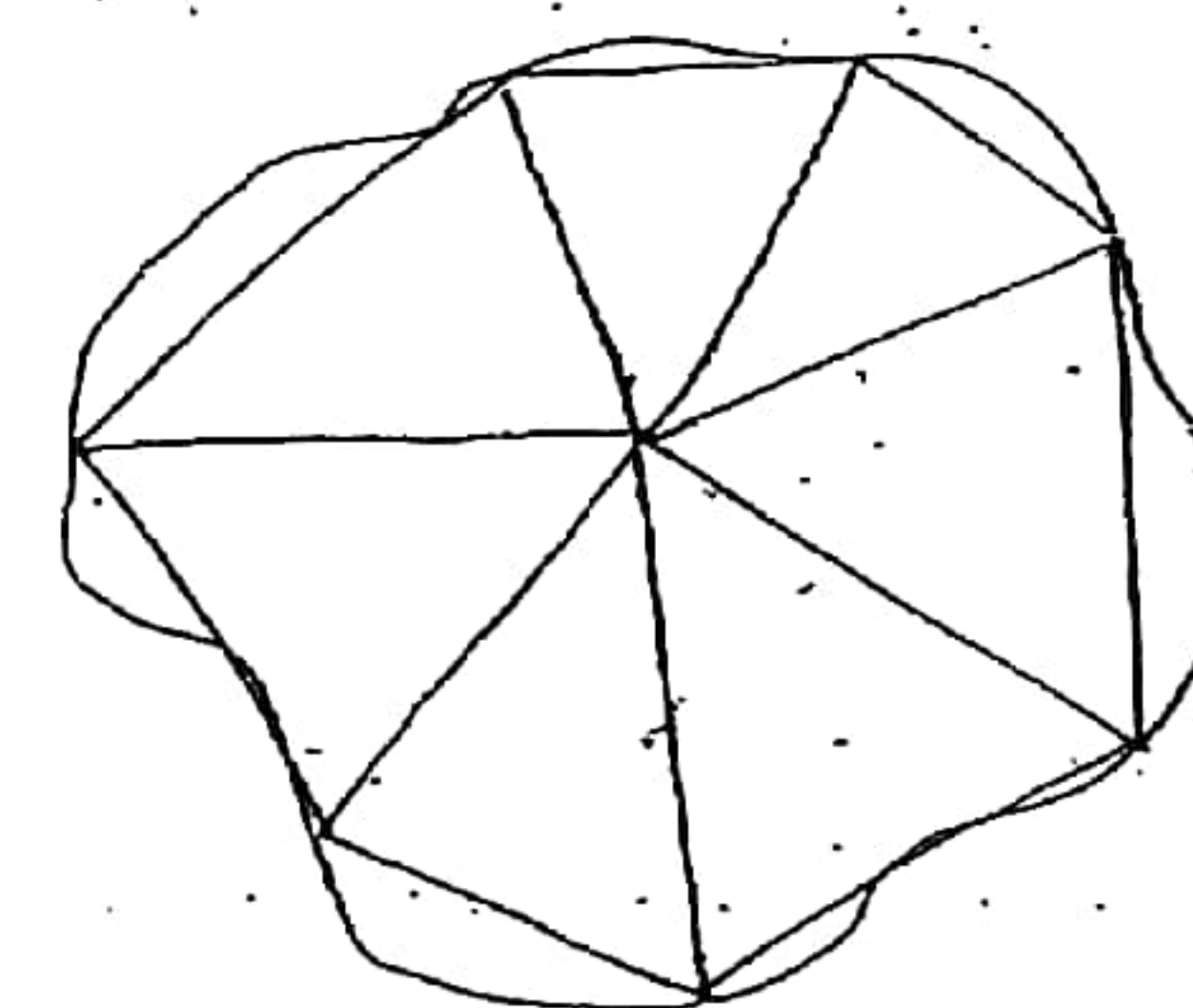
(A) By dividing into a no. of triangles.

Area of triangle —

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

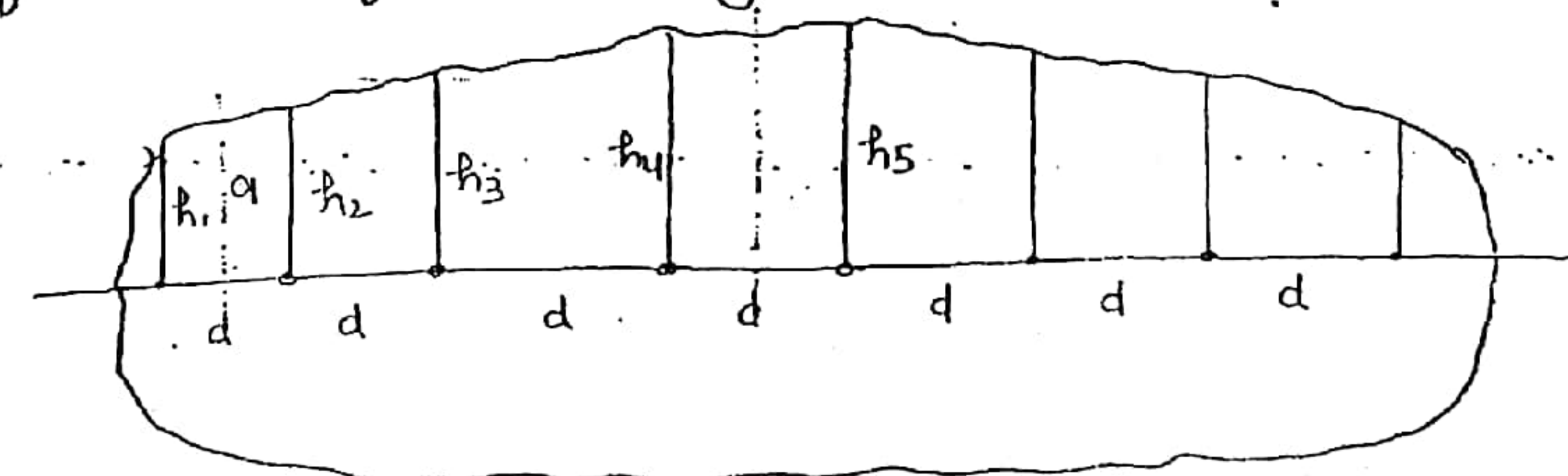
$$s = \frac{a+b+c}{2}$$

$$A = \frac{1}{2} ab \sin C = \frac{1}{2} bc \sin A = \frac{1}{2} ac \sin B$$



(B) offset Method

offset taken from a single line at equal interval.



(i) Average ordinate Rule

$$A = \left( \frac{h_1 + h_2 + h_3 + \dots + h_n}{n} \right) \times (n-1)d$$

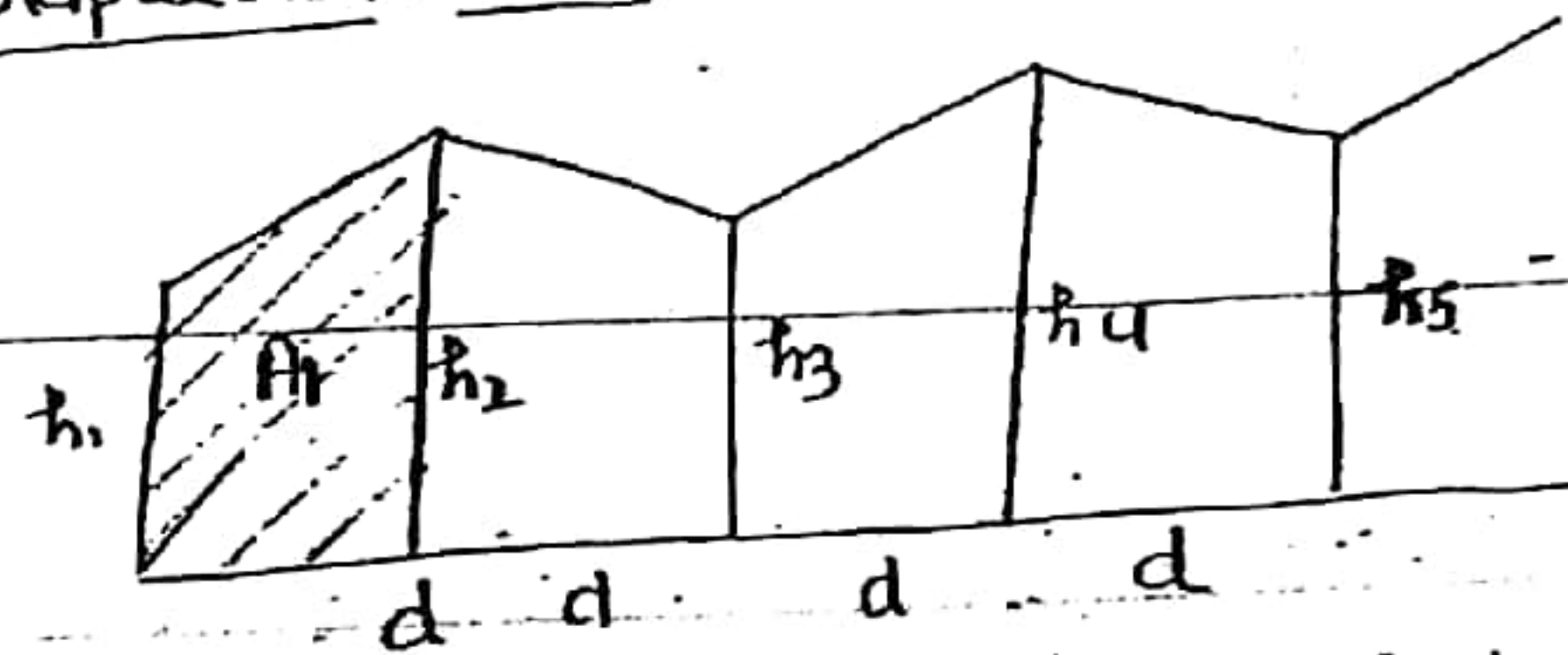
This is a rough method



(ii) Mid ordinate Rule :- If mid ordinates  $o_1, o_2, o_3, \dots$  are measured.

$$A = d \times (o_1 + o_2 + o_3 + o_4 + \dots + o_n)$$

(iii) Trapezoidal Rule :-



Area of one block

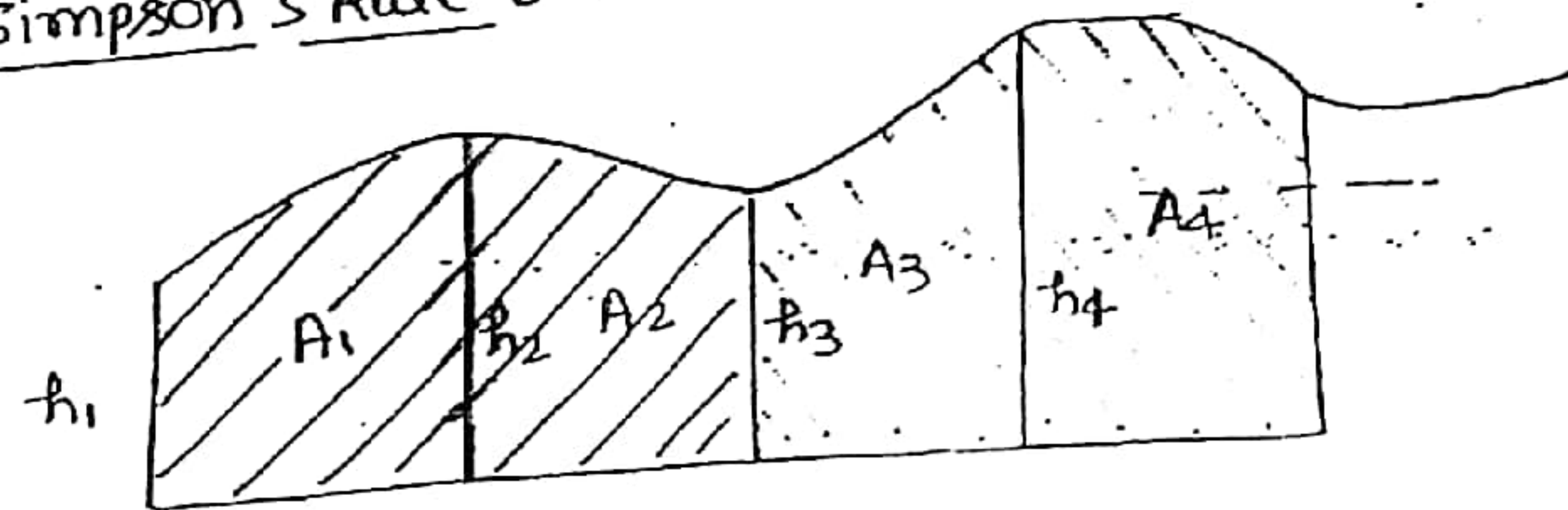
$$A_1 = d \times \left( \frac{h_1 + h_2}{2} \right)$$

$$A_2 = d \times \left( \frac{h_2 + h_3}{2} \right)$$

Total area

$$A = d \left[ \frac{h_1 + h_n}{2} + h_2 + h_3 + \dots + h_{n-1} \right]$$

(iv) Simpson's Rule :-



Area of two block

$$A_1 + A_2 = \frac{d}{3} (h_1 + 4h_2 + h_3)$$

$$A_3 + A_4 = \frac{d}{3} (h_3 + 4h_4 + h_5)$$

Total area

$$A = \frac{d}{3} \left[ (h_1 + h_n) + 4(h_2 + h_4 + h_6 + \dots) + 2(h_3 + h_5 + h_7 + \dots) \right]$$

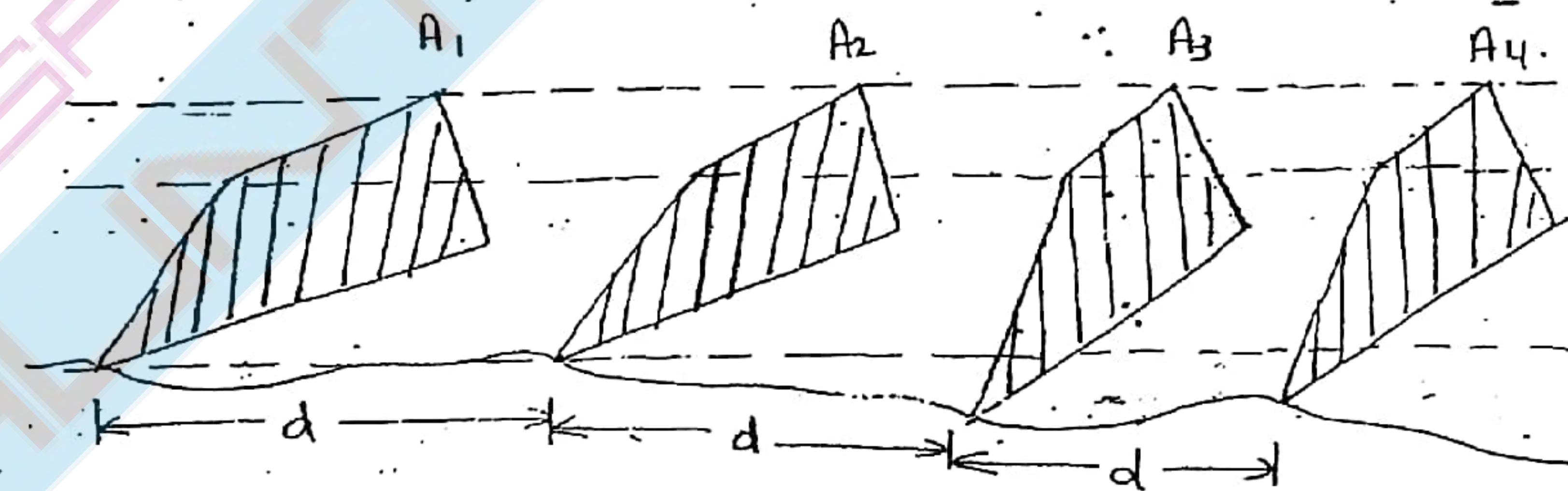
$\Rightarrow$  We need odd no. of offsets in Simpson's rule.

$\Rightarrow$  If there are even no. of offsets.

Calculate one area (1st or last) by Trapezoidal rule and add in total area calculated for other offset by Simpson's rule.

2) Volume :-

(A) Trapezoidal Rule :-



Volume

$$V = d \times \left[ \frac{A_1 + A_n}{2} + A_2 + A_3 + \dots + A_{n-1} \right]$$

Simpson's Rule

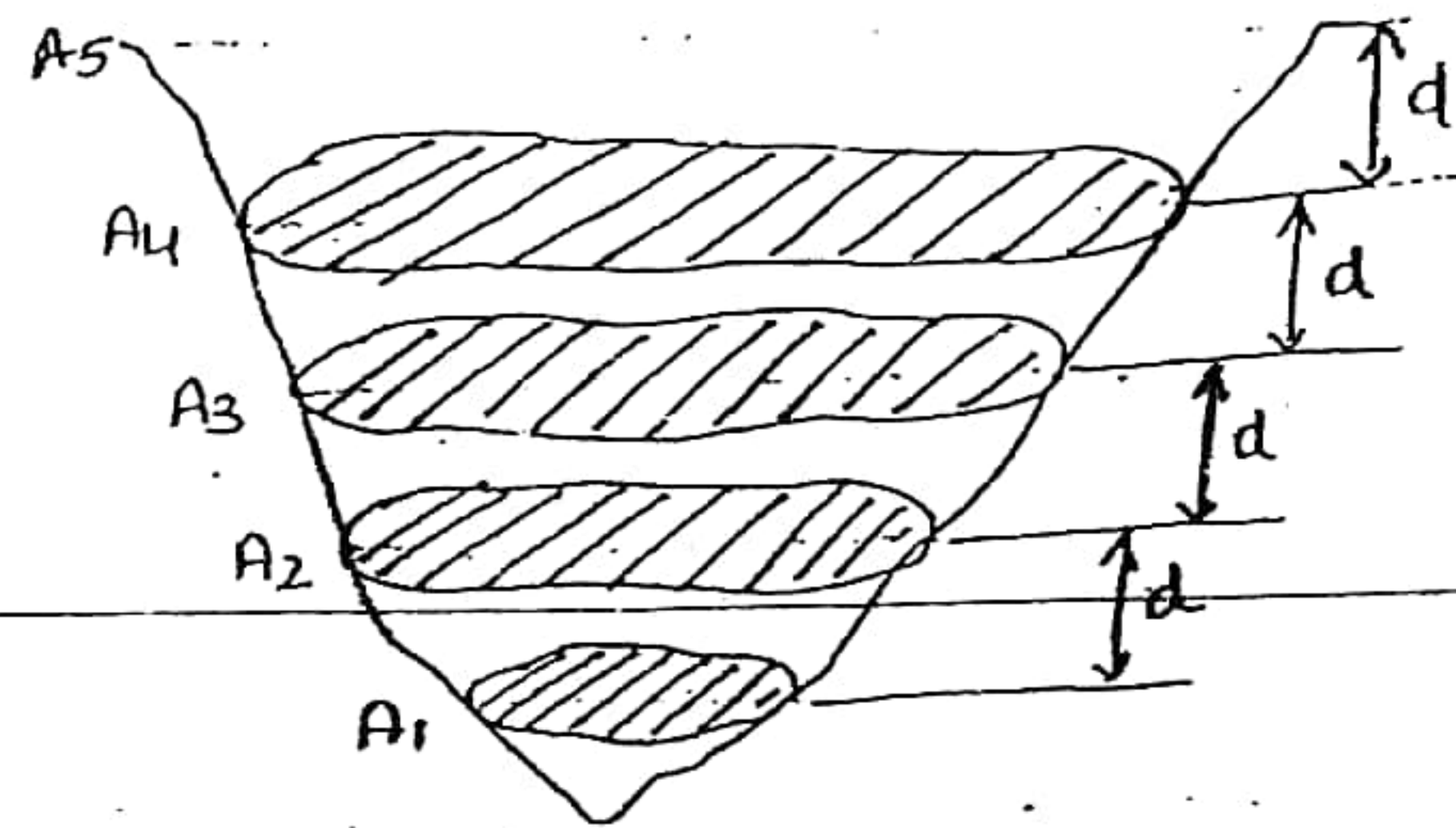
Volume

$$V = \frac{d}{3} \left[ (A_1 + A_n) + 4(A_2 + A_4 + A_6 + \dots) + 2(A_3 + A_5 + A_7 + \dots) \right]$$



Another Example -

Volume of water in a reservoir -



Ques: 6(a) In a proposed reservoir, the area contouring within ES-2006, the contours are.

Contours (in m)	100	95	90	85	80	75	70	65
Area (in ha)	32	26	24	18	15	13	7	2

$$1 \text{ hect} = 10^4 \text{ m}^2 \quad (100 \text{ m} \times 100 \text{ m})$$

Using the method of end areas calculate -

- Capacity of reservoir when it is full at 100m level.
- Elevation of water level when it is 60% full. Ignore the volume below 65 m level.

Soln: End Area Method is trapezoidal Rule -

- Volume of water when water is full up to 100 m.

$$\begin{aligned}
 V &= d \left[ \frac{A_1 + A_n}{2} + A_2 + A_3 + \dots \right] \\
 &= 5 \left[ \frac{32 + 2}{2} + 26 + 24 + 18 + 15 + 13 + 7 \right] \\
 &= 600 \text{ Ha-m}
 \end{aligned}$$

- When the reservoir is 60% full.

$$V = 0.6 \times 600$$

$$V = 360 \text{ Ha-m}$$

Contour	Area	Average Area	Volume	Total Volume
100	32	29	145	600
95	26	25	125	455
90	24	21	105	330
85	18	16.5	82.5	225
80	15	14.0	70.0	142.5
75	13	10	50.0	72.5
70	7	4.5	22.5	22.5
65	2	0		

Neglect

$$\text{For } 330 \text{ Ha-m} \rightarrow W \cdot T = 90 \text{ m}$$

$$\text{For } 455 \text{ Ha-m} \rightarrow W \cdot T = 95 \text{ m}$$

$$\text{For } 360 \text{ Ha-m} \rightarrow ?$$

$$W \cdot T = 90 + \frac{(95 - 90)}{(455 - 330)} (360 - 330)$$

$$W \cdot T = 91.2 \text{ m}$$

Ques: 6(b) A railway embankment is 16m wide with side slope 2:1 assume the ground to be level in the dir<sup>n</sup> transverse to the centre line. Calculate the volume contained in a length of 100m, the centre ht. at 20m

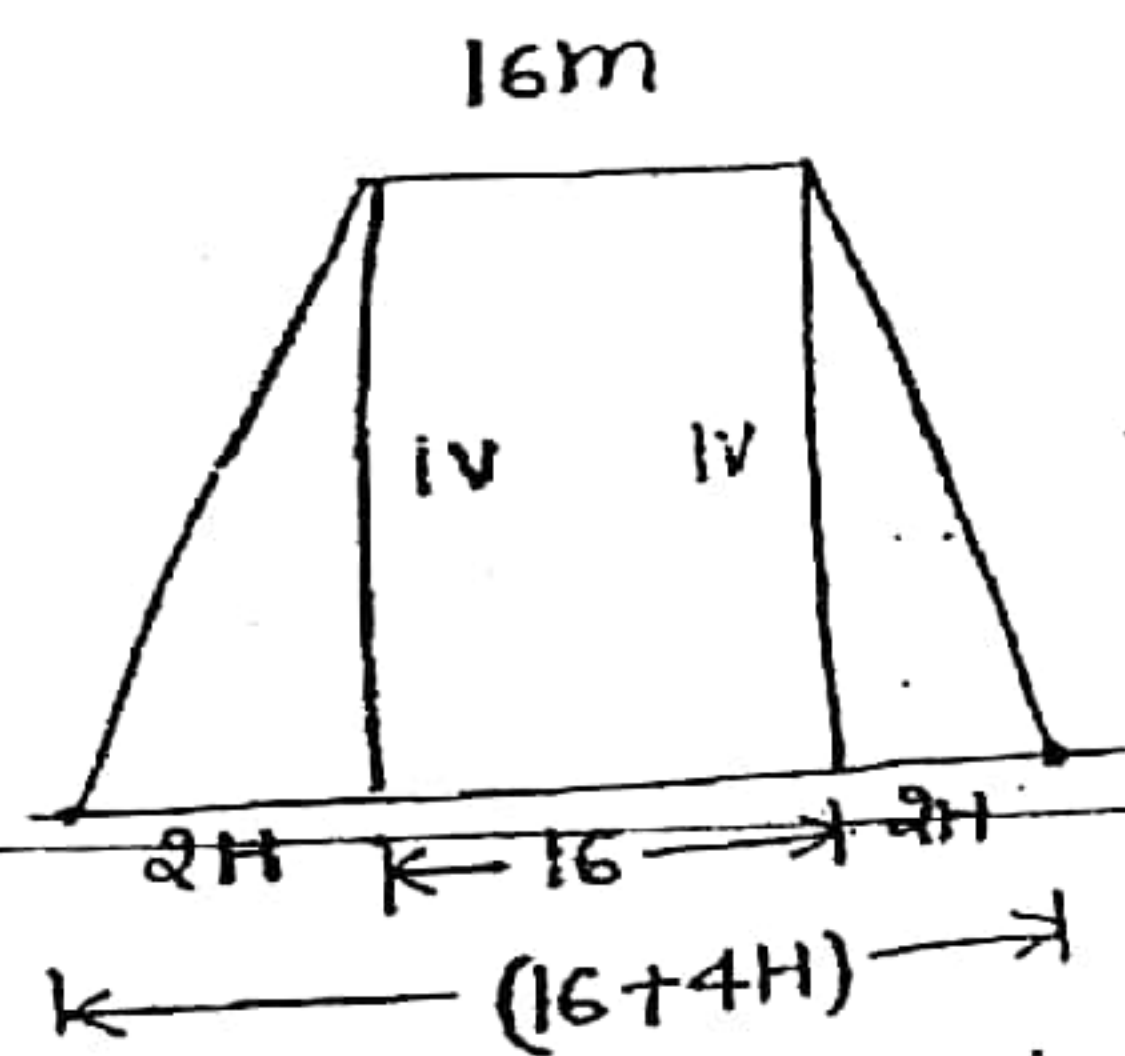


Interval being in meter —

2.0, 4.5, 4.0, 3.5, 2.5, 1.5

⇒ Area

$$A = \frac{(B + B_1)}{2} \times H$$



Distance	Height	Top width	Bottom width (16+4H)	Area
0m	2.0	16	24	40 m <sup>2</sup>
20m	4.5	16	34	112.5
40m	4.0	16	32	96
60m	3.5	16	30	80.5
80m	2.5	16	26	52.5
100m	1.5	16	22	28.5

$$A = \frac{16+24}{2} \times 2.0$$

Volume —

$$V = d \times \left[ \frac{A_1 + A_n}{2} + A_2 + A_3 + A_4 + A_5 + \dots \right]$$

$$= 20 \times \left[ \frac{40 + 28.5}{2} + 112.5 + 96 + 80.5 + 52.5 \right]$$

$$V = 7512 \text{ m}^3$$

Ques: (3) An excavation has been made as shown in figure. Calculate the quantity of earth excavated.

Use: (I) Trapezoidal Rule.

(ii) Simpson's Rule.

(I) Trapezoidal Rule:

$$\therefore V = A_1 = 70 \times 55 = 3850 \text{ m}^2$$

$$A_2 = 50 \times 35 = 1750 \text{ m}^2$$

$$V = d \left( \frac{A_1 + A_2}{2} \right)$$

$$= 10 \left( \frac{3850 + 1750}{2} \right)$$

$$V = 28000 \text{ m}^3$$

(II) Simpson's Rule:

$$A_1 = 70 \times 55 = 3850 \text{ m}^2$$

$$A_2 = 60 \times 45 = 2700 \text{ m}^2$$

$$A_3 = 50 \times 35 = 1750 \text{ m}^2$$

$$V = \frac{d}{3} [A_1 + 4A_2 + A_3]$$

$$= \frac{5}{3} \{ 3850 + 4 \times 2700 + 1750 \}$$

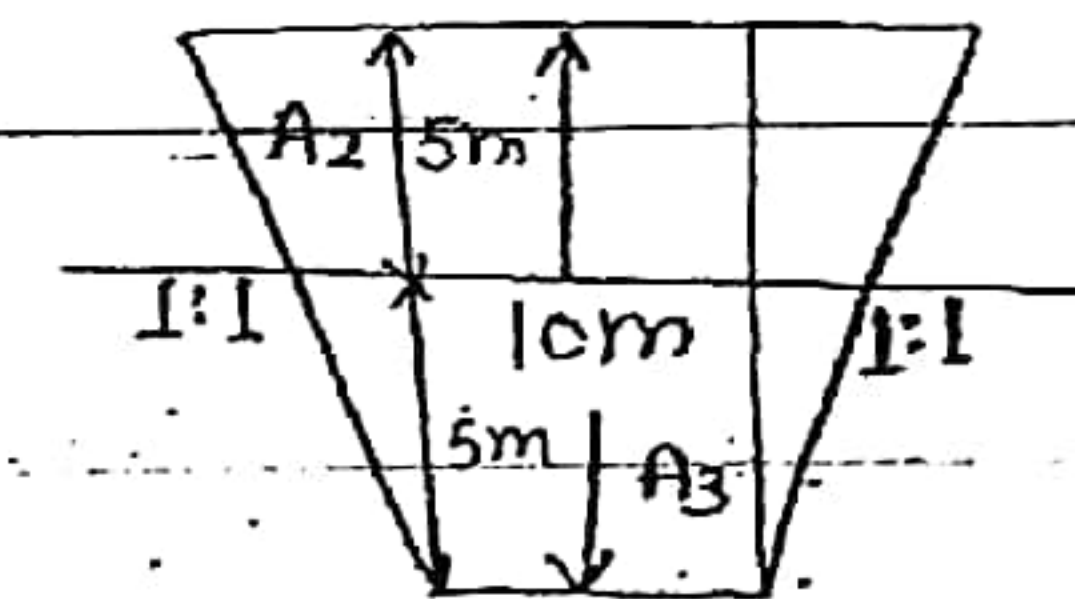
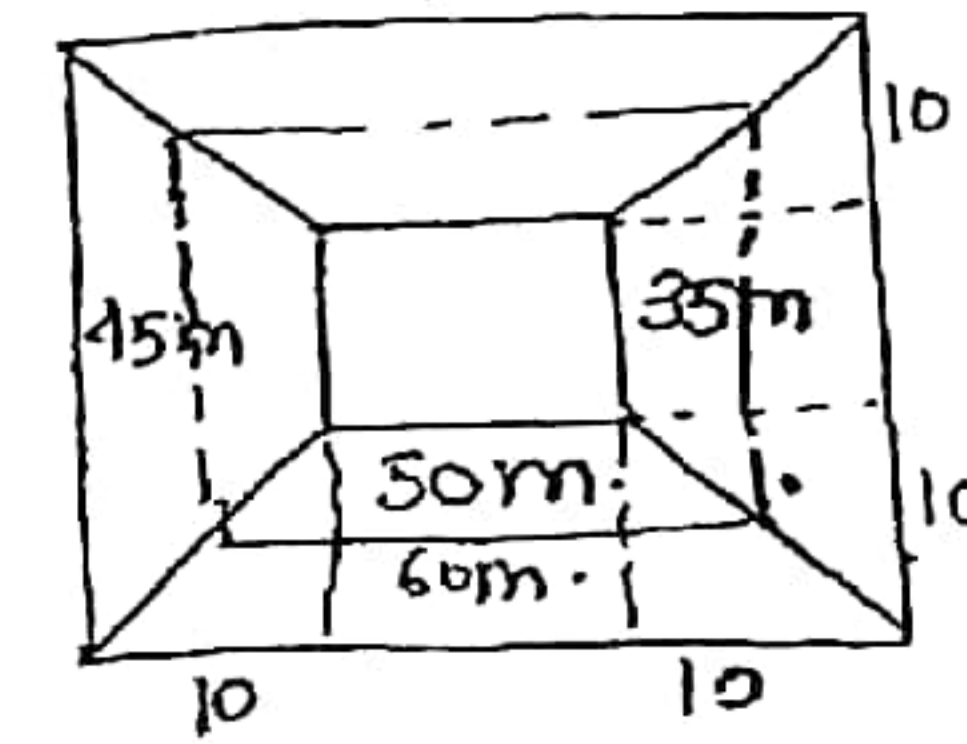
$$= 2733.33 \text{ m}^3$$

(III) Trapezoidal Rule

$$V = d \times \left\{ \frac{A_1 + A_3}{2} + A_2 \right\}$$

$$= 5 \times \left\{ \frac{3850 + 1750}{2} + 2700 \right\}$$

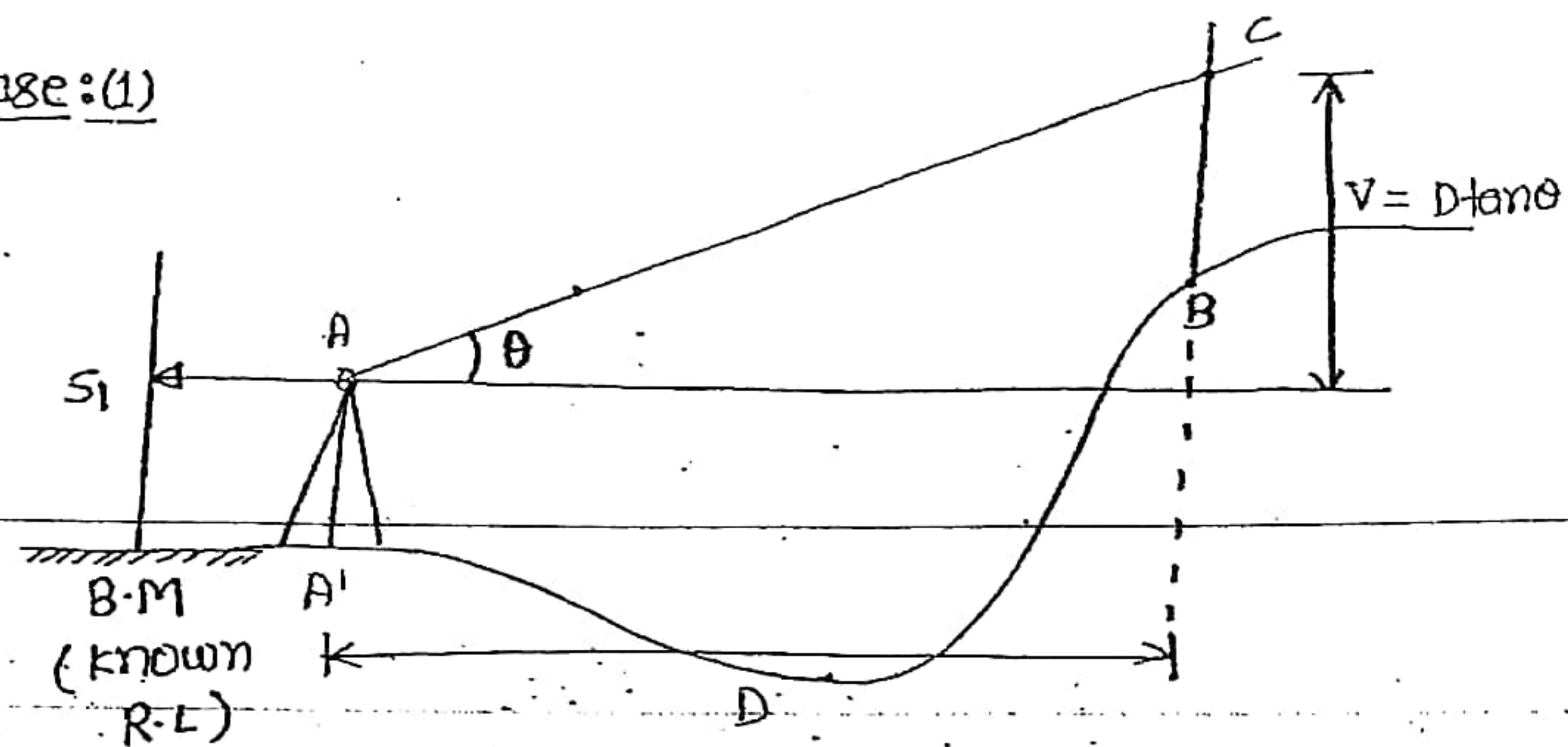
$$= 27500 \text{ m}^3$$





# TRIGONOMETRICAL LEVELLING

Case: (1)



Vertical height  $\Rightarrow$

$$V = D \tan \theta$$

Known Values :-

- (i) R.L. of B.M.
- (ii) Staff Reading B.M. =  $s_1$
- (iii) Angle =  $\theta$
- (iv) Distance =  $d$

Find out R.L. of C -

R.L. of C :-

$$\text{R.L. of C} = \text{R.L. of B.M.} + s_1 + V + C$$

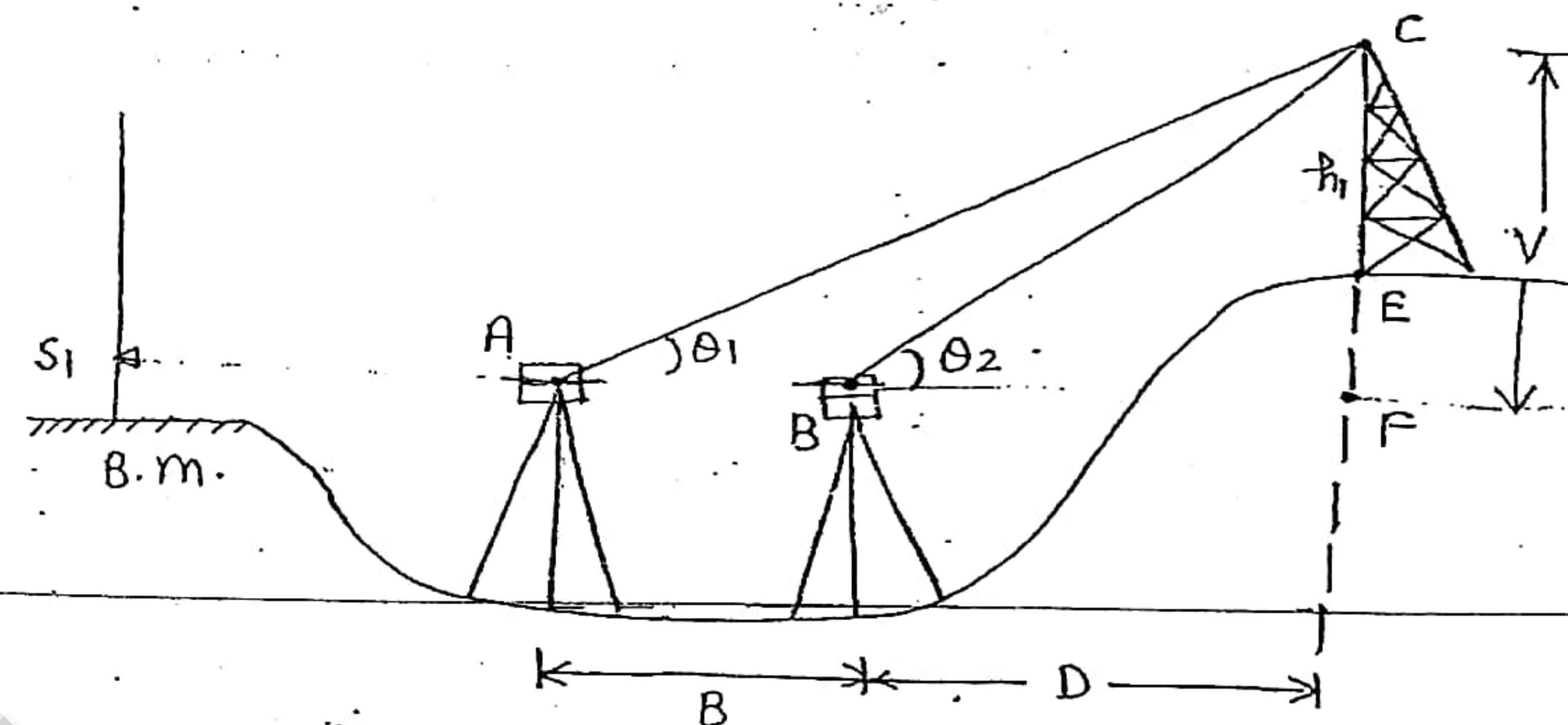
$$C = 0.0673 d^2$$

correction due to curvature and refraction.

Case: (2) If distance 'D' cannot be measured:

Known values

- (i) R.L. of B.M.
- (ii) Staff Reading at B.M. =  $s_1$
- (iii) Angle  $\theta_1$  &  $\theta_2$
- (iv) Distance B.



Find out :-

- (1) Distance 'D'
- (2) V
- (3) R.L. of C/E point

In  $\Delta ACF$

$$\tan \theta_1 = \frac{V}{B+D}$$

$$V = (B+D) \tan \theta_1 \quad \text{--- (1)}$$

In  $\Delta BCF$

$$\tan \theta_2 = \frac{V}{D}$$

$$V = D \tan \theta_2 \quad \text{--- (2)}$$

Put (2) in (1)

$$D \tan \theta_2 = (B+D) \tan \theta_1$$

$$D (\tan \theta_2 - \tan \theta_1) = B \tan \theta_1$$

$$D = \frac{B \tan \theta_1}{(\tan \theta_2 - \tan \theta_1)} \quad \text{--- (A)}$$

$$V = D \tan \theta_2$$

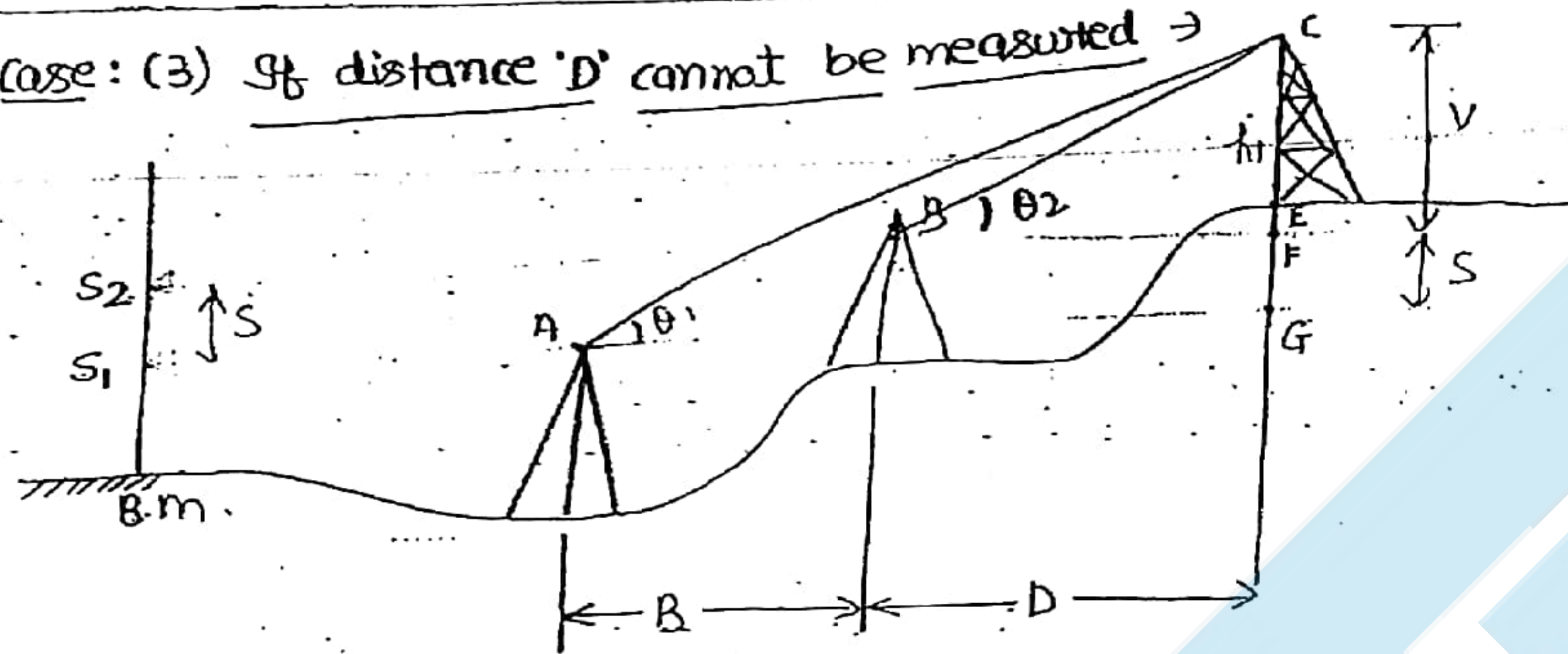


$$V = \frac{B \tan \theta_1 \cdot \tan \theta_2}{(\tan \theta_2 - \tan \theta_1)} \quad \text{--- (B)}$$

$$\begin{aligned} \text{R.L of point C} \\ = \text{R.L of Bm} + S_1 + V + C \end{aligned}$$

$$\begin{aligned} \text{R.L of E} \\ = \text{R.L of C} - h_1 \end{aligned}$$

Case: (3) If distance 'D' cannot be measured  $\Rightarrow$



Known value

Find out

- |                                       |                      |
|---------------------------------------|----------------------|
| (1) R.L of B.M.                       | (1) Distance 'D'     |
| (2) Staff Reading at B.M. = $S_1/S_2$ | (2) 'V'              |
| (3) Angle $\theta_1$ & $\theta_2$     | (3) R.L of C/E point |
| (4) Distance 'B'                      |                      |

In  $\Delta ACG$

$$\tan \theta_1 = \frac{V+S}{B+D}$$

$$(V+S) = (B+D) \tan \theta_1 \quad \text{--- (1)}$$

In  $\Delta BCF$

$$\tan \theta_2 = \frac{V}{D} \Rightarrow V = D \tan \theta_2 \quad \text{--- (2)}$$

At (1) in (1)

$$D \tan \theta_2 + S = (B+D) \tan \theta_1$$

$$D (\tan \theta_2 - \tan \theta_1) = B \tan \theta_1 - S$$

$$D = \frac{B \tan \theta_1 - S}{(\tan \theta_2 - \tan \theta_1)}$$

$$V = \frac{D}{B} \tan \theta_2$$

$$= \frac{(B \tan \theta_1 - S) \cdot \tan \theta_2}{(\tan \theta_2 - \tan \theta_1)} \quad \text{--- (B)}$$

R.L of point C

$$= \text{R.L of B.M} + S_1 + S + V + C$$

R.L of E

$$= \text{R.L of C} - h_1$$



ES-2005 310)

Ques: ① A flag post of ht. 2m ~~away~~ was erected on top of a building. Find the R.L. of top of flag post, if the vertical angle to bottom and top of it were measured as  $7^\circ$  and  $10^\circ$  resp. from a point. Staff reading on a B.M. from the same pt. at 0°0'0" was 1.245 m (R.L. of B.M. = 100.00m)

$$\tan 7^\circ = \frac{V}{D}$$

$$V = D \tan 7^\circ \quad \text{--- (1)}$$

$$\tan 10^\circ = \frac{V+2}{D}$$

$$V+2 = D \tan 10^\circ \quad \text{--- (2)}$$

$$D \tan 7^\circ + 2 = D \tan 10^\circ$$

$$D = 37.35 \text{ m}$$

$$V = D \tan 7^\circ$$

$$V = 37.35 \tan 7^\circ$$

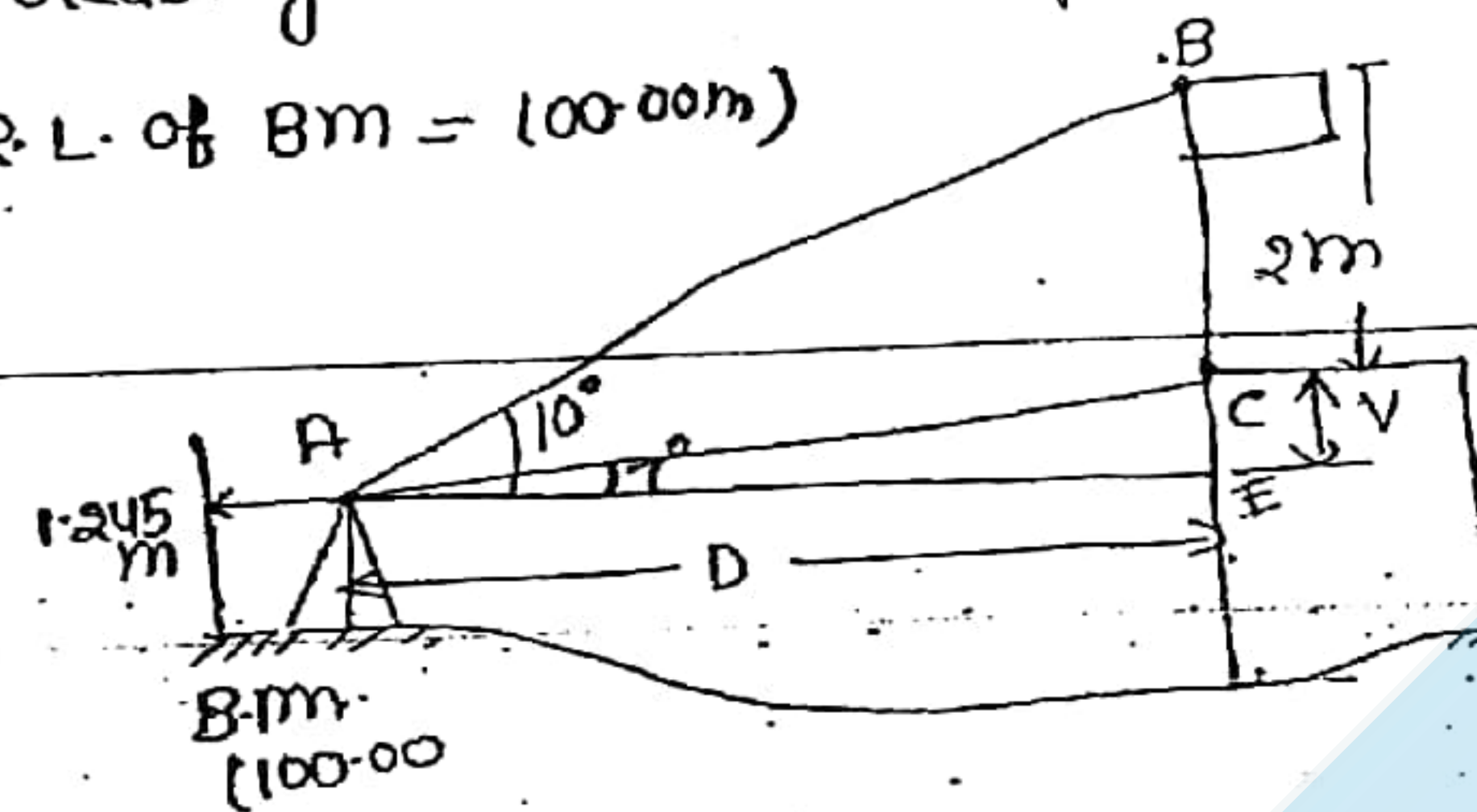
$$V = 4.59 \text{ m}$$

R.L. of top of the flag =

$$= \text{R.L. of B.M.} + S_1 + V + 2$$

$$= 100. + 1.245 + 4.59 + 2.0$$

$$= 107.83 \text{ m.}$$



Ques: (ES-2001) In order to determine the elevation of top A of (b)) a signal on a hill observations were made from 2 points P & R. All three points are in same vertical plane.

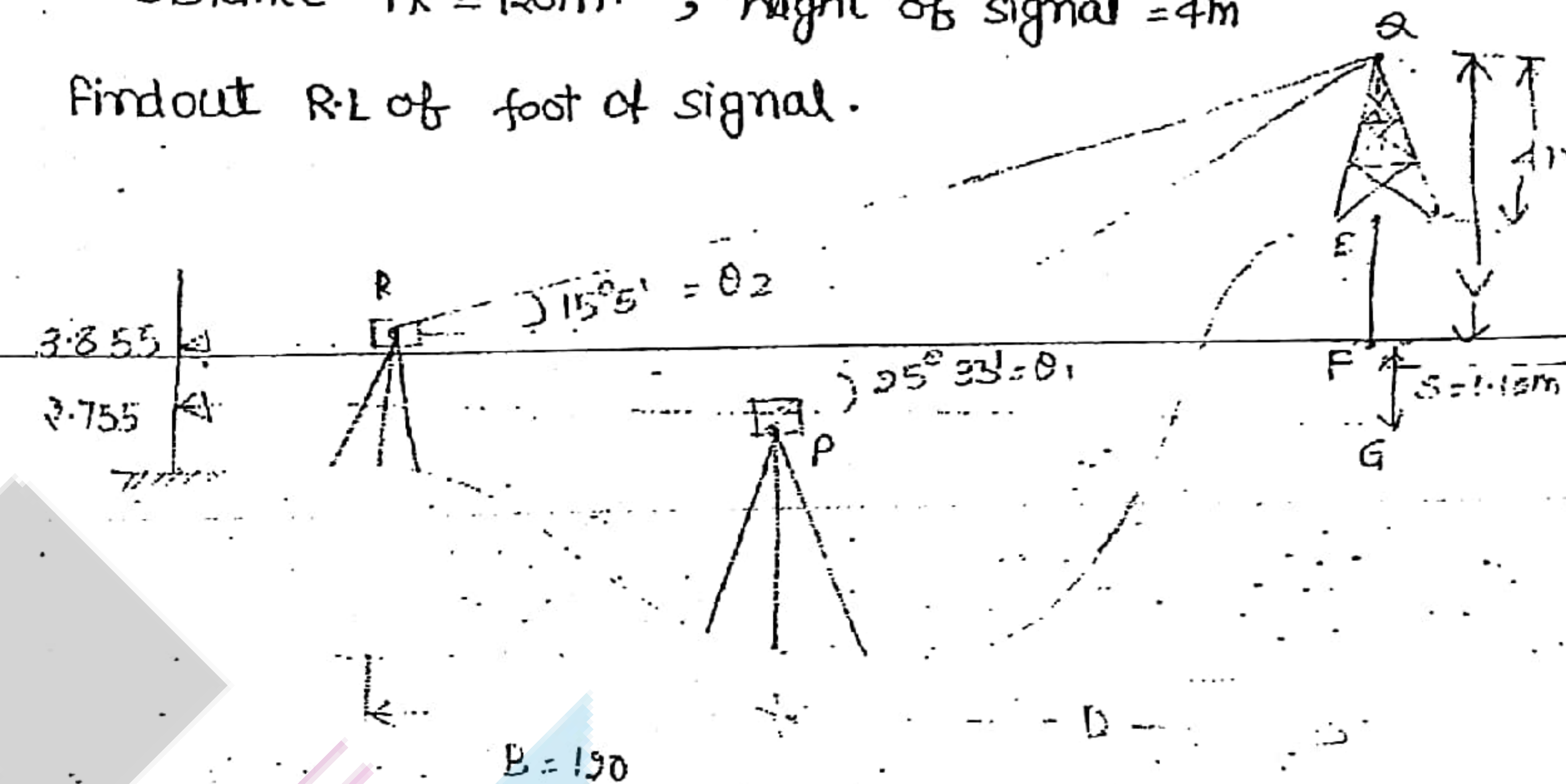
Angle of elevation to Q | from P =  $25^\circ 35'$   
R =  $15^\circ 5'$

Staff reading at B.M. | from P = 2.755m  
from R = 3.855m.

R.L. = 105.42m.

Distance PR = 120m. , height of signal = 4m

Find out R.L. of foot of signal.



$$\tan \theta_1 = \frac{V+S}{B+D}$$

$$V+S = D \tan \theta_1 \quad \text{--- (1)}$$

$$\tan \theta_2 = \frac{V}{B+D}$$

$$V = (B+D) \tan \theta_2 \quad \text{--- (2)}$$

Put in 1.

$$(B+D) \tan \theta_2 + S = D \tan \theta_1$$

$$D = \frac{B \tan \theta_2 + S}{\tan \theta_1 - \tan \theta_2}$$

$$= \frac{120 \tan 15^\circ 5' + 1.10}{\tan 25^\circ 35' - \tan 15^\circ 5'}$$

$$D = 159.81 \text{ m}$$



∴ in eq<sup>n</sup> ①

$$V = (B+D) \tan \theta_2$$

$$= (120 + 159.8) \tan 15^\circ 51'$$

$$V = 75.41 \text{ m}$$

R.L. of foot of signal —

$$= \text{R.L. of B.M.} + S_2 + V - 4.0$$

$$= 105.42 \text{ m} + 3855 + 75.41 - 4.0$$

$$= 180.68 \text{ m}$$

ES-1992

Ques  
(5(b))

Determine the R.L. of a church spire at C from the following observations taken from two stations A & B 50 m apart.  $\angle BAC = 60^\circ$ ,  $\angle ABC = 50^\circ$ . Angle of elevation to top of spire from A =  $30^\circ$ , from B =  $29^\circ$ .

Staff standing on a B.M. of R.L. = 25.0 m from A = 2.5 m from B = 0.50 m.

In plan :-

In  $\triangle ABC$

$$\frac{D_1}{\sin 50^\circ} = \frac{D_2}{\sin 60^\circ} = \frac{50}{\sin 70^\circ}$$

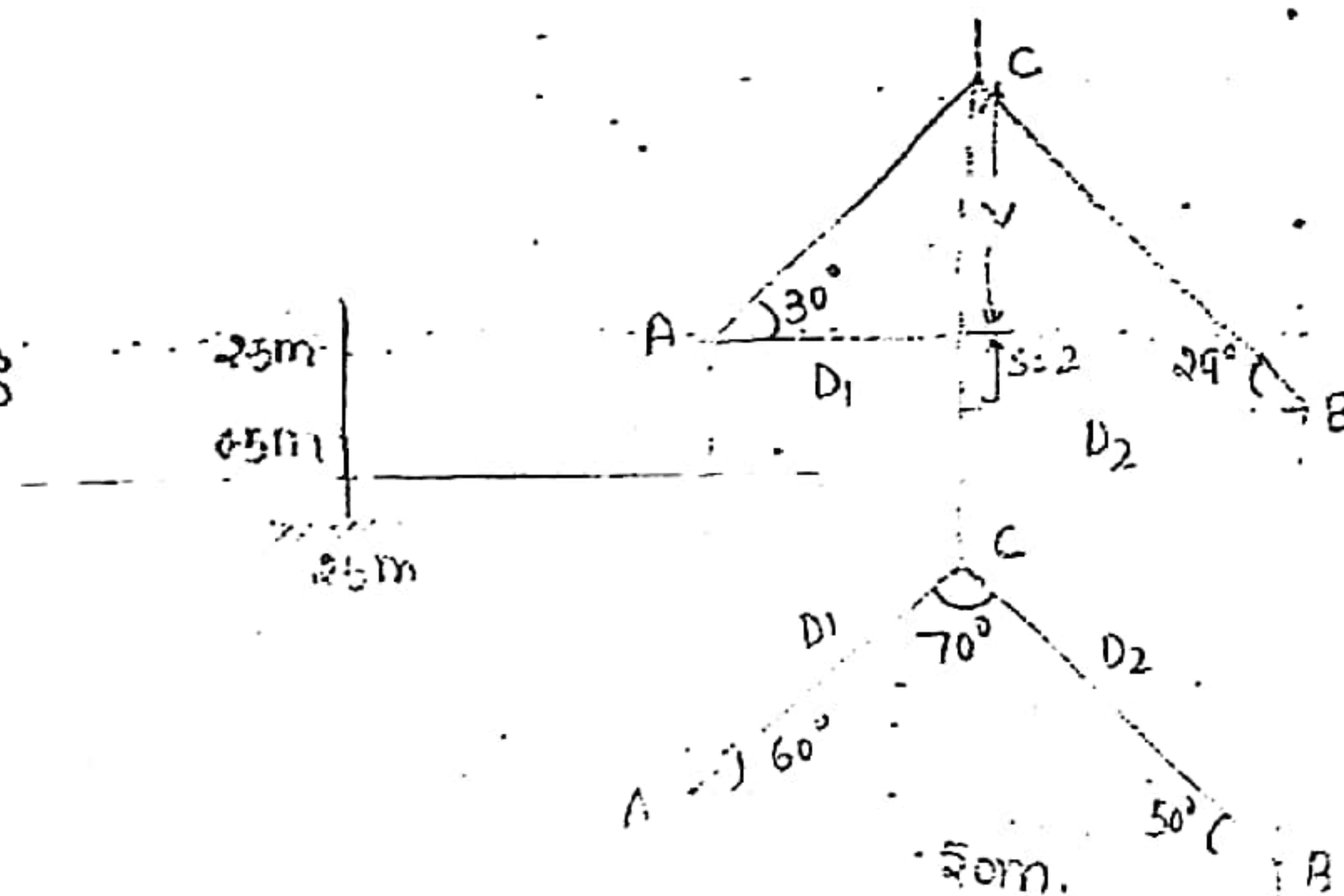
$$D_1 = 40.76 \text{ m}$$

$$D_2 = 46.08 \text{ m}$$

$$V = D_1 \tan 30^\circ$$

$$= 40.76 \tan 30^\circ$$

$$V = 23.53 \text{ m}$$



$$\text{R.L. of C} = \text{R.L. of B.M.} + 2.5 + V$$

$$= 25 + 2.5 + 23.53$$

$$\text{R.L. of C} = 51.03 \text{ m}$$

$$V + S = D_2 \tan 29^\circ$$

$$V = 46.08 \tan 29^\circ - 2.0$$

$$V = 23.54 \text{ m}$$

$$\text{R.L. of C} = 25.0 + 2.50 + 23.54$$

$$\text{R.L. of C} = 51.04 \text{ m}$$

Ques ES-2004-8- The top of a stack was sighted from 2 stations A & B 125 m apart & are in same vertical plane with top of the stack. Angle of the elevation — from A =  $35^\circ 20'$  of top of stack from B =  $22^\circ 28'$

The angle of elevation from B to a vane 1.75 m above the foot of staff held at station A was  $16^\circ 15'$  of instrument height at A = 1.856 m & at B = 1.565 m.

R.L. of station B = 100.00 m. Find out R.L. of top of the stack.

